DENT'S JUNIOR MATHEMATICS

BOOK THREE FOR GRADE NINE

1940 EDITION

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BOOK THREE FOR GRADE NINE

REVISED AND EDITED BY
WILLIAM GRAY, M.A.
AND
ELMER W. REID, B.A.

Authorized for Use in the Schools of British Columbia

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INTRODUCTION

To the Boys and Girls of Grade IX:

At the beginning of Grade IX you enter upon a new stage in your study of mathematics. This book is planned to assist you in learning the mathematics which you will need in school, and which you will need when you have become men and women.

The Uses of Mathematics

In almost everything you do, mathematics in some form or other is used. When you speak of your age, your height, your weight, or your rate of walking, you use numbers and measures. The foods you eat are measured as to quantities. Your hat, your clothes, and your shoes all are numbered as to size, and valued in terms of dollars and cents.

Without mathematics land could not be measured or sold, houses could not be constructed, distances could not be calculated, bridges could not be built, railway trains could not operate on the land or ships ply upon the sea. When you post a letter, send a telegram, use the long-distance telephone, order your food at a store, or buy a ticket at a theatre, you use mathematics in one form or another.

Informational Arithmetic

Many topics have been included in this book, not merely to help you to calculate with speed and accuracy, but to enable you to understand some of the interesting things about the business of the country and of the world at large.

Special Topics

The explanatory and problem material which appears in those chapters of this book which deal with the social aspects of arithmetic are based upon information obtained through questionnaires addressed to representative individuals and business firms in Western Canada. These chapters, as a result, offer a wide range of authentic information and the business practices outlined and the figures used are those of the actual world of commerce and industry about us. Yet some of you will want to read more extensively and to make special investigations about topics which interest you. You will, therefore, find in this book topics for class discussion and topics for special reports concerning things about which you will want to secure further information. In preparing these special reports, you will need to consult year books, geography texts, and other reference works.

One of the reasons why some pupils fail to do problems correctly is because they do not understand the language of mathematics. If you find in any problem a word that you do not understand, look it up in the dictionary, or ask your teacher to explain it, so that your progress may not be delayed.

In this book you will receive additional practice in the elements of geometry and algebra. This is introduced to make the work easier for you. You will feel great satisfaction in learning to use these tools easily and effectively.

NOTE TO TEACHERS: Some of the problems throughout the book have been starred. These are of special difficulty and need be attempted only by the brighter pupils.

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CHAPTER I

BEGINNING THE YEAR'S WORK

The Value of Mathematics

Number is woven into the pattern of our daily lives. Our activities are regulated by the clock and the calendar. Most things that pass from one person to another are measured and priced. Number relations are the guiding posts in our increasingly complex life. The extension of commerce and trade has greatly affected our civilization. The business of dollars and cents controls our share of food, clothing, shelter, amusements and often even our health. It is not the fault of mathematics that we do not stress sufficiently the noblest worth.

Pure mathematics deals with the truths that underlie natural laws and is fixed and unalterable. Its verdict is final, clothed in the beauty of truth, and can never be changed to suit the desires of man. And yet mathematics is our willing servant. By the principle of geometry the architect rears his lofty dome and the engineer constructs bridges that stand the stress of speeding trains, roaring on their way. The courses of heavenly bodies are determined by the use of algebraic symbols. By mathematics the air pilot plots his way through the heavens on the darkest night and the captain steers his course in unknown seas.

The study of mathematics is intensely practical and leads us on a pleasant journey in the discovery of truth, the exercise of reason, and the practice of self-discipline.

The Cost of an Education

In order that children may secure an adequate education, the citizens of Canada spend great sums of money each year to maintain their schools.

1. In a small western city the expenditure for schools in one year was $107,396. The enrolment was: high school, 237; junior high school, 328; elementary schools, 696. Find the average cost of schooling per pupil.

2. If the cost of operating the high school was $27,558.50, find the average cost per pupil.

3. If the annual cost of operating the junior high school was $31,488, what was the average cost per pupil?

4. From the facts given above, calculate the annual cost per pupil of operating the elementary schools.

Education is expensive. We should make
the best possible use of our time at school.

Review Tests

A review test is designed to discover how well you can do certain work in which you have had considerable practice. The result may show that you do not need further teaching or practice in this field or it may indicate a weakness that calls for remedial work.

The following test in addition will show how well you can add. After you have tried the test and checked results, decide what practice you need to be reliable in adding whole numbers.

If you were to add the following numbers two or three times you could, of course, find the correct totals, for you have been adding whole numbers for several years. The value of the test is in finding how many you can correctly total without checking your addition for mistakes. The teller in a bank always counts even the smallest sum of money twice, and you should have developed the habit of checking all your work in mathematics carefully. But in this particular instance, we are asking you to break a good habit in order to find how well you can add the first time.

Care should be taken in copying exercises 5, 6, and 7 accurately. In 8, 9, and 10 add the numbers as a whole: 25 and 37 make 62; 62 and 19 make ——'. In 11 and 12 shorten your labour by grouping in twos as in No. 11: —13—29—— etc.

Review Test in Addition

Add without copying:

(1)	(2)	(3)	(4)
78965	52936	56987	93768
99386	67986	68399	58937
27698	27698	81406	66429
58479	78965	39777	91760
66386	99386	93289	57688
73879	66386	76849	28793
67986	98767	32956	99399
98767	84803	76789	62876
52936	58479	30848	87685
84803	66386	68366	48598

Copy and add:

(5) 8,827 + 76,785 + 9,378 + 2,697 + 5,934 + 7,674 + 8,795 + 5,967 + 3,849 + 7,679 =

(6) 484 + 93,069 + 5,876 + 7,789 + 9,867 + 45,616 + 283 + 7,979 + 89 + 5,689 + 7,763 + 8,756 + 6,892 =

(7) 29,386 + 771,348 + 93,084 + 77,177 + 689,986 + 987 + 563,929 + 8,826,396 =

Add the numbers as a whole: Add by grouping in twos:

(8)	(9)	(10)	(11)	(12)
47	66	28	9837	97543
56	38	34	7676	87989
34	92	47	9837	88867
28	63	66	7676	99547
19	78	38	2889	98989
37	29	54	7676	99863
25	37	65	9837	77543
		29	2889	77989
			7676	89867
			9837	78563
				88989
				99847

Review Test in Subtraction

In subtraction the number from which another number is taken is called the *minuend.* The number subtracted is called the *subtrahend.*

In business practice, when numbers are already listed, it is often more convenient to subtract without copying the subtrahend under the minuend. This method is so opposed to that already acquired at school in subtracting the number below from the number above that it is an excellent test of one's ability to subtract.

In the test below there are three types of subtraction: (*a*) in which the subtrahend is placed under the minuend, (*b*) in which both are in a column with the minuend below the subtrahend, and (*c*) in which the subtrahend and minuend are opposite.

Write the differences without copying the figures *and do not check your work.* If you have no mistakes you are fairly accurate in subtracting. If you have errors, check them and try to find what your difficulty is.

A.	(1)	213,004,206 124,085,316	(2)	400,317,284 309,208,529
	(3)	823,334,212 559,606,413	(4)	724,136,000 27,464,526

B. Gold production in British Columbia:

1930	$3,475,811
1931	$3,310,886
1932	$4,656,849
1933	$6,955,716
1934	$10,965,416
1935	$13,747,994
1936	$15,418,594
1937	$17,680,972

(5) By how much did the output in 1937 exceed that of 1936?

(6) How much less was the value of gold in 1933 than in 1935?

(7) What was the increase of 1934 over 1932?

(8) How much was the value in 1930 below that of 1937?

C.	*Minuend*	*Subtrahend*
(9)	$407,216.00	$256,514.76
(10)	$1,126,324.00	$976,228.00
(11)	$513,102.00	$14,219.56
(12)	$2,000,000.00	$128,726.00

Review Test in Multiplication

Do not check your work. Tabulate your errors. Were they due to wrong addition, mistakes in carrying, wrong multiplying, or illegible figures?

Copy and multiply:

(1) $\begin{array}{r}2,789\\ \underline{\quad 29}\end{array}$ (2) $\begin{array}{r}162,300\\ \underline{\quad 180}\end{array}$ (3) $\begin{array}{r}74,629\\ \underline{4,007}\end{array}$

(4) $\begin{array}{r}78,769\\ \underline{\quad 12}\end{array}$ (Do not multiply by 2 and 1 but directly by 12).

(5) 8,296×27×36.

(6) 678×8×6×5×4×3 (Short method).

(7) $\begin{array}{r}497\\ \underline{2,869}\end{array}$ (8) $(789)^3$

(9) 538,496×7. (Set down the answer without copying).

Which of the following are wrong? Can you give a reason?

(10) 678×26+678×47=

$$\begin{array}{r}678\\ \underline{73}\\ 2034\\ \underline{4746}\\ 49494\end{array}$$

(11) 8,827×76:

$$\begin{array}{r}882700\\ \underline{24}\\ 35308\\ \underline{17654}\\ 211848\end{array}$$

670852 or 882700—211848

(12) The divisor is 67, the quotient 9873, and the remainder 16. Find the dividend.

$$\begin{array}{r}
9873 \\
67 \\
\hline
69111 \\
59238 \\
\hline
661491 \\
16 \\
\hline
661475
\end{array}$$

Review Test in Division

Do not check your work but count your errors.

(1) $68\overline{)290{,}132}$ (2) $76\overline{)450{,}627}$

(3) $\dfrac{94675}{29\times7}$ (4) Divide 80,164 by 14 and then by 27.

(5) $289\overline{)76{,}204{,}127.}$ (Set down the remainder).

(6) What other factors has 44,863 besides 29?

(7) $602{,}136{,}413 \div 453$.

(8) 38 pupils made a total attendance of 7,030 days in a school year of 190 days. What was the average actual attendance per day?

(9) $97\overline{)613{,}209{,}487}$.

(10) The quotient is 2,649 and the dividend is 8,242,016. Find the remainder.

Review Test in Addition of Fractions

(1) $\begin{array}{r} \frac{1}{4} \\ \frac{1}{3} \\ \underline{\frac{1}{2}} \end{array}$ (2) $\begin{array}{r} \frac{5}{6} \\ \frac{2}{3} \\ \underline{\frac{1}{2}} \end{array}$ (3) $\begin{array}{r} \frac{4}{9} \\ \frac{3}{4} \\ \underline{\frac{1}{3}} \end{array}$ (4) $\begin{array}{r} \frac{7}{12} \\ \frac{5}{8} \\ \underline{\frac{1}{4}} \end{array}$

(5) $\frac{2}{3}+\frac{1}{2}+\frac{5}{6}$ (6) $\frac{5}{8}+\frac{1}{6}+\frac{3}{4}$

(7) $\begin{array}{r} 5\frac{1}{3} \\ 2\frac{1}{2} \\ \underline{\frac{7}{9}} \end{array}$ (8) $\begin{array}{r} 4\frac{1}{5} \\ 6\frac{3}{10} \\ \underline{6\frac{2}{3}} \end{array}$ (9) $\begin{array}{r} 8\frac{5}{9} \\ \underline{\frac{3}{5}} \end{array}$

(10) $\begin{array}{r} 7\frac{3}{8} \\ 9\frac{1}{3} \\ 2\frac{5}{12} \end{array}$ (11) $\begin{array}{r} 3\frac{15}{16} \\ \frac{1}{2} \\ \frac{7}{8} \\ \underline{\frac{3}{4}} \end{array}$ (12) $\begin{array}{r} \frac{7}{9} \\ 9\frac{7}{9} \\ 2\frac{7}{8} \\ \underline{\frac{3}{4}} \end{array}$

(13) 6 ft. 7 in. + $2\frac{1}{3}$ ft. + 4 ft.
11 in. + $3\frac{3}{8}$ ft.

(14) 9 hr. 26 min. + 7 hr. 35 min.
+ $5\frac{7}{15}$ hr.

Review Test in Subtraction of Fractions

(1) $\begin{array}{r} \frac{7}{9} \\ \underline{\frac{1}{2}} \end{array}$ (2) $\begin{array}{r} 5\frac{5}{8} \\ \underline{2\frac{1}{4}} \end{array}$ (3) $\begin{array}{r} 9\frac{2}{5} \\ \underline{3\frac{3}{4}} \end{array}$ (4) $\begin{array}{r} 8 \\ \underline{\frac{7}{8}} \end{array}$

(5) $\begin{array}{r} 7\frac{1}{4} \\ \underline{\frac{5}{6}} \end{array}$ (6) $\begin{array}{r} 8\frac{5}{6} \\ \underline{2} \end{array}$ (7) $9\frac{1}{3}-\frac{5}{8}+1\frac{1}{4}$

(8) $5-\frac{3}{4}-\frac{2}{3}$ (9) $12\frac{1}{2}-\frac{5}{8}-\frac{1}{4}$ (10) $10\frac{2}{5}-2\frac{1}{2}-3\frac{1}{3}$

Review Test in Multiplication of Fractions

(1) $\frac{5}{9} \times \frac{5}{9}$

(2) $\frac{2}{3} \times \frac{5}{8} \times \frac{7}{10}$

(3) $6\frac{3}{4} \times 2\frac{1}{2}$

(4) $1\frac{1}{4} \times 2\frac{1}{3} \times 3\frac{1}{3}$

(5) $\frac{5}{9} \times 6 \times 8\frac{5}{6}$

(6) $2\frac{2}{3} \times 80 \times \frac{5}{12}$

(7) $17 \times 96\frac{4}{9}$

(8) $\frac{7}{8}$ of $112\frac{3}{5}$

(9) $2\frac{7}{8} \times 33\frac{1}{3} \times 67$

(10) $70\frac{1}{5} \times 82\frac{3}{4} \times 6\frac{7}{8} \times 0$

Review Test in Division of Fractions

(1) $\frac{5}{6} \div \frac{1}{3}$

(2) $2\frac{7}{8} \div \frac{3}{4}$

(3) $\dfrac{8}{\frac{1}{2}}$

(4) $\dfrac{12\frac{1}{2}}{2\frac{1}{3}}$

(5) $\dfrac{7\frac{1}{4} \times 9\frac{2}{5}}{5\frac{2}{9}}$

(6) $6)\underline{5000\frac{2}{3}}$

(7) $\frac{5}{8})\underline{7200}$

(8) $27.9 \div \frac{3}{4}$

(9) $6\frac{2}{3} \times 7\frac{2}{5} \div 3\frac{5}{6}$

(10) $\frac{1}{3} \times 43560 \times \dfrac{62\frac{1}{2}}{2000}$

NOTE: In dividing a mixed number by a whole number, as in No. 6, the remainder will be either a fraction or a mixed number. Place it over the divisor and simplify.

Review Test in Multiplication of Decimals

(1) $8 \times .0026$

(2) 6.7×5.9

(3) $.78\frac{1}{2} \times 9.4$

(4) $.025 \times .00876$

(5) 1.25×3.375

(6) 8.7 times 65 hundredths

(7) $.02 \times .006 \times 19.8$

(8) $\$17.49 \times .067$

(9) $\frac{3}{5} \times 26.78$

(10) 8.4×37.015

Note: The product must contain as many decimal places as are found in both the multiplier and multiplicand.

Review Test in Division of Decimals

(1) $\frac{6}{7}$. (Express as a five-place decimal).

(2) $26\frac{5}{9}$. (Express as a decimal to the nearest thousandth).

Find the answers to three places of decimals:

(3) $78\overline{)200}$

(4) $.56\overline{)427}$

(5) $9.6\overline{).7236}$

(6) $.093\overline{)3.8}$

(7) $8.4\overline{)77\frac{1}{2}}$

(8) $.69\overline{)10.727}$

(9) $\dfrac{6.7 \times 8.9}{.34}$

(10) $\dfrac{24\frac{1}{2} \times 6.8}{.092}$

Note: The divisor must be changed to a whole number and the dividend increased accordingly.

See Improvement Tests on pages 335-341.

Review of the Three Types of Percentage

TYPE ONE

Exercises (oral):

What common fraction is:

(1) $8\frac{1}{4}$ of 100?
(2) $6\frac{3}{5}$ of 100?
(3) $4\frac{1}{4}$ of 100?
(4) $9\frac{2}{3}$ of 100?
(5) $11\frac{1}{4}$ of 50?
(6) $7\frac{3}{7}$ of 50?
(7) $6\frac{2}{9}$ of 50?
(8) $4\frac{2}{3}$ of 200?

Example:

To find 26% of 375.

$$\begin{aligned} &= \frac{26}{100} \text{ of } 375. \\ &= .26 \times 375. \\ &= 97.5. \end{aligned}$$

Exercises (written):

(Estimate the approximate answers before working).

(1) 76% of 1,280
(2) 127% of 976
(3) 48% of 1,684
(4) 29% of 588
(5) 66% of 916
(6) $8\frac{1}{4}$% of 760
(7) $6\frac{3}{5}$% of 19
(8) 20.7% of 256
(9) $7\frac{3}{7}$% of 800
(10) 12.45% of $2,450

TYPE TWO

To find what per cent 28 is of 112.

$$\begin{aligned} 28 &= \frac{28}{112} \text{ of } 112. \\ &= .25 \text{ of } 112 \\ &= 25\% \text{ of } 112. \end{aligned}$$

Exercises (*written*):

First estimate your answers.

What per cent is :

(1) 64 of 320? (2) 19 of 25? (3) 128 of 72? (4) 2 of $1\frac{1}{4}$?
(5) $12\frac{1}{2}$ of $62\frac{1}{2}$? (6) 119 of 336? (7) $18\frac{1}{2}$ of 90? (8) 478 of 329?
(9) $\frac{5}{8}$ of 1? (10) 28.4 of 180?

Research work:

(1) Find what per cent the number of pupils in your class is of the number of pupils in the school.

(2) What per cent of the population of Canada is that of British Columbia?

(3) What per cent of the members of the House of Commons does each political party have?

TYPE THREE

Exercises (*oral*):

(1) $\frac{63}{.3} = ?$ (2) $\frac{84}{.21} = ?$ (3) $\frac{48}{.16} = ?$
(4) $\frac{65}{1.3} = ?$ (5) $4 = 50\%$ of ? (6) $10 = 20\%$ of?
(7) $18 = 33\frac{1}{3}\%$ of ? (8) $24 = 75\%$ of?

63 is 30% of what number?

$$\begin{aligned} 30\% \text{ of number} &= 63? \\ .3 \text{ of number} &= 63 \\ \text{number} &= \frac{63}{.3} \\ &= 210 \end{aligned}$$

Check: 30% of 210 = 63.

Exercises (*written*):

First estimate the approximate answers.

(1) 84 = 45% of what number?

(2) 78 = 150% of what number?

(3) $125 = 33\frac{1}{3}\%$ of what number?

(4) $69 = 25\%$ of what number?

(5) $647 = 135\%$ of what number?

(6) 99 is $66\frac{2}{3}\%$ of what number?

(7) Of what number is 750 80%?

(8) 76% of what number is 628.52?

(9) 93% of what number is 725.4?

(10) $5\frac{1}{2}\%$ of what number is 35.2?

(11) 20% of a certain number is 168. What per cent is 294 of the same number?

(12) 3% of a certain number is $16\frac{2}{3}\%$ of 918. What is the number?

The Value of Improvement Tests

In almost every game you play, you try to beat your opponent. It gives you a feeling of pride and power to surpass him. But the finest game one can engage in is to compete against one's own record. This is part of the lure of golf. One watches one's own score and tries to improve it.

To add, subtract, multiply, and divide with accuracy and speed, two conditions must be fulfilled. There must be constant practice and there must be a definite rate of progress. For these purposes the Improvement Tests on pages 335-344 have been devised. For all the tests there is the same time, ten minutes. In that time see how many you can have correct. Make a record sheet on which to mark your score on each attempt. The column for the date is very important, as it should indicate regularity. Then there should be a graph on which to record the progress you are making.

In working addition and subtraction tests do not copy the examples, but use a folded paper on which to record your answers. In working tests in multiplication or division or other tests that have to be copied, the copying should be completed before the time allowance for the test begins.

CHAPTER II

GENERALIZED ARITHMETIC

Powers and Roots

The formula $A=s^2$ enables you to find the area of a square when the length of its side is known. Thus a square whose side is 8 inches long is found by replacing s with 8.

$$A=8^2 \text{ or } 64$$

This formula may be changed to $s=\sqrt{A}$, enabling us to find the side of a square when its area is known. Thus the side of a square whose area is 225 sq. in. may be found by replacing A with 225. Thus:

$$s=\sqrt{225} \text{ or } 15.$$

The volume of a cube is given by the formula

$$V=e^3,$$

where e is the length of its edge. This formula may be changed into another,

$$e=\sqrt[3]{V},$$

enabling us to estimate the edge of a cube when its volume is known.

The small numbers ^{2} and ^{3} written at the top right-hand side of the letters in s^2 and e^3 are called *exponents* or indices. Exponents, in mathematics, indicate an operation. Thus the exponent or index ^{2} in 5^2 tells us to multiply two 5's together, and the exponent in 7^3 indicates that we are to multiply three 7's together.

So far we have been able to explain expressions such as s^2 and e^3 practically by means of a square and a cube. There are, however, expressions of a similar kind which have no geometrical interpretation, such as x^4, a^5, b^{10}. These quantities, as well as squares and cubes, are known as powers, x^4 being the fourth power of x, usually read as x to the fourth, a^5 the fifth power of a, or a to the fifth, and b^{10} the tenth power of b, or b to the tenth. s^2 is the second power of s, but is generally called the square. e^3 is the third power of e, and is called e cubed.

(NOTE: $1^3=1$; $1^x=1$; $\sqrt[3]{1}=1$; $\sqrt[2]{1}=1$. a means a^1. Exponent is 1).

The generalized expression for a power has two letters, as a^m. In this expression, as in the simpler cases which we have considered, three names are used. a is the *base*, m is the *exponent* (or index), and the whole quantity a^m is known as the *power*. In this particular case a^m is the mth power of a, and is read a to the mth.

It means $a\times a\times a\times$ — — — — —, the a being taken m times.

In the same manner, in dealing with roots, we use a generalized term to include all cases. For example, $\sqrt[n]{a}$ is the nth root of a. The sign $\surd$ is called the *radical sign* or sometimes the root sign. The small number ^{n} is called the index of the root.

In dealing with square roots, the index ^{2} is omitted.

Thus $\sqrt[2]{a}$ is written simply $\sqrt{a}$.

$\sqrt{a}$ is one of the two equal factors of a.

For example, $\sqrt{49}=7$ because the two equal factors of 49 are 7 and 7.

$\sqrt[3]{a}$ is one of the three equal factors of a.

Thus $\sqrt[3]{125}=5$ since $125=5\times5\times5$.

$\sqrt{49}$ may be written $\sqrt{7^2}$ which equals 7.

So also $\sqrt[3]{125}=\sqrt[3]{5^3}=5$
$\sqrt{10^2}=10$;
$\sqrt[3]{8}=\sqrt[3]{2^3}=2$

Thus the index of the root cancels or neutralizes the exponent when both are the same.

Generally then, $\sqrt[n]{x^n}=x$.

The use of powers higher than the third is shown by the following:

$10{,}000=10^4$; $1{,}000{,}000=10^6$;
$20{,}000=2\times10^4$; $300{,}000=3\times10^5$.

Exercises (Oral)

1. Indicate the base and the exponent of each of the following powers:

$$5^2;\ 3^4;\ b^3;\ c;\ .4^3;\ (\tfrac{3}{4})^5;\ (ab)^2;\ \left(\frac{2x}{3}\right)^3.$$

2. In the expressions $3x^2$, $2a^3$, $5a^{10}$, name the exponents.

3. Give the answers for the following:

$$15^2;\ 9^3;\ (4x)^2;\ 2^4;\ (3b)^3;\ (2y)^5.$$

4. Find the roots as indicated:

$$\sqrt{225};\ \sqrt[3]{216};\ \sqrt[5]{32};\ \sqrt{4x^2};\ \sqrt[3]{8y^3};\ \sqrt{121c^2}.$$

5. Which of the following statements are correct?

$$10^4=1{,}000;\ 10^6=1{,}000{,}000;\ \sqrt[3]{1{,}000}=10;$$
$$\sqrt[3]{10^3}=10.$$

Square Root in Arithmetic

Some of the following methods of finding the square root of arithmetical numbers have been previously taken up:

1. *From memory.* Thus $\sqrt{225}=15$; $\sqrt{324}=18$.

2. *By factoring.* Since the square root of a number is defined as one of the equal factors of the number, we break up the number into pairs of equal factors.

Thus
$$\begin{aligned}\sqrt{1089}&=\sqrt{3\times3\times11\times11}\\&=\sqrt{3^2\times11^2}\\&=3\times11\\&=33\end{aligned}$$

$$\begin{array}{r|r}3&1089\\\hline 3&363\\\hline 11&121\\\hline &11\end{array}$$

$$\begin{aligned}\sqrt{1764}&=\sqrt{2\times2\times3\times3\times7\times7}\\&=\sqrt{2^2\times3^2\times7^2}\\&=2\times3\times7\\&=42\end{aligned}$$

$$\begin{array}{r|r}2&1764\\\hline 2&882\\\hline 3&441\\\hline 3&147\\\hline 7&49\\\hline &7\end{array}$$

3. *From the Table of Roots.* The table on page 370 gives the square roots of all whole numbers from 1 to 100. Only ten of these whole numbers are complete squares, so that the square roots of the other numbers are given approximately. In this table the roots are given to three decimal places. Thus $\sqrt{65}=8.062$.

Read off the following: $\sqrt{22}$; $\sqrt{41}$; $\sqrt{78}$; $\sqrt{98}$.

Solve these equations: $x^2=32$; $x^2=75$; $x^2=87$.

Exercises:

The following eight numbers are perfect squares. Find their square roots by factoring:

(1) 3969 (2) 6084 (3) 7056 (4) 2601
(5) 5776 (6) 3249 (7) 4356 (8) 17424

From the Table of Roots find:

(9) $\sqrt{37}$ (10) $\sqrt{120}$ (11) $\sqrt{496}$ (12) $\sqrt{1000}$

4. A more exact method for determining approximately the square root of a number, whole or fractional, is now given. The following four examples indicate the method, but as the steps are rather difficult to follow from a written explanation, it is left to your teacher to explain to you.

(*a*) To find $\sqrt{7569}$

$$\begin{array}{r|l} & \;\;8\;7 \\ \cline{2-2} 8 & \overline{7569} \\ 8 & 64 \\ \hline 167 & 1169 \\ 7 & 1169 \\ \hline \end{array}$$

$\sqrt{7569}=87$

(*b*) To find $\sqrt{15625}$

$$\begin{array}{r|r} & 1\;2\;5 \\ \cline{2-2} 1 & \overline{15625} \\ 1 & 1\;\;\;\; \\ \hline 22 & 56\;\; \\ 2 & 44\;\; \\ \hline 24\;5 & 1225 \\ 5 & 1225 \\ \hline \end{array}$$

$\sqrt{15625}=125$

(*c*) To find $\sqrt{914}$

	3 0. 2 3
3	914.0000
3	9
602	1400
2	1204
6043	19600
3	18129

$\sqrt{914} = 30.23$ approx.

(*d*) To find $\sqrt{5274.342}$

	7 2. 6 2
7	5274.3420
7	49
142	374
2	284
1446	9034
6	8676
14522	35820
2	29044

$\sqrt{5274.342} = 72.62$ approx.

Exercises:

Find by the last method:

(1) $\sqrt{484}$ (2) $\sqrt{961}$ (3) $\sqrt{1156}$
(4) $\sqrt{1089}$ (5) $\sqrt{9409}$ (6) $\sqrt{12996}$

Find to two decimal places the square root of:

(7) 735 (8) 825 (9) 629
(10) 1028 (11) 2118 (12) 43729
(13) 54.62 (14) 163.4 (15) 234.58

In the case of the following fractions, change each to the decimal form before finding the root:

(16) $\sqrt{\frac{1}{2}}$ (17) $\sqrt{\frac{3}{4}}$ (18) $\sqrt{\frac{4}{5}}$
(19) $\sqrt{1\frac{1}{2}}$ (20) $\sqrt{\frac{7}{22}}$ (21) $\sqrt{\frac{63}{2}}$

(22) A square pane of glass has an area of 150 square inches. What is the length of each side?

(23) A square garden plot has an area of 1000 square feet. Find its length to the nearest foot.

Formulas

In your work in mathematics during the past year, you employed certain formulas for estimating the perimeters and areas of certain geometrical figures. Try the following exercise to see how much you have remembered. Review and memorize those which you have forgotten:

Exercises:

Match the following, as shown by the first example:

1. Area of parallelogram
2. Area of triangle
3. Volume of a rectangular solid
4. Volume of a cube
5. Total surface of a cube
6. Circumference of a circle
7. Area of a circle
8. Perimeter of a rectangle

( ) $A=\frac{1}{2}ab$.
( ) $V=lwh$.
( ) $S=6e^2$.
( ) $A=\pi r^2$.
( ) $P=2(l+w)$.
( ) $P=4s$.
( ) $V=e^3$.
(1) $A=ab$.
( ) $C=2\pi r$.

One extra formula is given. What does it represent?

Exercises:

1. Evaluate c^2+d^2 when $c=4$ and $d=8$.
2. Find a^3-b^3 when $a=9$ and $b=3$.
3. If $p=2(l+w)$, find p when $l=32.5$ and $w=16.2$.
4. Evaluate a^2+2b-3 when $a=5$ and $b=\frac{1}{2}$.
5. In the formula $S=6e^2$ find S when $e=0.25$.
6. In $A=\frac{1}{2}ab$, find b when $A=30$ and $a=23.1$.
7. Evaluate $2x^2-3xy+4y^2$ when $x=y=5$.

*8. $A=P(1+rt)$. Find A when $P=3{,}500$, $r=5\%$, $t=3$.

*9. $A=\dfrac{\pi d^2}{4}$. Find A when $d=6$ and $\pi=3.14$.

*10. $l=a+(n-1)d$. Find l when $a=4$, $n=8$, $d=3$.

New Formulas

The cube is a solid figure, occupying space or volume. There are other *solids* with which we shall now deal, namely, the cylinder, the cone, and the pyramid.

The Surface of a Cylinder

Roll a sheet of paper around the curved cylindrical surface of a cylindrical can. Cut the sheet so that it exactly covers the curved surface of the can. Unroll the sheet of paper, and note that it has the shape of a rectangle, its length being the circumference of the can, and its width being the height of the can. From this we see that:

The area of the curved surface of a right cylinder is equal to the circumference of the base times the height.

This may be written as $A = Ch$, or, since $C = 2\pi r$, as $A = 2\pi rh$.

Exercises:

(1) If you were required to find the entire outside surface of the can, how would you find the area of the two ends?

(2) Suggest a formula for the area of the entire surface of a cylinder.

(3) Find the area of the curved surface of a cylinder having a circumference of 24 ft. 6 in., and a height of 12 ft.

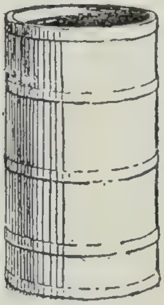
Find the area of the curved surfaces of cylinders having the following dimensions:

	Diameter	*Height*		*Diameter*	*Height*
(4)	14 ft.	8 ft.	(5)	3 ft. 6 in.	1 ft. 8 in.
(6)	24 ft.	9 ft.	(7)	$4\frac{2}{3}$ ft.	$2\frac{1}{5}$ ft.
(8)	$4\frac{1}{12}$ ft.	$3\frac{1}{4}$ ft.	(9)	6.3 in.	2.9 in.

* (10) Find the cost of painting the outside of a cylindrical smokestack, which has a diameter of 29 ft., and a height of 406 ft., at $.34 per sq. yd.

* (11) A cylindrical gasoline tank has a diameter of 63 ft., and a height of 32 ft. Find the cost of painting the curved surface and the flat top of the tank, at $.28 per sq. yd.

The Volume of a Cylinder



1. In Mr. Baker's barn there is a cylindrical water-tank which has an inside diameter of 8 ft. and is 14 ft. high. Mr. Baker wishes to find the capacity of the tank.

We find the volume of a cylinder just as we find the volume of a rectangular solid. The volume of a rectangular solid is equal to the area of its base times its altitude. *The volume of a right cylinder is equal to the area of its base times its altitude.*

The base of a cylinder is a circle. The formula for the volume of a cylinder is $\pi r^2 \times h$, or $\pi r^2 h$.

We first find the area of the circle.

Area of base $= \pi r^2$, or $\frac{22}{7} \times 4 \times 4$ sq. ft. $= \frac{352}{7}$ sq. ft.

Volume of tank $= \frac{352}{7} \times 14$ cu. ft. $= 704$ cu. ft.

Since 1 cu. ft. contains $6\frac{1}{4}$ gal., the capacity of the tank is $704 \times 6\frac{1}{4}$ gal. $= 4{,}400$ gal.

2. Using the formula $V = \pi r^2 h$, find the value of V in the following:

(*a*) $r = 7$ in., $h = 10$ in.
(*b*) $r = 1$ ft., $h = 14$ ft.
(*c*) Diameter $= 4$ cm., height $= 3$ cm.
(*d*) Diameter $= 1\frac{3}{4}$ ft., height $= 2$ ft.

3. The water in a cylindrical well is 12 ft. deep. The diameter of the well is $3\frac{1}{2}$ ft. How many cubic feet of water are contained in the well? How many gallons of water are in the well?

4. A silo has an inside diameter of 12 ft. and a height of 32 ft. How many cubic feet of silage will it contain when it is half full? If a ton of silage occupies a space of 50 cu. ft., how many tons of silage are required to fill the tank?

The Volume of a Cylinder

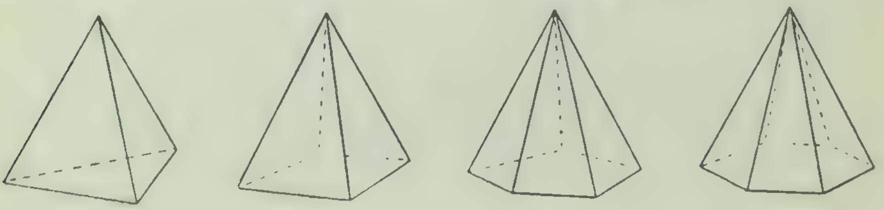
5. A large wheat elevator contains 16 cylindrical storage tanks, each 28 ft. in diameter and 60 ft. high (inside measurements). How many cubic feet of storage space does the elevator contain?

***6.** If one bushel of wheat requires $1\frac{7}{25}$ cu. ft. of space, how many bushels of wheat can be stored in the elevator described in Problem 5?

***7.** Two cylindrical tanks are each 12 ft. high. One has an inside diameter of 7 ft., while the other has an inside diameter of 14 ft. Compare the volumes of the two tanks.

Pyramids

A *regular* pyramid is one whose base is a regular polygon, and whose faces are equal isosceles triangles.



Pyramids are named according to the shape of their bases.

A regular pyramid with:

(*a*) a triangular base is called a *triangular pyramid;*

(*b*) a square base is called a *square pyramid;*

(*c*) a pentagonal base is called a *pentagonal pyramid;*

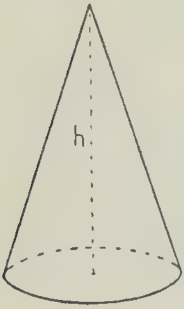
(*d*) a hexagonal base is called a *hexagonal pyramid.*

The *height,* or *altitude* of a pyramid is the *perpendicular distance* from its vertex to its base. This perpendicular passes through the centre of the base. The slant height of a regular pyramid is the perpendicular distance from the vertex to the base of one of the *equal isosceles triangles.*

Pyramids

The area of one of these triangles may be found by applying the formula $A = \frac{1}{2}ba$. The result is then multiplied by the number of triangles. This gives the lateral surface of the pyramid. In the case of a square pyramid the area of the base is found from the formula $A = s^2$. At this stage, it is not necessary for you to memorize a formula for the surface of a pyramid.

Cones



The cone with which you will have to deal is the *right circular cone.* In shape it resembles the familiar ice-cream container. It is a solid, whose base is a circle. The altitude, which is the perpendicular distance from its vertex to the base, meets the base at its centre. No formula need be memorized for estimating the surface of a cone .

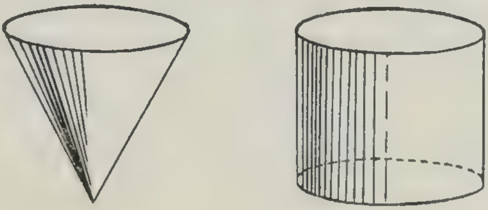
Volume of Pyramid and Cone

Volume of pyramid. Take a hollow rectangular (prism) vessel, also a hollow pyramid, both with equal bases and of the same altitude. Fill the pyramid with water and pour it into the rectangular vessel. You will find that the volume of the pyramid is just one-third of the volume of the rectangular vessel.

∴ The volume of a pyramid $= \frac{1}{3} \times$ (base $\times$ height). $= \frac{1}{3}bh$

Volume of cone. Take a hollow cylinder and a hollow cone vessel with equal bases and equal altitudes. Do as you did in the case of the pyramid. You should find the volume of the cone to be one-third the volume of the cylinder.

∴ The volume of a cone $= \frac{1}{3} \times$ (base $\times$ height). $= \frac{1}{3}\pi r^2 h$



Exercises:

1. Find the volumes of pyramids having the following dimensions:

   (*a*) Base 6 in. by 6 in.; height 7 in.
   (*b*) Base 2.1 in. by 2.1 in.; height 3.6 in.
   (*c*) Base 4 in. by 6 in.; height 4.3 in.

2. Find the volumes of cones having the following dimensions:

   (*a*) Diameter 7 in., height 6 in.
   (*b*) Radius 4 in., height 7 in.
   (*c*) Diameter 14 in., height 12 in.
   (*d*) Radius 4.2 in., height 3.5 in.

3. Find the volume of a conical monument whose base has a diameter of 28.7 in. and whose height is 9 ft.

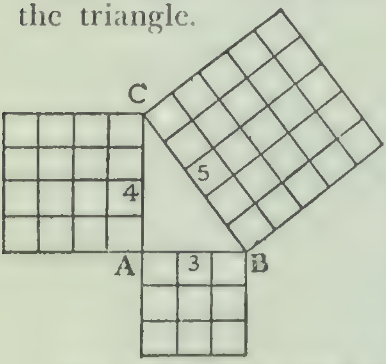
4. Find the number of cubic feet of air space in a conical tent which is 10 ft. high, with a base whose diameter is 12.6 ft.

*5. A pile of sand in the shape of a cone has a circumference of 52.8 ft., and is $4\frac{1}{2}$ ft. high. How many cubic feet of sand are in the cone?

*6. A cone and a cylinder have the same radius, but the cylinder is twice as high as the cone. Compare their volumes.

The Right-angled Triangle

A *right-angled* triangle is so called because one of its angles is 90°. Sometimes it is called a *right triangle*. The side opposite the right angle is called the *hypotenuse*, and is the longest side. The other sides are often called the *legs* of the triangle.



The sides of the triangle shown in the figure are 3, 4, and 5 units, respectively. On each side, squares have been drawn and each square divided into smaller equal squares. The square on BC has 25 small squares, that on AB has 9, and that on AC has 16. Since $25=9+16$ we write $BC^2=AB^2+AC^2$.

We shall see if this is true for other right triangles.

Exercise:

Draw a number of right triangles. Measure the three sides of each, square each result, and tabulate.

	1	2	3	4	5	6	7	8
Square on hypotenuse	25							
Square on side	9							
Square on side	16							
Sum of squares on sides								

Compare the square on the hypotenuse with the sum of the squares on the other two sides. You should find them approximately equal. We may say then:

In a right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.

If a, b, c represent the sides of a right triangle, we have the formula

$$a^2=b^2+c^2.$$

Tradition tells us that this truth was discovered by a Greek philosopher, Pythagoras, about 540 B.C. It is sometimes called the *Pythagorean Theorem.* Carpenters make use of this truth in laying out the 'square corner' of the foundation for a house, in cutting rafters, etc.

Exercises:

1. Two sides of a right-angled triangle are 6 ft. and 8 ft. Find the length of the hypotenuse.

$$a^2=b^2+c^2=6\times6+8\times8=36+64=100$$
$$a^2=100$$
$$a=\sqrt{100}=10$$

Length of hypotenuse=10 ft.

2. The two legs of a right-angled triangle are 5 in. and 12 in. Find the length of the hypotenuse.

3. The hypotenuse of a right-angled triangle is 25 ft., and one leg is 15 ft. Find the length of the other leg.

4. Find the length of the diagonal of a 12-in. square.

5. By using the formula, find which of the following sets of numbers could be used for constructing right triangles:

(*a*) 2, 3, 4
(*b*) 6, 8, 10
(*c*) 10, 24, 26
(*d*) 7, 24, 25
(*e*) 1, 2, 3
(*f*) $1\frac{1}{2}$, 2, $2\frac{1}{2}$.

6. Find the distance from first base across to third base in a baseball field, if it is 90 feet from first to second base.

*7. How much shorter is it to go diagonally across a lot which is 140 feet by 50 feet in size, than to go around the two sides?

*8. Find the height and the area of an equilateral triangle, the side of which is 2 inches.

Review Exercises on Chapter II

1. Write expressions for: (*a*) the cube of w; (*b*) the fourth root of t; (*c*) the fourth power of y; (*d*) m to the tenth; (*e*) the nth power of a; (*f*) the nth root of d.

2. Using the table of squares, find the value of (*a*) 84^2+38^2; (*b*) 95^2-21^2; (*c*) $5\,(32)^2$.

3. Find, by factoring, the value of: (*a*) $\sqrt{7744}$; (*b*) $\sqrt{4761}$.

4. The number 2,000 may be written as 2×10^3 since $2{,}000=2\times1{,}000$, and $1{,}000=10^3$. Write in similar form the numbers 600, 4,000, 300,000.

5. (*a*) Find to one decimal place the square root of 3296.

(*b*) Find to two decimal places $\sqrt{194.3}$.

(*c*) Find to three places of decimals $\sqrt{\frac{2}{3}}$.

6. Find, to two places of decimals, the length of the diagonal of a square whose side is 3 inches long.

7. Find the value of A if $A=2\pi rh$, when $r=2\frac{1}{2}$in. and $h=1$ ft. Use π as $3\frac{1}{7}$.

8. Find the total surface of a cylinder 2 ft. long and 6 in. in diameter.

9. Find the weight of a solid iron cylinder 1 in. in diameter and 1 yd. long. One cubic foot of iron weighs 450 lb.

10. Find the weight of a solid cone made of the same material as in Question 9, if the diameter of the base is 5 in. and the altitude 14 in.

*11. A square pyramid of marble has a base whose sides are 2 ft. 3 in., and a vertical height of 5 ft. If one cubic foot of marble weighs 185 lb., find the weight of the pyramid.

*12. A square pyramid has a vertical height of 12 in. and a slant height of 15 in. If the edge of its base is 6 in., find its volume, and its total surface.

Vocabulary Test

Explain each of the following, giving an example in each case:

Power
Root
Exponent
Base
Formula
Cylinder
Pyramid
Cone
Hypotenuse
Index of a root

Achievement Test

1. Find, by the method of factoring, the value of $\sqrt{7056}$.

2. By using the table of squares, find the value of 82^2-56^2.

3. Find the value of $\sqrt{3.269}$ to two decimal places.

4. Give expressions for:

(*a*) The fifth power of t. (*b*) The cube of y.
(*c*) The fourth root of k. (*d*) The xth root of b.

5. Find the total outside surface of a tank in the shape of a closed cylinder, 2 ft. 6 in. in diameter and 8 ft. long.

6. If one cubic foot contains about $6\frac{1}{4}$ gallons, how many gallons of water will the tank in Question 5 hold?

7. The base of a triangular pyramid is $8\frac{1}{2}$ sq. in. and the altitude is 1 ft. Find its volume.

8. The base of a cone has a diameter of 12 cm., and the height of the cone is 14.5 cm. Find its volume.

9. If a cubic foot of copper weighs 560 lb., find the weight of 30 ft. of copper wire $\frac{1}{4}$ in. in diameter.

10. A cylindrical silo has a base of 28 ft. How high must it be made to hold 900 cu. ft. of silage?

11. The two sides containing the right angle of a right triangle are 22 cm. and 120 cm., respectively. Find the length of the hypotenuse.

12. Find, to the nearest tenth of an inch, the diagonal of a square of one inch side.

CHAPTER III

MEASUREMENT

Steps in Solving Problems

If you have difficulty in solving problems, the suggestions given below may help you:

1. Read the problem thoughtfully, making sure that you understand the meaning of every word and phrase in it.

2. Try to picture in your mind the scene or situation which the problem describes, making a diagram when possible.

3. Find what question you are to answer.

4. Look for the facts "given" or stated in the problem.

5. Consider how you will use these facts in order to find the answer. Will you add, subtract, multiply, or divide?

6. Estimate about how large your answer will be.

7. Do the work carefully, by the shortest method, writing your figures neatly, lest you mis-read them later on.

8. Check all your calculations.

9. Compare your answer with your estimate. Is your answer a reasonable one?

10. To find whether your answer is right, use it to work back to what is given.

Example:

A farmer bought 480 acres of land at $12.50 per acre. He sold one-third of it at $10 per acre. For how much per acre must he sell the remainder in order not to lose on the whole transaction?

1. Read the problem. Do you understand the word transaction? If not, look it up. In this case it means *purchase and sale of land.*

2. Picture the problem in your mind. Draw a diagram.

3. Answer required:

   Selling price per acre of $\frac{2}{3}$ of 480 acres.

4. Facts given:

   He paid $12.50 per acre.
   He bought 480 acres.
   He sold $\frac{1}{3}$ of 480 acres at $10 per acre.

5. Use of facts:

   Multiply $12.50 by 480; find $\frac{1}{3}$ of 480; multiply the result by 10; subtract the product from the first product above; divide the remainder by $\frac{2}{3}$ of 480.

6. Estimate of answer:

   One-third of the land sells at $2.50 less than cost. Two-thirds of the land would have to sell for about $1.25 per acre more than cost, or at about $13.75 per acre.

7. Computation:

   $\frac{1}{3}$ of 480 acres=160 acres. 480—160=320.
   Cost of land=480×$12.50 =$6000
   Selling price of 160 acres = 1600
   For 320 acres he must receive $4400

   For 1 acre he must receive $\frac{1}{320}$ of $4,400, or $13.75.

8. Check:

   320×$13.75=$4,400. 160×$10=$1,600. Total $6,000. $6,000÷480=$12.50.

Some Problems to Solve

1. A farmer exchanged 68 bushels of seed wheat worth 77¢ a bushel for seed oats worth 28¢ a bushel. How many bushels of oats did he obtain?

2. At an average speed of 97.6 miles an hour, how long will it take an aeroplane to fly 1,500 miles?

3. If a nautical mile is 1.15 miles, express in miles per hour the rate of a steamer that travels 65 nautical miles in $2\frac{1}{2}$ hours.

4. In a city it was estimated that 18,762 automobiles passed over a certain bridge between 6:30 a.m. and 5 p.m. What was the average number of cars crossing per hour?

5. A schoolroom is 24 ft. wide, 30 ft. long, 12 ft. high. If each pupil requires 240 cu. ft. of air space, how many pupils will this room accommodate?

6. Find the average yearly cost per pupil in a class of 40 pupils, if the teacher's salary is $1,400 annually, supplies $45 annually, janitor's service $15 a month, and extras $8.50 a month. (Allow 10 months for the school year.)

7. If a lake is frozen to a depth of 12 in., and if 1 cu. ft. of ice weighs 56 lb., how many tons of ice can be obtained from an acre of the lake's surface?

8. If a cubic foot of water weighs $62\frac{1}{2}$ lb., and ice is .9 times as heavy as water, find the weight of a cake of ice 2 ft. 3 in. long, 23 in. wide, and 17 in. high?

9. Find the weight of a sheet of lead 7 ft. long, 4 ft. wide, and .42 in. thick. (Lead is 11.5 times as heavy as water.)

10. How many gallons of water will be required to cover a skating rink 100 ft. long and 60 ft. wide to a depth of 3 in.? (1 cu. ft. of water contains $6\frac{1}{4}$ gallons.)

The Building of Problems

Oral Work for Class Discussion

1. A farmer's crop of wheat was grown on 60 acres of summer-fallowed land which produced 18 bushels per acre, and 90 acres of stubble which produced 15 bushels per acre. What is summer-fallow? What is stubble-wheat? Why does summer-fallow give a better crop than stubble? From the above facts make three problems.

2. An automobile travels 20 miles on a gallon of gasoline, which costs 25¢ a gallon. The automobile was driven 190 miles. Make two problems based on the trip. Supply an extra fact and make an additional problem.

3. A train travels 45 miles per hour. Make three problems whose answers would be: (*a*) 144 miles; (*b*) 6⅓ hours; (*c*) 72 ft.

4. A room is 28 ft. long, 22 ft. wide, and 10 ft. high. From these facts make two problems dealing with area, one dealing with perimeter, and one with volume.

5. A famous cow, May Echo, averaged 5 gallons of milk daily for a year. Add one more fact and make four problems based on all the facts you have in this problem.

6. A man's taxes are $120 on a house assessed for $4,800. Make three problems from these facts.

7. Using 4,672 lb. of wheat and 68¢, construct four problems.

8. A skating rink is 80 ft. long and 40 ft. wide. The ice is 1 ft. deep. With this material make four problems.

9. A newsboy bought papers at the rate of 50 papers for $1.25, and sold them at the rate of $.03 each. Make two problems about these facts.

Review of Units of Measurements

Exercises (oral):

1. How many inches are in $5\frac{1}{2}$ ft.?

2. How many square inches are in three-quarters of a square foot?

3. Express 1 pint as a fraction of a gallon.

4. What fraction of 1 yd. is $2\frac{1}{2}$ ft.?

5. Express 6 lb. 7 oz. in ounces.

6. What decimal of an hour is 18 seconds?

7. If a section of land is a mile long and a mile wide, how long and how wide is a quarter-section?

8. What fraction of 10 minutes is 8 seconds?

9. What fraction of a ton is 12 cwt.?

10. How many feet are in $2\frac{3}{4}$ miles?

Exercises (written):

How many of the following can you work without referring to the tables at the end of the book?

1. What per cent of a gallon is 3 pints?

2. Express m tons$+n$ cwt. in pounds.

3. The area of a floor is 162 sq. ft. and the length is twice the width. Find the length in feet. (Solve with x).

4. The perimeter of a room is 114 ft. and its length is twice its width. Find the area of the floor in square feet.

5. A room is 18 ft. long, 16 ft. wide, and 9 ft. high. How many square yards are in the area of the four walls?

6. How many bushels of oats at 35¢ a bushels should a farmer give in exchange for 3 tons of seed wheat at 78¢ a bushel?

7. When a sprinter runs 100 yds. in 10 seconds, how many miles an hour is he going?

8. Sixty square rods are what per cent of $2\frac{1}{2}$ acres?

9. When cream sells at 9¢ a quarter-pint, what is the price by the gallon?

10. In a rainfall of $3\frac{1}{4}$ in., how many tons of water will fall on an acre?

Units of Measure

THE IMPERIAL GALLON

1. The Imperial gallon used in Canada is one-fifth larger than the American gallon. If an automobile driver makes 24 miles on a Canadian gallon of gasoline, how many miles to the American gallon should his car make?

2. Which is cheaper, and by how much, American gasoline at $20\frac{1}{2}$¢ a gallon or Canadian gasoline at 27¢ a gallon?

3. What would be the correct price a quart for milk in the United States to correspond with 12¢ a quart in Canada?

4. If an Imperial gallon of water weighs 10 lb., how many pounds does an American quart of water weigh?

THE LONG TON

The long ton or English ton contains 2,240 lb. The tonnage of English ships and coal from our mines are computed in long tons.

1. Which is the better freight rate from Vancouver to Liverpool: 15s. per long ton, or 18¢ per cwt. regular ton? For convenience consider the English pound (£) as $5 Canadian money.

2. In 1937 British Columbia coal mines produced 1,444,687 long tons of coal valued at $6,139,920. How many regular tons of coal were produced, and for how much a long ton did it sell at the mine?

*3. A coal dealer bought a 40-ton scow-load of coal at Nanaimo for $4.50 a long ton. He allowed 10% for waste in handling, 95¢ a long ton for transportation, and $3.50 a short ton for overhead and cost of delivery. If he sold the coal at $11 a ton, how much profit did he make on the scow-load?

THE NAUTICAL MILE

A nautical mile contains 6,080 ft. When a ship goes 12 nautical miles an hour, her speed is 12 knots. The word "knot" came into use in the days of sailing-ships, when a block of wood, attached by a rope to the ship, was thrown overboard to measure the rate of sailing. Knots tied in the rope were counted, and so the ship was timed in passing them.

1. Express a nautical mile as the decimal of a mile.

2. When a battle-cruiser has a speed of 30 knots, what figure would the speedometer of an automobile travelling at the same rate show?

*3. The French liner Normandie crossed from Cherbourg to New York in 3 days 22 hours and 7 minutes. If the distance from Cherbourg to New York is 2,907 miles, what was her average speed in knots?

*4. The *Queen Mary* crossed from Cherbourg to New York in 3 days 21 hours and 48 minutes. How many knots did she average on the voyage?

THE FATHOM

The fathom is 6 ft. and is the unit that is used at sea for measuring depth and for measuring rope. For small ships the depth is found with a weighted line called a sounding line. A new device, the depth sounder, used on larger ships, measures the distance to the bottom of the sea by the time it takes sound to travel to the ocean floor and back. This device, similar to the radio, working by electricity, flashes the depth on a chart which is marked in fathoms.

1. In 1826, on Lake Geneva, it was discovered that sound travelled through water a distance of 44,250 ft. in 9.4 seconds. How many fathoms per second does sound travel in water?

2. If the depth sounder registers 8 fathoms, and the ship draws 22 ft. of water, how much clearance is there?

THE CHAIN

In measuring land, surveyors use a chain 66 ft. in length.

1 chain = 100 links.

1. How many chains are in a mile?
2. How many rods are in a chain?
3. How many square chains are in an acre?
4. Find the cost of clearing a roadway $7\frac{1}{4}$ miles long and 1 chain wide at 50¢ a square chain.
5. How many inches are contained in a link?
6. Read in chains and links 66.865 chains.

Measuring Gas in Cubic Feet

From where is the gas obtained that is piped into city homes?

What is the difference between natural and manufactured gas?

The present rates on gas for furnaces in Vancouver homes are: $1.58 for the first thousand cubic feet, 45¢ per thousand for the next 24,000 cubic feet, and 40¢ per thousand for additional amounts. Readings are made monthly.

1. From the 12th of December to the 12th of January Mr. Wilson's furnace consumed 46,000 ft. of gas. What was his gas bill for the month, according to the above rates?

2. Mr. Ferguson's gas bill for his furnace for one winter month was $32.50. How many feet of gas did his furnace consume?

3. During the winter a furnace used the following quantities of gas: October, 12,800 ft.; November, 20,400 ft.; December, 32,600 ft.; January, 38,200 ft.; February, 31,000 ft.; March, 28,500 ft.; April, 8,800 ft.

(*a*) What was the total cost of gas, according to the above rates?

(*b*) Make a line graph to record the amounts of gas used.

Review of Electrical Units

The watt hour is the unit of measuring electricity. The price of the current is counted by the 1,000-watt or kilowatt hour (K.W.H.). 746 watts are equal to 1 horse-power.

1. Mrs. Graham has a radio which uses 90 watts, a washing-machine which uses 250 watts, a vacuum-cleaner which uses 70 watts, and an electric range with an oven using 1,250 watts. At 4¢ a K.W.H. what was the total cost of operating her radio for $6\frac{1}{2}$ hours, the washing-machine for $1\frac{1}{2}$ hours, the vacuum-cleaner for 1 hour, and the oven for 3 hours and 20 minutes?

2. At 2¢ a K.W.H. what would it cost to run an 80-horsepower dynamo for 8 hours.

Research Work:

1. Make a drawing of the four dials on an electric meter in your home to show 8,756 K.W.H.

2. Check the electric appliances in your home, and compute the cost per hour if all are operating at the same time. (See the back of a monthly bill for rates.)

Lumber Measure

A board foot of lumber is an amount equal to 1 ft. square and 1 in. thick, or to 144 cu. in.

How many board feet does a cubic foot contain?

The rule for finding the number of board feet is: Multiply the length in feet by the width in inches by the thickness in inches and divide by 12.

M is the abbreviation for 1,000 ft.

Example: How many board feet does a stick of timber contain, which is 18 ft. long and 6 by 8 in.?

$$\frac{18\times 6\times 8}{12}=72 \text{ ft.}$$

Practice problems:

1. How many feet of lumber are in the following?

28 pieces 2 in. by 4 in.	16 ft.
37 pieces 2 in. by 8 in.	12 ft.
84 ft., 6 in. by 6 in.	
23 planks 2 in. by 12 in.	24 ft.

2. Find the cost of the following:

2,450 ft., ship lap 1 in. by 8 in.	at $18.50 per M.
980 ft., 2 in. by 4 in.	at $17.80 per M.
1,680 ft., 2 in. by 6 in.	at $20.75 per M.

The Metric System

The *metric system* is a system of weights and measures that originated in France during the French Revolution. Nearly all the civilized countries of the world, with the exception of the British Empire and the United States, have adopted it.

The advantages of the system are:

1. Being a decimal system, it is easy to use.

2. It eliminates all reduction, addition, subtraction, multiplication, and division of compound denominate numbers.

3. Its adoption by all countries would greatly facilitate commerce.

The disadvantages are:

1. There being no simple relationship between yards, feet, and inches and the metric measures, much of our machinery, which is standardized in inches and fractions of an inch, would have to be altered, on renewal, and all our present weighing scales would be useless.

2. Since the units 10, 100, and 1,000 are not divisible by 3, 6, 12, etc., we should have to discard fractions like $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, etc., in dealing with weights and measures, which we would find difficult.

Measures of Length

In measuring length, the unit is the metre which equals 39.37 in. Other units are derived from this. You will remember that the name of each unit shows its relations to the meter by means of its prefix.

Measures of Length

10 millimetres (mm.) = 1 centimetre (cm.) = .01 metre
10 centimetres = 1 decimetre (dm.) = .1 metre
10 decimetres = 1 metre (m.) = 1 metre
10 metres = 1 dekametre (Dm.) = 10 metres
10 dekametres = 1 hektometre (Hm.) = 100 metres
10 hektometres = 1 kilometre (Km.) = 1000 metres

The centimetre is used for measurements which we would express in inches. The metre is used where we would use a yard, and the kilometre where we would use a mile.

The following equivalents are important. Learn them:

1 yd.=.91 metres. 1 metre=39.37 in.

1 Km.=.62 mi.

In using the metric system, express all the quantities in one unit. Thus:

2 Km. 35 m. 98 cm.=2,035.98 m.

Metric Measures of Length

Exercises (*oral*):

1. What difference do you notice between the abbreviations for units less than a metre and those for units greater than a metre?

2. Express as metres:

(*a*) 300 dm. (*b*) 80 Hm. (*c*) 5,000 mm. (*d*) 5 Dm.
(*e*) 4.56 Km. (*f*) 43 cm. (*g*) 1.27 Hm. (*h*) 3 Km. 40 dm.

3. Read 694.785 m., giving the denomination of each figure.

4. Measure the length and width of the room in metres.

Exercises (*written*):

1. Write 11 Km. 7 dm. 22 mm. as metres.

2. Add 7.47 m., 1.37 Km., 23.6 dm., 16.4 cm.

3. From 7 Km., take .3 Km. 23.7 cm.

4. John walked 3 Km. 45m., and Henry walked 2 Km. 850 m. How many kilometres farther did John walk than Henry?

5. A train travels at 42.38 Km. per hour. How long will it take to go 825.4 Km.?

6. A French and British plane have a race. The distance is 500 miles. The French plane travels at 136.25 Km. per

hour and the British plane at 150 miles per hour. Which wins?

7. Tom runs at the rate of 5.27 Km. per hour, and Henry runs at 3.8 miles per hour. Who went farther in 2 hours? By how many miles?

Metric Measures of Area

The area of the square at the right is 1 sq. cm., because it is 1 cm. long and 1 cm. wide. Since 1 cm.=10 mm., the square contains 100 sq. mm. In the same way the whole table of measures of area is built up.

MEASURES OF AREA

100 square millimetres =1 square centimetre
100 square centimetres =1 square decimetre
100 square decimetres =1 square metre
100 square metres =1 square dekametre
100 square dekametres =1 square hektometre
100 square hektometres =1 square kilometre

The *square metre* is used in measuring surfaces.

The *square kilometre* is used in measuring large surfaces.

The unit used for *land measure* is the *are.*

1 are=1 square dekametre (sq. Dm.)
100 ares=1 hektare (Ha.)

Two useful equivalents are:

1 acre=.4 hektares. 1 hektare=2.5 acres.

Example: Find the area, in hektares, of a city block 3.24 Hm. square. How many acres does it contain?

Area of block=3.24×3.24 sq. Hm.=10.5 sq. Hm.=10.5 Ha.
10.5 Ha.=10.5×2.5 acres=26.25 acres

Metric Measures of Area

Exercises (*oral*):

1. What metric unit would you use in giving the area of the walls of a room?

2. What metric units are used to measure areas compared to: the square yard; the square inch; the square mile?

3. What is the relation between a square dekametre and an are; between a square hektometre and a hektare?

4. Which is the larger, a square kilometre or a hektare?

5. Express as square metres or decimals of a square metre:

(*a*) 62.5 sq. dm. (*b*) 37.5 sq. dm. (*c*) .75 sq. Dm.
(*d*) 2.34 sq. Km. (*e*) 28.3 sq. Hm. (*f*) 1,000 ares

Exercises (*written*):

1. What is the area of the top of your desk measured in square centimetres?

2. What is the area of the floor of your classroom measured in square metres?

3. The area of a rectangular piece of land is 18 Ha. What is its width if the length is 60 Dm.?

4. The area of a window is 1.44 sq. m. If the length is 18 dm. how many millimetres wide is it?

5. A farmer owns a quarter section of land. What is its area measured in hektares?

The Olympic Games

The Olympic Games are international competitions which take place every four years and in which the leading athletes of many nations contend in friendly rivalry. All the measurements are made in the metric system. In 1928 the games were held at Amsterdam, Holland. On that occasion a boy from British Columbia won first place in the 100-metre and 200-metre dashes.

In 1936 the games were held in Berlin and the following problems are based on this Olympic.

1. A B.C. runner in the preliminaries at Montreal ran 100 metres in 10.6 seconds. At the same rate of speed what would have been his time for 100 yards?

2. At Berlin a New Zealand runner won the 1,500-metre race in 3 minutes 47.8 seconds. What per cent of a mile was the distance?

1. An American won the 100-metre race in 10.3 seconds. How many feet did he run per second?

4. Another American was first in the 400-metre race, his time being 46.5 seconds. At the same rate of speed, what would have been his time for 440 yards?

Metric Measures of Volume and Capacity

A cube 1 *metre long,* 1 *metre wide, and* 1 *metre deep* has a volume of 1 *cubic metre.*

Since 1 metre=10 decimetres, the volume of the same cube may be given as 1,000 *cubic decimetres.*

Therefore 1,000 cu. dm.=1 cu. m.

In this way we can write down the *table of volumes.*

1,000 cubic millimetres =1 cubic centimetre
1,000 cubic centimetres =1 cubic decimetre
1,000 cubic decimetres =1 cubic metre

Units larger than the cubic metre are seldom used.

For measuring wood, excavations, etc., the *stere* is used.

1 *stere*=1 *cubic metre.*

In measuring *liquids* the unit is the *litre* (l).

1 litre=1,000 cu. cm.=1 cu. dm.

For fluids in larger quantities, and for *grains, fruits,* etc., the *hektolitre* (hl.) is used.

You will find the following equivalents useful:

1 cu.m.=1.3 cu. yards.
1 litre =.89 quarts
1 stere=.29 cords
1 Hl. =2.75 bu.

Metric Measures of Weight

The standard unit for measuring weight is the *gram.* It was established by taking the *weight of* 1 *cubic centimetre of distilled water* at 4° C. at the sea-level in the latitude of Paris.

The *gram* is used for weighing *medicines, precious metals,* etc., and for many *scientific purposes.*

The *kilogram* is used for weighing heavier objects.

The *metric ton,* 1,000 *Kg.,* is used for weighing bulky articles such as *coal, hay,* etc.

1 kilogram=2.2 pounds. 1 metric ton=2,200 lb.=1.1 tons

Exercises (oral):

1. What is the weight, in grams, of 1 cu. cm. of water?

2. What is the weight of 1 litre of water?

3. Which is heavier, John, who weighs 85 lb., or Henry, who weighs 40 Kg.?

Metric Measures

Some useful equivalents are:

1 in.=2.54 cm.
1 m.=39.37 in.; 1.1 yd.
1 Km.=.62 mi.; $\frac{5}{8}$ mi.
1 sq. m.=1.2 sq. yd.
1 Ha.=2.5 acres
1 cu. m.=1.3 cu. yd.
1 litre=.88 qt.
1 Hl.=2.75 bu.
1 Kg.=2.2 lb.
1 metric ton=1.1 tons

Review

1. One half of a half is how many quarters of a quarter?

2. Read to the nearest hundredth:
13.3694 42.3548 9.597

3. $6\,\overline{)\,40376\frac{5}{8}}$

4. 8 minus 16 thousandths.

5. $9\,\overline{)\,51872\frac{5}{6}}$

6. Find the average of $6\frac{1}{4}$, $7\frac{2}{3}$, $8\frac{5}{6}$, and $4\frac{1}{2}$.

7. Read to the nearest thousandth:
5.3726 8.0079 12.3589 7.2596

8. Find $\frac{1}{4}$ of 2 hours, 25 minutes and 40 seconds.

9. $2\frac{1}{3}\times4\frac{1}{4}\div5\frac{5}{6}$.

10. $36\frac{3}{4}\times.25$.

Standard Time

We say that the sun rises in the east and sets in the west. Primitive man thought that the sun encircled the earth every 24 hours, giving us daylight and darkness. We now know that it is the earth turning on its axis that gives us day and night. Every 24 hours the earth makes a complete revolution, or turns through 360 degrees.

When the sun is due south, or, to be more exact, on our meridian, we say it is 12 o'clock noon.

When it is 12 *noon in Edmonton, it is past noon in Regina. In Vancouver it is before noon* because the sun is not yet on the meridian at Vancouver.

It may readily be seen that if each place used its 'true' or 'sun' time much confusion would result. In 1883 a Canadian, Sir Sanford Fleming, devised a system of *Standard Time* which is now used throughout the continent.

In this system North America is divided into five *Time Belts,* as shown in the accompanying map. All places in the same zone use the same time. The time in each zone differs by 1 hour from the time in the next zone. This difference corresponds to a change in longitude of 15°.

Standard time is now used all over the world. Starting from Greenwich, England, on the Prime Meridian, each 15 degrees of longitude has a difference in time of 1 hour.

On this continent, the time zones are:

(*a*) Atlantic, the centre of the zone being the 60th meridian.
(*b*) Eastern, " " " 75th "
(*c*) Central, " " " 90th "
(*d*) Mountain, " " " 105th "
(*e*) Pacific, " " " 120th "

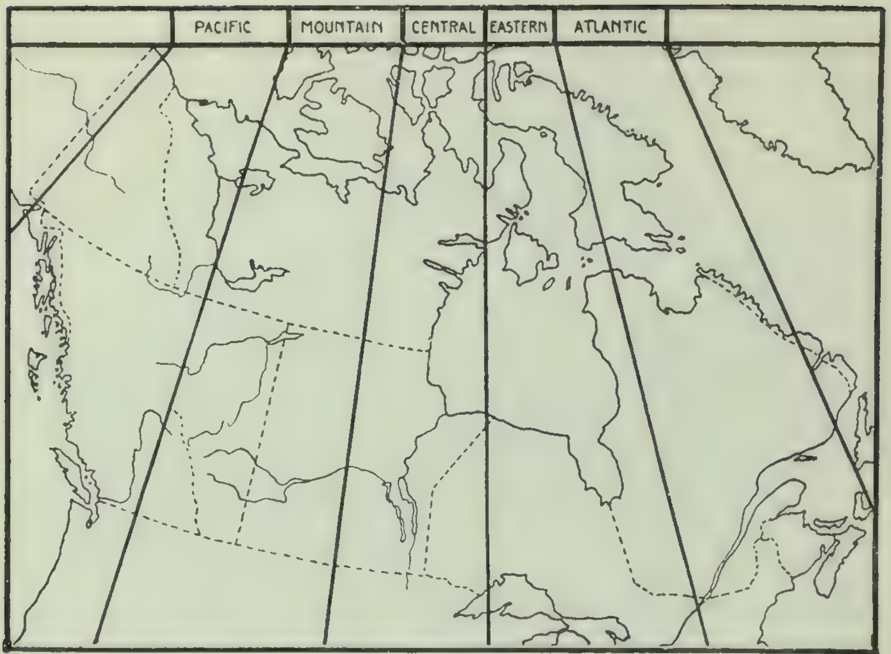
Exercises (*oral*):

1. What is "sun time," and what is "standard time"?

2. Explain why each place with a different longitude has a different sun time.

3. About how long has standard time been in use in North America? Who originated the idea?

4. Suggest as many advantages as you can that standard time has over sun time.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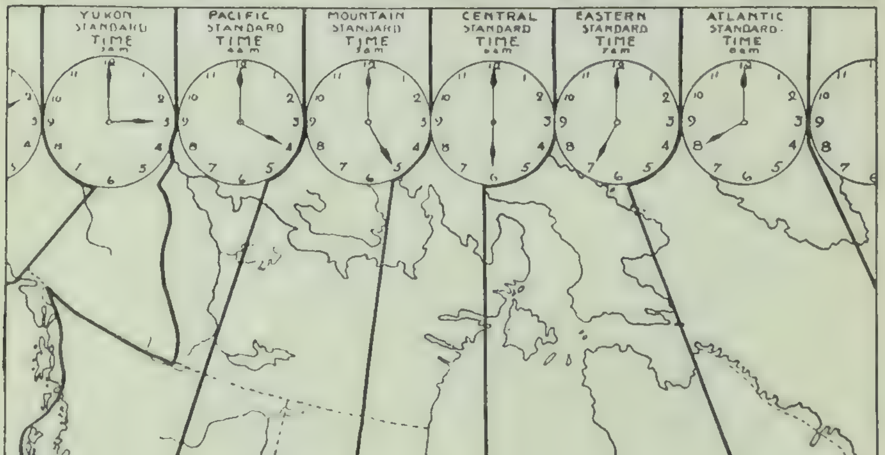
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MOUNTAIN STANDARD TIME 5 a m
CENTRAL STANDARD TIME 6 a m
EASTERN STANDARD TIME 7 a m
ATLANTIC STANDARD TIME 8 a m

5. If you go east of the central meridian of a time zone is the real time ahead or behind the standard time?

6. Why are the time zones taken as 15° in width?

7. As you go west of the central meridian of a time zone is the sun time ahead or behind the standard time?

8. What is the difference in time between places having a difference in longitude of 30°, 105°, 175°, 45°, 60°?

9. If you were travelling from Vancouver to Montreal, how many times would you have to set your watch? How much would you change it each time? Would you set it ahead or backward?

10. If you were travelling east from Vancouver to Winnipeg, how many times would you set your watch? Would you move it ahead or backward?

Example 1. When it is 5 p.m. in Montreal, what time is it in Vancouver?

Vancouver uses Pacific time, 120th meridian.
Montreal uses Eastern time, 75th meridian.
Difference of the time zones in longitude$=45°$.
15° in longitude corresponds to 1 hour in time.
45° in longitude corresponds to $\frac{45}{15}=3$ hours.
Vancouver is west of Montreal. Time at Vancouver is 2 p.m.

Example 2. When it is 7:20 a.m. on May 19 at Hong Kong (120° E.), what time is it in Montreal?

Hong Kong uses time of 120° E. Montreal uses time of 75° W.
Difference in longitude (through Greenwich)$=195°$.
195° longitude means a difference of $\frac{195}{15}$ hours$=13$ hours.
Montreal (through Greenwich) is west of Hong Kong.
Montreal time is 13 hours earlier than Hong Kong time.
Time in Montreal is 13 hours before 7:20 a.m., or 6:20 p.m. on May 18th.

Exercises (written):

1. What is the difference in time between Regina and Edmonton; Halifax and Winnipeg; Quebec and Victoria; Calgary and Montreal?

2. If it is 6 p.m. in each of the first named cities, what time is it in the others?

3. When it is 5 p.m. in Montreal, what time is it in Vancouver?

4. The Premier of Canada made an important address in Ottawa at 8:30 p.m. At what hour could he be heard on the radio at Vancouver? At Calgary? At Winnipeg? At Halifax? At a point 1,000 miles south of Ottawa?

5. An important news item was telegraphed from Moncton, N.B., at 5:49 p.m. on March 14. Allowing no time for transmission, at what times did the news reach Winnipeg? Calgary? Vancouver?

6. A train left Montreal at 11:15 p.m. on Tuesday, and arrived in Calgary at 8:40 a.m. on Saturday. In how many hours and minutes did the train actually make the journey?

7. An aeroplane left Lethbridge at 4:30 p.m. on Friday, and reached Toronto at 3:37 a.m. on Saturday. How long did it take to make the journey?

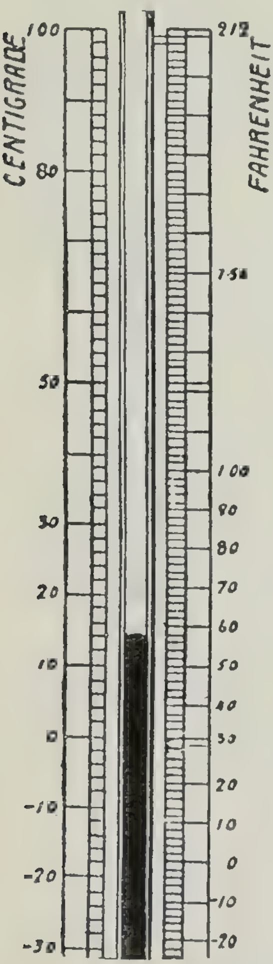
8. An aeroplane disaster was cabled from Edmonton to Winnipeg, Calgary, and Vancouver. The message left Edmonton at 12:15 p.m. Allowing no time for transmission, at what time did it reach the other cities?

9. A cable was received at 3:15 p.m. from Cairo (33° E.), reporting a serious storm. If the point of reception was Ottawa, at what time did the cable leave Cairo if no allowance was made for time of transmission?

10. A report regarding the aeroplane race from London to Melbourne (145° E.), was sent from the latter place at 6:12 p.m. If two minutes were necessary to transmit the message, at what time did this report reach London?

11. A ship calls at Rio de Janiero at 6:40 a.m. standard time. The ship's chronometer shows Greenwich time to be 9:40 a.m. From what time meridian does Rio take its time?

12. A ship was wrecked near Sydney, Australia (150° E.), at 3:43 a.m. on October 7. Allowing $3\frac{1}{2}$ minutes for transmission, calculate the time at which the news reached Edmonton.



Measures of Temperature

The instrument used for measuring temperature is called a *thermometer.* Temperature is usually measured by one of two scales, the Fahrenheit scale is the one ordinarily used. The Centigrade scale is used by scientists. At the left you will see how the two scales compare. The differences are in the size of the units and in the location of the freezing and boiling-points.

Fahrenheit	*Centigrade*	
212°	100°	Boiling-point
32°	0°	Freezing-point

Notice that on the Centigrade scale 100° covers the same change in temperature as 180° on the Fahrenheit scale. Therefore:

180 F. degrees = 100 C. degrees, and
1 F. degree = $\frac{100}{180}$ or $\frac{5}{9}$ C. degrees
1 C. degree = $\frac{9}{5}$ F. degrees

Remembering that freezing-point on the Fahrenheit scale is 32° above zero, we get the following formula for changing temperatures from one scale to the other:

$$C=\frac{5}{9}(F-32) \text{ and } F=\frac{9}{5}(C+32).$$

NOTE: The formula at foot of page 49 should read:

$$C=\tfrac{5}{9}(F-32) \text{ and } F=\tfrac{9}{5}C+32.$$

Example: Change 59° F. to Centigrade, and 80° C. to Fahrenheit.

$$\begin{aligned} C&=\tfrac{5}{9}\,(F-32) & F&=\tfrac{9}{5}\,C+32 \\ &=\tfrac{5}{9}\,(59-32) & &=\tfrac{9}{5} \text{ of } 80+32 \\ &=\tfrac{5}{9} \text{ of } 27 & &=144+32 \\ &=15^\circ \ \textit{Ans.} & &=176^\circ \ \textit{Ans.} \end{aligned}$$

Exercise:

The temperatures on a certain day were given as follows. Change the Fahrenheit readings to Centigrade readings and the Centigrade to Fahrenheit:

(1)	Victoria	44° F.	(6)	Paris	30° C.
(2)	Kamloops	36° F.	(7)	Berlin	20° C.
(3)	Swift Current	8° F.	(8)	Vienna	9° C.
(4)	Winnipeg	22° F.	(9)	Leningrad	2° C.
(5)	Edmonton	10° F.	(10)	Copenhagen	17° C.

Aliquot Parts

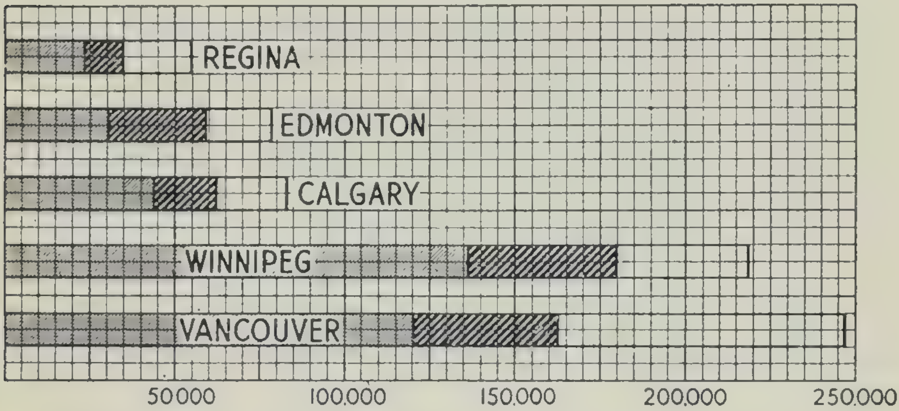
Test your knowledge of the following facts, by covering the columns of each table with a paper on which you write the equivalents of the fractions in the column not covered.

Fraction	*Decimal*	*Per Cent*	*Fraction*	*Decimal*	*Per Cent*
1. $\frac{1}{2}$	.50	50%	13. $\frac{1}{10}$	.10	10%
2. $\frac{1}{4}$	.25	25%	14. $\frac{1}{5}$	.20	20%
3. $\frac{3}{4}$	.75	75%	15. $\frac{3}{10}$	.30	30%
4. $\frac{1}{8}$	.125	$12\frac{1}{2}$%	16. $\frac{2}{5}$	.40	40%
5. $\frac{3}{8}$	.375	$37\frac{1}{2}$%	17. $\frac{3}{5}$	.60	60%
6. $\frac{5}{8}$	.625	$62\frac{1}{2}$%	18. $\frac{7}{10}$	.70	70%
7. $\frac{7}{8}$	.875	$87\frac{1}{2}$%	19. $\frac{4}{5}$	.80	80%
8. $\frac{1}{3}$	$.33\frac{1}{3}$	$33\frac{1}{3}$%	20. $\frac{9}{10}$	.90	90%
9. $\frac{2}{3}$	$.66\frac{2}{3}$	$66\frac{2}{3}$%	21. $\frac{1}{12}$	$.08\frac{1}{3}$	$8\frac{1}{3}$%
10. $\frac{1}{6}$	$.16\frac{2}{3}$	$16\frac{2}{3}$%	22. $\frac{1}{9}$	$.11\frac{1}{9}$	$11\frac{1}{9}$%
11. $\frac{5}{6}$	$.83\frac{1}{3}$	$83\frac{1}{3}$%	23. $\frac{1}{16}$	$.06\frac{1}{4}$	$6\frac{1}{4}$%
12. $\frac{1}{200}$	.005	$\frac{1}{2}$%	24 $\frac{1}{50}$	.02	2%

If you hesitate over any of the equivalents, take time to memorize them.

Bar Graphs

The graph below shows the growth in population in the five largest cities of Western Canada over a period of three decades, or 30 years. The shaded portion shows the growth from 1901 to 1911; the black part from 1911 to 1921; and the unshaded part from 1921 to 1931.



The population of Regina in 1931 was 55,000.

1. Copy and complete the following:

(*a*) The largest city in Western Canada is ——. Its population which in 1911 was about —— increased to —— in 1921 and to —— in 1931.

(*b*) Winnipeg had a population in 1911 of ——, which increased to —— in 1921 and to —— in 1931.

(*c*) Calgary in 1911 had —— people. By 1921 it had increased its population to —— and by 1931 to ——.

(*d*) Edmonton in 1911 had —— fewer people than Calgary. By 1921 its population had increased to —— and by 1931 to ——.

(*e*) Regina, which in 1911 had a population of ——, increased by only —— people in the following decade, but thereafter increased rapidly until in 1931 it had —— people.

(*f*) List the population of these cities according to the 1941 census.

2. The growth in population of Montreal and Toronto over three decades is shown by the following population figures:

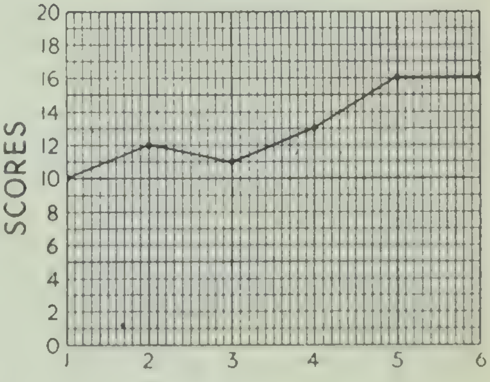
Population in:	1911	1921	1931
Montreal	490,504	618,506	818,577
Toronto	318,833	521,893	631,207

Draw a bar graph to illustrate these facts.

The Broken-line Graph

This kind of graph is used to show changes or variations of the same thing at different times, or under different conditions, such as temperature of the same place at different days; production of crops from year to year; scores made in the same subject on different days.

Graph showing the scores made by a student in six successive tests in *algebra*



The construction of such a graph is somewhat like that of the bar graph. The days or times are generally placed at equal spaces along the *horizontal axis*, and the scores, or temperatures, or number of bushels, etc., placed on the *vertical axis*.

NOTE: It is not necessary always to begin with 0 at the foot of the vertical axis. For instance, if the range of scores is from 55 up to 125, you might begin with 50. This enables you to have a larger unit, and, of course, the larger the unit you have, the better will the changes or variations be shown.

Exercises on Broken-line Graphs

1. The figures given below show changes in the population of several provinces and Canada as a whole in decades from 1871 to 1931: (Figures are to the nearest thousand).

	1871	1881	1891	1901	1911	1921	1931
Man.	25,000	62,000	152,000	255,000	461,000	610,000	700,000
B.C.	36,000	49,000	98,000	178,000	392,000	525,000	694,000
Alberta	———	———	———	73,000	374,000	588,000	731,000
Sask.	———	———	———	91,000	492,000	757,510	922,000
Canada	3,690,000	4,325,000	4,833,000	5,371,000	7,206,000	8,788,000	10,374,000

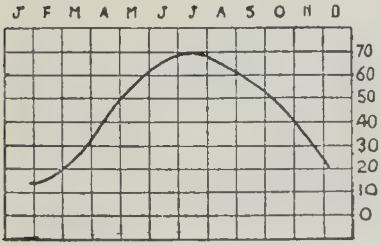
Draw separate line-graphs showing the increase in population of: (*a*) Man.; (*b*) B. C.; (*c*) Alberta; (*d*) Sask.; (*e*) Canada.

2. The statistics below show the changes in Canada in the employment index-numbers, with the year 1926 taken as 100 or normal:

Year	1926	1927	1928	1929	1930	1931
Index No.	100	105	112	119	113	103
Year	1932	1933	1934	1935	1936	1937
Index No.	88	83	96	99	104	114

Draw a broken-line graph to illustrate these facts.

A curved line is sometimes used to show a continuous change, such as a change in temperature. The curved-line graph at the left shows the mean daily temperature at Montreal for a twelve-month period.



Make the table on which the graph is based.

Replot the graph, beginning with July and ending with June.

Exercises:

1. The school nurse kept a record of Fred Moody's weight for six successive years. Here is the record: age 11, 82 lb.; age 12, 96 lb.; age 13, 102 lb.; age 14, 120 lb.; age 15, 135 lb.; age 16, 164 lb. Make a broken-line graph of Fred's weights.

2. Make a curved-line graph based on the following record of daily temperatures: 1 a.m., 23°; 4 a.m., 26°; 7 a.m., 35°; 10 a.m., 56°; 1 p.m., 65°; 4 p.m., 54°; 7 p.m., 42°; 10 p.m., 29°.

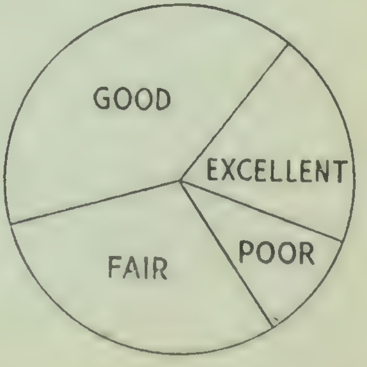
3. The following is Manitoba's population for each of the last seven census years: 1871, 25,228; 1881, 62,260; 1891, 152,506; 1901, 225,211; 1911, 461,397; 1921, 610,118; 1931, 700,139. Show the growth of population in Manitoba by a broken-line graph and a curved-line graph.

Circle Graphs

You have used bar graphs and broken-line graphs to show the relation between separate number facts. For showing the relation of parts to the whole, the *circle graph* may be used to greater advantage. The circle graph is made by the use of compasses, ruler, and protractor.

Example: Of 120 Grade IX pupils in a school, 24 were rated excellent, 48 good, 36 fair, and the rest poor. This circle graph clearly shows these facts.

A circle contains 360°.
120 pupils are represented by 360°.
24 pupils $= \frac{24}{120}$ or $\frac{1}{5}$ of 120 pupils.
$\frac{1}{5}$ of $360° = 72°$.
48 pupils $= \frac{2}{5}$ of 120 pupils.
$\frac{2}{5}$ of $360° = 144°$.
36 pupils $= \frac{36}{120}$ or $\frac{3}{10}$ of 120 pupils $= \frac{3}{10}$ of $360° = 108°$.



Exercises:

1. In a school 20% of the pupils were under age, 40% were at age, and the rest over age. Show these facts by circle graph.

2. Show by a circle graph that 5% of the pupils of grade 8 made A grade; 20% B grade; 48% C grade; 25% D grade; and the rest E grade. What per cent made E grade?

3. A family divided its income so that it spent 20% of its income for rent, 30% for food, 10% for fuel, 25% for clothing. Show how much remained.

4. In one city the total enrolment in all schools was 50,000 pupils. Of this number 25,000 were in the Elementary grades, 15,000 in the Junior High School, and the remainder in the Senior High School. Show these facts by a circle graph.

5. A manufacturer of shoes found that for every pair of a certain last he spent $1.20 for raw material, $.80 for labour, $1 for overhead expenses, and made a profit of $.60. Find the sale price of the shoes. Show these facts by a circle graph.

6. Of 5,000 pupils promoted to High School in one year, 300 took the academic course, 1,800 the commercial course, and the rest the technical course. Show the distribution of pupils in the several courses by a circle graph.

Class Activity on Graphs

1. Make a collection of graphs from newspapers and magazines. Arrange them according to types, post and read.

2. Stretch a roll of narrow white wallpaper across the back of the classroom. Make a temperature line graph, charting daily the temperature in your district and the temperature of a city in Central Canada. Mark the graph in two colors.

Exercises:

1. $8 \div \frac{1}{4}$.
2. $2\frac{1}{2}\%$ of \$80.
3. 9—.016.
4. Express $16\frac{2}{3}\%$ as a fraction.
5. What percent is 56 of 60?
6. 5 to 12 is how much to 100?
7. Find the cost of a section of land at \$12.50 an acre.
8. $4 \times 2\frac{1}{2} \times 19 \times 0 \times 6\frac{2}{3}$.
9. How many square inches are in a square yard?
10. Circle A contains 8 square feet. What is the area of a circle twice the diameter of Circle A?
11. 16% of 80 is what per cent of 14.4?
12. From $15\frac{1}{2}$ take the product of 6 and $2\frac{1}{2}$.

Problems

1. How many 4-inch squares will fill a pattern of linoleum 5 yds. long and 4 yds. wide?

2. During 5 months a family spent for groceries \$28.50, \$32.20, \$24.80, \$36.15, and \$30.25. How much can they spend for groceries during the sixth month in order to average \$30 a month?

3. A refrigerator was sold for \$200 with \$50 cash and the balance in 12 equal monthly payments. If interest at the rate of 7% on the full amount was added to the principal until the last payment was made, what was each monthly payment?

4. The plan of a house was drawn in the scale of 1:12. How many inches long and wide would the dining room 12′×14′ show in the plan?

5. A team of horses weighs 3240 pounds. One weighs 50 pounds more than the other. Find the weight of each.

Review Exercises on Measurement

CHOOSE THE BEST COMPLETION

A. **1.** Land is most commonly measured in square yards, square feet, acres, square rods, square chains.

2. The one who measures gravity most frequently is the farmer, engineer, meat-seller, weather-man, sea-captain.

3. The bushel measure has been largely replaced by the gallon, 100-lb. sack, ton, peck.

4. The chain measure is used mostly by the farmer, surveyor, logger, seaman, blacksmith.

5. The Centigrade thermometer is used most by a science professor, doctor, weather-man, ship's engineer, surveyor.

6. The metre is used more largely in Asia, Europe, Africa, America.

7. Standard Time means: (*a*) that every place has its own time; (*b*) areas on the north have a different time; (*c*) all places between certain meridians are given the same time; (*d*) each province has its own time.

B. 1. How many acres are in a field 880 ft. long and 660 ft. wide?

2. How many short tons and pounds in 6 long tons?

3. How many bushels of wheat will weigh the same as 238 bushels of oats?

4. A board is 8.6 metres long. Express its length in feet and inches.

5. How many inches must be cut from a board 8 in. wide to make $3\frac{1}{2}$ sq. ft.?

6. If the Canadian gallon of gasoline sells for 27¢, what would be the price of American gasoline selling at the same rate?

7. What per cent of a land mile is a nautical mile?

8. A ship drawing 18 ft. of water finds in sounding that the depth is 7½ fathoms. How many feet of clearance has the ship?

9. Express 67° Fahrenheit in the Centigrade system.

10. When it is 9:20 a.m. in Toronto, what time is it at Calgary?

11. A room 12 ft. long and 10 ft. 6 in. wide contains 750 cu. ft. of space. Find the height?

12. An automobile travelling 40 miles an hour is going how many feet per second?

13. Find one-eighth of 12 lb. 9 oz.?

14. What is the average of 2 hours 38 minutes 24 seconds and 3 hours 28 minutes 46 seconds?

15. What per cent on the cost is gained in buying coal by the long ton at $4 a ton and selling it at $8.50 per short ton?

16. An automobile travels 80 kilometres in 2 hours. What is its average speed in miles per hour?

17. Find the area of the walls and ceiling of a room 18 ft. 6 in. long, 16 ft. wide, and 9 ft. high. The answer should be in square yards.

18. What fraction of 4 miles 110 rods is 3 miles 60 rods?

19. Find the average of the following Fahrenheit temperatures: 16°, —20°, 18°, —4°, and 12° below freezing.

20. What per cent of ¾ is ⅞?

C. Review carefully the tables of measures at the end of the text.

1. How do you find the number of acres in a field when you know its length and width in feet?

2. By what number would you divide long tons to change them to short tons?

3. If the weight of a bushel of oats is .566 of a bushel of wheat, how can this information be used in finding how much 90 bushels of oats weigh, when a bushel of wheat weighs 60 lb.?

4. If a metre is 39.37 in., express the decimal relation of a yard to a metre and a metre to a yard.

5. Express .86 yds. in feet and inches.

6. If you know the area of a rectangular surface and the length of one side, how do you find the length of the other side?

7. Change 67°C to Fahrenheit.

8. How many miles an hour is equivalent to 30 feet per second?

9. When two fractions have equal denominators, what fraction represents their comparative size?

10. What is the shortest method of finding the area of the four walls of a room?

Review Test

1. How many tons of water will fall on $2\frac{1}{2}$ acres in a rainfall of 4 in.?

2. What length of time transpires between 9:25 a.m. on Monday and 7:12 p.m. on Tuesday?

3. Express 18° Fahrenheit in the Centigrade scale.

4. In constructing a circle graph to designate 203, 296, 274, 489, and 178, how many degrees would represent the second number?

5. Find the cost of cutting a ditch $2\frac{1}{2}$ miles long, 4 ft. wide, and 6 ft. deep, at 27¢ a cu. yd.

6. A ship sails at the rate of 23 knots. How many miles per hour does she travel?

7. A bushel of wheat occupies $1\frac{7}{25}$ cu. ft. How many bushels of wheat will fill a bin 14 ft. long and 12 ft. 6 in. wide to a depth of 7 ft.?

8. Light travels at the rate of 186,000 miles a second. If the light from the sun reaches the earth in 8 minutes, what is the distance in miles from the earth to the sun?

9. Divide the time from 8:45 p.m. on Thursday to 11:53 a.m. on Saturday into 8 equal parts.

10. Find, in pounds, the weight of water that fills a box 1 m. long, 3 dm. wide, and 2 dm. high.

CHAPTER IV

SOCIAL ASPECTS OF ARITHMETIC, PART I

Commission

Commission is the money paid for selling goods that are not retailed through a store. Agents selling insurance, stocks, bonds, and real estate are not paid regular salaries, but receive a commission on the sales made. There are two classes of commission:

1. Wholesale produce companies maintain warehouses and sell farmers' produce on a commission of 10% to 15% to grocers and vegetable and fruit stands. The shippers pay the freight, and the wholesalers fix the price according to supply and demand. The wholesaler usually charges a commission in per cent of the selling-price, although in the Prairie Provinces the practice is to charge $70 a carload on berries sent from British Columbia. Fruit and vegetables from California are not sold on commission in this province, but are bought outright by the wholesalers. As soon as the produce is sold, the wholesaler deducts his commission and remits the net proceeds to the farmer or marketing-board, but he carries the grocer's accounts for 30 or 60 days.

2. The sale of automobiles, radios, washing-machines, and many other kinds of merchandise, as well as insurance, is greatly increased by agents, who make a list of possible buyers and solicit their business. Wholesale houses also send commercial travellers through the country to sell groceries, hardware, and dry goods to the various stores. These

salesmen are paid regular salaries, travelling expenses, and a small commission on certain lines of goods. Agents selling for local dealers are paid a straight commission of 15% or 20%. In selling automobiles, they are paid a 5% commission. Certain large companies pay their salesman a commission organized on a sliding scale.

Problems for discussion:

1. Why are goods sold from door to door usually high priced?
2. Why can not wholesale grocers afford to pay a large commission to their commercial travellers?
3. What would be the main items in the expense accounts of travelling salesmen?
4. Why would business firms rather pay a straight commission than a regular salary to salesmen?
5. What would you list as the most important qualifications for an insurance agent?
6. Why would a druggist rather sell certain goods on commission?
7. Why do book-stores prefer to sell magazines on commission?
8. Why does an insurance agent make more regular sales than an automobile agent?
9. Why does a wholesale-produce company need to charge a larger commission than a wheat brokerage company does?
10. Why do wholesalers of farm produce prefer that the farmer pays the freight?

Exercises (oral):

1. 1½% of $200
2. 3¼% of $400
3. ½% of $2,000
4. 2¼% of $800
5. $1,200 less ½%
6. $1,500 less 2%
7. $80 less 1%
8. $300 less 1½%
9. 99% of $600
10. 98½% of $1,400.

Problems

1. An agent for a real estate company sold a house for \$7,200. The commission was 5% on the first \$6,000 and 2½% on the balance. His company kept 40% of the commission.

   (*a*) How much did the agent receive for selling the house?

   (*b*) What were the net proceeds paid the owner?

2. A carload of strawberries shipped from Gordon Head, Vancouver Island, to Regina, contained 800 crates of 24 boxes each. 25% of the berries were sold for \$2.80 a crate, 40% for \$2.40 a crate, and the remainder for \$2.15 a crate. Express the commission in per cent, if it was \$70 for the carload.

3. The Vancouver Stock Exchange has the following scale of commissions: for stocks under 30¢ a share, ½¢; 30¢ to 75¢, \$.01; 75¢ to \$1, \$.01½. How much would a brokerage firm charge for buying 350 shares of stock listed at 40¢; 800 shares listed at 69¢; and 680 shares listed at 95¢?

4. An agent received a salary of \$12.50 a week, an allowance of \$18 a week on his travelling expenses, and was given 6% on all orders sanctioned by his firm of wholesalers. In six weeks he sold \$1,920 of goods. If the merchandise cost the firm 50% of the sale price, how much profit did they make on the goods, when the cost of shipping was \$179.78?

5. A salesman worked on a sliding scale of 3% commission for the first \$1,000 sales per week, 5% on the next \$1,000 sales, 7% on the next \$500 sales, and 10% on any amount above \$2,500. For six weeks his sales were as follows:

First week	\$950	Fourth week	\$1,870
Second week	\$1,480	Fifth week	\$1,900
Third week	\$2,650	Sixth week	\$2,100

Make a line graph showing his earnings during the six weeks.

Trade Discount

Wholesale merchants and manufacturers usually catalogue or mark their goods at certain fixed prices, and then allow deductions from these prices to purchasers.

The price given in the catalogue is frequently referred to as the *list, catalogue,* or *invoice* price.

The allowance or deduction made from the list price is called *commercial* or *trade discount.* These allowances are made for one or more of the following reasons:

(*a*) To meet competition.
(*b*) To induce buying in quantity.
(*c*) To meet price changes in raw material or labour.
(*d*) To sell out *dead* stock or discontinued *lines* of goods.
(*e*) To make the preparation of a new catalogue unnecessary when prices change.

The *list price,* less the discount or discounts, is called the *net price.* This is the price actually paid for the goods.

A *discount* is usually expressed as a certain per cent fraction of the list price.

Example: If knives are listed at \$18 a dozen, with a discount of 20%, what is the net price per dozen?

List price of knives = \$18.00
Discount 20% of \$18 = .20 of \$18 = 3.60
Net price = \$18 — \$3.60 = \$14.40

Exercises (*oral*):

Find the discount and net price on each of the following:

List Price	*Rate of Discount*	*List Price*	*Rate of Discount*
1. \$200	10%	5. \$150	33⅓%
2. \$60	5%	6. \$600	40%
3. \$75	2%	7. \$80	2½%
4. \$15	1%	8. \$140	25%

Trade Discount

1. What is due on a bill of $4.80 subject to a discount of 12½%?
2. Find the net price of goods listed at $175, less 15%.
3. One wholesale house lists baseballs at $16.80 a dozen, with a discount of 16⅔%. Another lists the same quality of baseballs at $18.20, with a discount of 25%. Which is the cheaper offer, and how much cheaper is it?
4. A merchant ordered a bicycle listed at $40, 3 footballs listed at $7.50 each, and a basketball listed at $10.50. On this order he was allowed a discount of 20%. Find the price he paid for the goods.
5. How much cheaper per pair are shoes listed at $78.30 a dozen pairs, with a discount of 10½%, than shoes listed at $87.60 per dozen pairs, with a discount of 15%.

Successive Discounts

Sometimes two or more discounts are allowed by manufacturers and wholesalers. These discounts are called *successive discounts.* The first is a discount from the list price, the second from the remainder, and so on.

Example: A manufacturer marked his goods at $60 with three successive discounts of 20%, 10%, and 5% off. Find the net price.

List price of goods = $60.00
First discount = 20% of $60 = .20 of $60 = 12.00
First reduced price = $60 — $12 = 48.00
Second discount = 10% of $48 = 4.80
Second reduced price = $48 — $4.80 = 43.20
Third discount = 5% of $43.20 = 2.16
Net price = $43.20 — $2.16 = $41.04

Notice that the second and third discounts are not taken from the *list price,* but from the $48 and from the $43.20 respectively.

Successive Discounts

Exercises: Find the net price of each item below:

List Price	*Discounts*	*List Price*	*Discounts*
1. $360	25%, 10%	6. $1,500	$33\frac{1}{3}$%, 20%, 5%
2. $100	20%, 10%, 5%	7. $50	10%, 5%
3. $75	20%, $16\frac{2}{3}$%	8. $25	10%, 1%
4. $150	$33\frac{1}{3}$%, 25%, 10%	9. $6,000	10%, 10%
5. $500	10%, 2%	10. $3,500	$14\frac{2}{7}$%, 2%

1. A radio was listed at $264, with successive discounts of $16\frac{2}{3}$% and 5%. Find the net price.

2. Find the net price of a piano listed at $600, with a discount of $33\frac{1}{3}$% and another discount of 10% for cash. Find the cash price.

3. What is the difference on an invoice of $340 between a discount of 40% and successive discounts of 25% and 15%?

4. In one catalogue a stove was listed at $120, with discounts of 25% and 10%. In another catalogue the same type of stove was listed at $120, with discounts of 10% and 25%. Was there any difference in the net price of the two stoves?

Profit and Loss

It is the work of the merchant to secure—frequently from other parts of Canada and from foreign countries—and place in his store, goods which people need.

The price which the merchant pays for the goods is called the *net cost*. If the goods are imported, the merchant must pay freight charges, cartage, customs duties, and other expenses. In selling goods, certain expenses may occur, such as rent of store, fuel, light, delivery service, insurance, and wages of employees. Such expenses are usually called *overhead.* The net cost plus the expenses constitutes the *gross cost* of the goods.

The price at which the merchant wishes to sell the goods is called the *marked price.* The price at which he sells the goods is the *selling price.* When the selling price is greater than the gross cost, the merchant makes a *profit* or *gain.* When the selling price is less than the gross cost, the merchant suffers a *loss.*

The rate per cent of profit is sometimes calculated on the cost price and sometimes on the selling price. Large firms and wholesale houses usually count the profit as a per cent of the selling price, but wholesalers encourage small grocers and other small retailers to reckon profit on the cost price. Which method makes a better showing?

1. A hardware merchant bought a saw for \$1.20 and sold it for \$1.80. Find: (*a*) his per cent of profit figured on the cost price, and (*b*) his per cent of profit calculated on the selling price.

2. Mr. Smith bought 30 boxes of apples at \$1.50 a box. He sold them at a gain of 25% on the cost. Find: (*a*) the amount of gain; (*b*) the selling price.

3. I bought a bicycle for \$25, and sold it at a gain of 20% on the selling price. Find: (*a*) the selling price; (*b*) the amount of gain.

(*a*) 80% of selling price=\$25

$$100\% \text{ `` `` ``} = \frac{\$100 \times 25}{80} = \$31.25$$

(*b*) Gain=20% of selling price

$=.2\times\$31.25=\$6.25$

or $\$31.25-\$25=\$6.25$

Finding Profit and Loss on the Cost Price

Problems:

1. A merchant buys hats at \$25 per dozen and sells them at \$3 each. How much does he gain per dozen?

2. I bought a radio for \$150 and sold it at a loss of 25%. Find: (*a*) the amount of loss; (*b*) the selling price.

3. A grocer buys 400 boxes of apples at \$1.50 per box. He sells half of them at \$1.80 per box, quarter of them at \$1.90 per box, and the rest at \$2 per box. What is his gain?

4. A farmer buys a farm of 320 acres at \$12 per acre. He spends \$1,500 on buildings, \$300 on fences, and clears and breaks 200 acres at \$15 per acre. He sells the farm at \$30 per acre. Find the amount of his gain.

5. A wholesaler bought 5 tons of tea at \$25 per cwt. The freight cost him \$50. Find his gain if he sold the tea at 40¢ per pound.

6. How much more is gained in buying 40 yds. of cloth for \$1 per yd. and selling it for \$1.35 a yd., than in buying 60 yds. of cloth for \$1.15 a yd. and selling it for the same price as the other?

Profits computed on the Selling Price

Some business men calculate their profits on the selling price, as it is often difficult to separate the exact cost from the operating expenses.

1. A shoe seller bought shoes at \$6.30 a pair, and sold them at \$9.80 a pair. His expenses averaged \$1.20 a pair. What per cent of the selling price was his profit?

$$\text{Profit } \$9.80-(\$6.30+\$1.20) \qquad =\$2.30$$

$$\text{Rate of profit}=\frac{230}{980}=.234 \qquad =23.4\%$$

2. Mr. Trimble paid $472 for a consignment of hats, which he sold for $830. If his expenses were $192, what was his profit? What was the per cent of profit on the sales?

3. In one year a store sold for $172,800 hardware which cost $101,376. The operating expenses amounted to $57,600. Find the profit and the per cent profit on the sales.

4. In one month a store's sales amounted to $13,440. If the goods cost $6,960, and the overhead was $4,800, find the per cent of profit on sales.

Find the profit or loss, and the per cent of profit or loss on the selling price:

	Cost Price	*Expenses*	*Selling Price*		*Cost Price*	*Expenses*	*Selling Price*
5.	$250	$30	$320	9.	$20.50	$8.20	$32.80
6.	$3,620	$80	$4,200	10.	$415.20	$193.80	$680
7.	$27,680	$1,670	$25,000	11.	$276.70	$99	$450
8.	$20,500	$8,200	$32,800	12.	$3,904	$2,196	$7,320

13. A canoe which cost $135 was sold for $185. The expenses of selling were $23. What per cent of the selling price was profit?

*Finding Rate Per Cent of Profit on the Selling Price

Example: By selling a dress for $9.60, a salesman gained $1.20. Express his rate of gain as per cent.

Let x=the rate per cent of gain.

$$\text{Then} \quad \frac{x}{100} \times 9.60 = 1.20$$

$$\text{or} \quad \frac{9.6x}{100} = 1.2$$

$$\text{Then} \quad 9.6x = 120$$

$$x = \frac{120}{9.6}$$

$$= 12.5$$

Thus the rate is $12\frac{1}{2}\%$.

Solve the following problems by equations:

1. What was the rate per cent of gain when a wagon, which cost a merchant $3.75, was sold so as to gain $1.25?

2. A jeweller sold a watch for $64, and thereby gained $16. Express this gain as a per cent fraction.

3. Calculate the per cent rate of profit from the sale of a doll for $4.75, if it cost $3.

4. By selling overcoats for $30, a salesman gained $12. Find the per cent rate of gain.

5. A piano costing $880 was sold for $1,075.20. If $60 was paid out for advertising and other expenses, what per cent profit was made on the cost?

6. A boy bought peaches at 3 for 10¢, and sold them at 5 for 20¢. Find his rate per cent of gain.

7. A flour dealer bought 300 barrels of flour at $5 per barrel, and sold $\frac{2}{3}$ of it at $8 per barrel, and the remainder at the cost price. What was the rate per cent of gain?

8. A man bought 2 horses, paying $100 for one and 10% more for the other. He sold the first at a gain of 20% and the second at a loss of 10%. Find his gain or loss, and his gain or loss expressed as a per cent fraction.

9. If brooms are bought at $7.50 a dozen, what per cent profit will be made by selling them at 85¢ each?

Calculating Overhead Expenses

At the end of the year merchants often compute the year's expenses as a per cent of the sales. As the books show the amount of sales and the expenses, this is easily done. By computing profits and expenses on sales, they know what percentage of each dollar taken in is used for expenses, how much goes to cover the cost, and how much is profit.

1. One year the total sales of a general store amounted to $48,000. The total overhead expenses were $19,200. What per cent of the selling price went to expenses?

The expenses were $\frac{19,200}{48,000}$ or .4, or 40% of sales.

2. Mr. Linn found that his total sales were $172,800, his total expenses $57,600, and the cost of merchandise $101,376. What per cent of the sales went for expenses? For cost? For profit?

3. A merchant bought goods for $68,000. His overhead expenses were $27,200 and his profit was $13,600. What per cent of the selling price was used for overhead? What per cent was profit? What per cent was cost?

In the following, the expenses are what per cent of the sales?

	Sales	*Expenses*		*Sales*	*Expenses*
4.	$20,000	$5,814	7.	$78,480	$26,160
5.	$43,750	$11,000	8.	$301,800	$90,540
6.	$67,500	$18,225	9.	$25,600	$6,400

10. Mr. Binns operates a shoe store. His expenses during one year were: $85 per month for rent, $175 per month for wages, $18 per month for light, $240 per year for advertising, and $350 per year for other expenses. His total sales for the year were $30,200. The expenses were what per cent of the total cost?

INSURANCE

Property Insurance

Companies that hold mortgages on homes insist that the property be insured against fire. Why do they take this precaution? Thus, in Canada, most houses in cities and towns, and buildings on farms, carry fire insurance. Although barns on farms are not heated with stoves, many take fire. What reason can you give for this? Even though city houses are close together, a house in the country is more liable to be destroyed by fire than a house in the city. What is the cause of this?

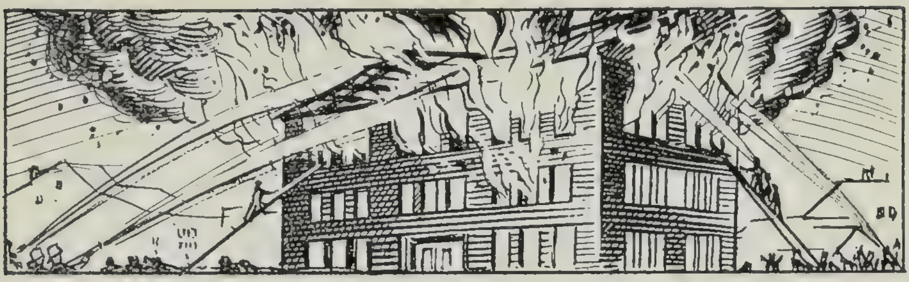
When an insured house is damaged by fire, the insurance company pays the cost of repairing it. The rate of insurance on a house in the city depends upon its distance from other houses and from areas of fire hazards. What conditions in a city would constitute likely dangers of fire?

Why is one not allowed to insure his house for more than 80% of its value or take out full insurance on it in more than one company?

Buildings are not insured against fire by lightning or destruction from earthquakes.

1. John Redmond insured his house for three years with a $3,000 policy, the premium being $20. If his house should be destroyed by fire, he would receive from the company the face value of the $3,000 policy. For this protection he pays a premium of $20 every three years. What per cent of the policy is the yearly premium?

Furniture and personal belongings can be insured against fire or theft. Obtain the rates for such insurance from an agent and submit a problem for the class to work.



Fire—Friend or Enemy?

It would be difficult to imagine life without fires. If we had no fires we should have to do without cooked foods, heated homes, warm baths, clean clothes, and many of the things that make for comfort and happiness. Fire is a good friend but a bad enemy—a good servant but a poor master.

It is estimated that losses by fire cost the people of Canada an average of $50,000,000 per year. Much of this loss is due to carelessness. Some people are careless with matches, or they leave rubbish in attics and basements, so that fire hazards are increased.

1. In 1939 insurance companies collected $42,445,000 in insurance premiums in Canada. Taking the population of that year as 10,500,000, what was the average cost per person in Canada for fire insurance?

2. If the companies paid $17,356,000 that year in fire losses, what per cent of the premium collected was paid out for fire losses?

3. If the amount paid out for fire losses represented 75% of the value of the insured property destroyed by fire, what was the full value of the insured property destroyed by fire?

4. Not all the property destroyed by fire each year is insured. If the $21,655,460 which was paid to cover fire losses for insured properties in 1933 represented 40% of the total fire loss, find the total fire loss for that year.

5. Of the premiums collected for fire insurance in 1937, $16,000,000 was paid to Canadian companies, $30,000,000 to British companies and $25,000,000 to foreign companies. Draw a circle graph to illustrate these facts.

Five-point Automobile Insurance

The following statement appears on an insurance policy for 1938:

Perils	*Limits and Amounts*	*Premium*
1. Legal liability for bodily injury or death	$10,000 for one person $20,000 for two or more	$18.07
2. Legal liability for damage to property of others	$1,000 any one accident	8.00
3. Collision—Sum payable by insured each time $100	Cash value at time of loss	3.00
4. Fire and transportation	Cash value at time of loss	2.00
5. Theft	Cash value at time of loss	3.75
6. Passenger hazard		5.65

1. What are the five points of the insurance?

2. What is the total premium?

3. The same policy the year before was sold for $32.56. Why was there a general advance in the cost of automobile insurance across Canada at the beginning of 1938? What per cent was the increase on the above policy?

4. What is meant by passenger hazard? Why was the premium so large for this item in 1938? It has been reduced. Why?

5. Notice that in the above policy the owner pays the first hundred dollars of damage to his own car in a collision. If $25 were inserted in place of the $100, how much more do you think the policy would cost?? Can you obtain this information?

6. Do you think a careful driver might omit some point in the policy to reduce the premium?

Life Insurance

Mr. Wilson has a wife and several young children. He earns a very small salary, and, should he die, his family would suffer want because Mrs. Wilson could not go out to work and leave the children.

To insure, or guarantee, that his family shall have sufficient money to live on, in case of his death or illness, Mr. Wilson makes a written agreement, called a *policy,* with a company, called an *insurance company.*

This *insurance company* agrees that if Mr. Wilson, the *policy holder,* will make a yearly payment of $46, called the *premium,* it will pay to Mrs. Wilson, called the beneficiary, $2,000, called the *insurance* or *face* or *amount of the policy,* should Mr. Wilson die.

Insurance rates are usually expressed as so much money each year for $1,000 of insurance. In the case of Mr. Wilson, the rate was $23 yearly premium per $1,000 of insurance. The rate of insurance depends largely upon the age of the insured person. Why?

Example. At age 25, the yearly premium is $21.30 per $1,000 of insurance. Find the premium paid for $4,000 insurance.

On $1,000 insurance, the premium is $21.30
" $4,000 " " " " 4×$21.30=$85.20

Insurance covering loss of life, sickness, and accident, is called *personal insurance.* Life insurance is the most important form of personal insurance. In Canada today, nearly a million people carry life insurance, totalling over $6,257,200,000.

Different Plans of Life Insurance Policies

There are four general types of life insurance policies:

(1) Ordinary Life
(2) Limited-payment Life
(3) Endowment
(4) Term Insurance

In an *Ordinary Life* policy, the premiums are paid every year as long as the insured person lives and wishes to continue his insurance.

In the *Limited Life* policy, the premiums are to be paid for a fixed number of years, such as twenty years, after which the policy is *paid up,* and no further premium payments are necessary. The object of this form of insurance is to allow the insured to pay up his insurance while his earning capacity is greatest. The face of the policy is payable only at death.

In the *Endowment Policy,* the premiums are paid for a fixed number of years, such as ten, fifteen, twenty, or thirty. Should the policy-holder die within the stated period, the full amount of the policy is paid. The face of the policy is payable when fully paid up or at death.

In the *Term Insurance* policy, the premiums are paid for a fixed period. The insurance is paid only if the insured dies within the stated period. At the end of that time, the contract ceases.

Which of the above kinds of insurance do you think would have the lowest premiums? The highest? Why?

ORDINARY LIFE PLAN PREMIUMS PER $1,000 INSURANCE

Age	*Premuim*	*Age*	*Premium*	*Age*	*Premium*	*Age*	*Premium*
20	$19.00	28	$23.00	36	$28.80	44	$37.45
22	19.85	30	24.25	38	30.60	46	40.30
24	20.75	32	25.60	40	32.60	48	43.50
26	21.85	34	27.10	42	34.85	50	47.05

1. From the above table, how would you find the premium for a person at the age of 21 years?

2. What is the premium on a policy of $8,000 taken at age 34?

3. What is the premium on $6,000 taken at age 25?

4. Find the premium on $2,500 taken at age 38.

5. Mr. Clarke took $3,000 in an ordinary life policy at the age of 28, and a $2,000 policy at the age of 36. What is the total of his yearly premiums?

Other facts worth remembering about life insurance are:

(*a*) After the policy has been in force for three years, it has a *loan value*; that is, a certain per cent fraction of the premiums which have been paid may be borrowed from the company by the insured. Interest is paid on such a loan.

(*b*) Insurance remains in force only as long as premiums are paid promptly, unless the policy has a loan value.

(*c*) After three years the insured person may no longer desire to be insured, and may *surrender* his policy to the company and receive back from them a part of the premiums paid. The money received by the insured is called *cash surrender value*. *Term* insurance has no cash surrender value.

(*d*) Interest is paid to the insured by the insurance company in the form of *dividends* on each policy. These dividends are estimated on the loan value of the policy at that time and on the financial earnings of the company.

Problems without Numbers

1. For what reasons does a person take out life insurance?

2. What do we mean by the following terms; premium, beneficiary, face of the policy?

3. Why is it necessary to have a medical examination before one can be insured?

4. Name four common forms of life insurance.

5. Which form of insurance policy has no cash surrender value?

6. What is the standard sum of money on which life insurance rates are based?

7. Why does it cost more to take out insurance as one becomes older?

8. Why is an insurance company able to pay more on an endowment policy than it receives in premiums?

Different Plans of Life Insurance

ANNUAL PREMIUMS PER $1,000 INSURANCE

Age	*Ordinary Life*	*20-payment Life*	*20-year Endowment*	*5-year Term*
20	$19.00	$27.40	$48.00	$10.00
25	21.30	30.00	48.50	10.00
30	24.25	33.25	49.40	11.60
35	27.95	36.95	50.55	13.50
40	32.60	41.35	52.25	15.50
45	38.85	46.95	55.05	19.60
50	47.05	54.10	59.45	25.30

NOTE: Rates vary slightly in different companies.

Using the rates given in the table, solve the following:

1. What is the premium on a 20-year endowment policy for $8,000 at age 35?

2. A man took out a 20-payment life policy for $5,000 at the age of 30. He died at the age of 44. What amount had he paid in premiums?

3. Wilbur White, at the age of 25, took out a 20-year endowment policy for $4,000. If he lived to cash in his policy, how did the amount he had paid in premiums compare with the face of the policy?

4. Which type of insurance is best for a young man who has no dependents and is anxious to save money?

5. James White, at the age of 30, took out an ordinary life policy for $5,000 and a 20-payment life policy for the same amount. He died at the age of 57. How much more did he pay in premiums on one policy than on the other?

6. Mr. Ferguson took out an ordinary life policy for $12,000 at the age of 35. He died at the age of 56. What was the difference between the amount paid in premiums and the insurance received by his family?

Life Insurance Problems

1. Mr. Walton, at the age of 40, took out a 20-year endowment policy. He is now 52 years old, and has paid $2,137.75 in premiums. What is the amount of his policy?

2. John Schofield is 35 years old. He has about $180 a year that he can spend on life insurance. What amount of insurance can he buy if he takes: (*a*) Ordinary life? (*b*) 20-payment life? (*c*) 20-year endowment?

3. How much a week should one set aside to pay the premium on a 20-payment life policy for $3,000, taken out at the age of 25?

4. The annual premium on a policy of $3,800 is $44.08. What is the rate per $1,000? Per $100? Per cent?

5. A man, 25 years old, took out a 20-payment life policy for $3,000, and lived the full term. Find the total premium.

6. How much per year will a $5,000 ordinary life policy cost a man who insures at the age of 25? How much more per year will it cost him if he insures at the age of 40?

7. A man of 30 takes out an ordinary life policy for $1,000. In 15 years he surrenders it and receives $276.02. How much more, or less, is this than he paid out?

8. A man who insured for $10,000 at 30 years of age on a 20-year endowment policy, may surrender it at the end of 15 years and receive $6,748.50. Compare this with what he paid in premiums.

9. The total insurance carried in Canada in one year was $2,657,084,000. The premiums paid were $91,005,093. What was the average premium per $1,000.

10. An insurance company took a risk at $2,500 for an annual premium of $76.50. Find the rate per $1,000; the rate per cent.

11. $360 was the yearly premium paid on a policy at the rate of $48 per $1,000. What was the face of the policy?

Health and Accident Insurance

1. James Farley belongs to a lodge that has a sick-benefit plan for its members. For 15 years he has paid $2.50 a week into the fund. Last year a long illness kept him from his work, at which he earned $160 a month. While he was ill during 15 weeks, he drew from the lodge 60% of his salary, the amount of his doctor's bill, $112, and a hospital bill for 28 days at $4 a day.

(*a*) How much money has he paid into the fund?

(*b*) How much money did he derive from the fund, allowing a month as four weeks?

(*c*) How much money did he lose on account of illness?

NOTE: Accident policies can be purchased from insurance companies.

2. The Workmen's Compensation Act. Workmen in British Columbia pay 1¢ a day or part day worked to the Compensation Board, and this money goes into a Medical Aid Fund. The amount contributed by workmen is about two-fifths the cost of this service, the balance being paid by employers. In case of injury while working, men receive time-loss while disabled, amounting to 62½% of earnings.

John Gibson fell from a new building on which he was working as a carpenter at the union wage of 90¢ an hour for a 40-hour week. He was laid off work for 2½ weeks by the accident.

(*a*) How much compensation did he receive?

(*b*) How much of his compensation was supplied by the employers of the province?

Class Activity: Gather information about the Dominion Old Age Pension Act and discuss it.

Taxation

In Canada we have three different forms of government: dominion, provincial, and municipal. Each has to find a large sum of money to spend for the welfare of the people. There are two kinds of taxation, direct and indirect. Direct taxation is a tax on income or land, while indirect taxation is a tax on what one buys.

The Dominion Government expends money on soldiers' pensions, war debts, defence, provincial grants, old-age pensions, relief, and the many departments of government. Its main sources of revenue are a sales tax, income tax, duties on imported goods, and excise duty on some goods made in Canada.

The Provincial Government maintains roads, assists schools, administers law courts, gives grants to municipalities, and keeps up prisons and asylums as well as sharing in relief costs and mothers' pensions. It receives revenue from an income tax, liquor profits, taxes on theatres, horse-racing, gasoline tax, motor licenses, etc.

The main source of revenue for the municipality is a direct tax on land.

How much is the Dominion Sales Tax? How is it collected? Discuss this tax with the business firms in your locality.

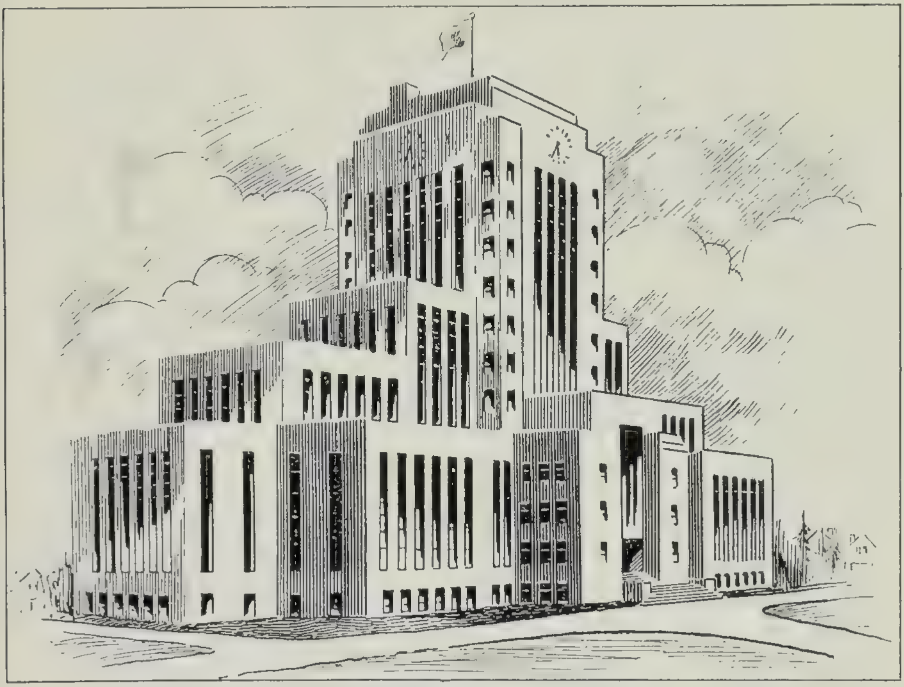
Class Activity—Provincial Income Tax

Procure a provincial income-tax sheet and study the scale of taxation for different salaries. Every one earning money from wages or salary has 1% deducted from his earnings by his employer, who forwards it to the Provincial Government. Those whose pay is in the lower brackets may have all or part of their 1% returned by the Government.

1. How much extra income tax will a man have to pay who is married, has no children, and earns $300 a month?

2. How much of his income tax will be refunded to a married man with two children, ages 8 and 10, if his salary is $125 a month?

3. A single woman earns $24 a week, and has to support her mother. How much of the 1% tax will she receive back, or how much additional must she pay?



VANCOUVER CITY HALL

Civic Budgets

All the provinces contain smaller areas which are responsible for carrying on their own local affairs. Regardless of size, these are called *municipalities,* but there are different kinds of municipalities, depending upon the population of the area.

Manitoba, Alberta, and Saskatchewan have cities, towns, villages, and rural municipalities. British Columbia has cities, villages, and district municipalities.

Each municipality must secure money to meet expenses. An estimate of revenue and expenditure, called a *budget,* is made at the beginning of each year.

Explanation of Terms

1. *Tax sale lands.* When a landowner is unable to pay his taxes, the municipality has the power to sell the property in order to obtain the amount of the taxes.

2. *Departmental revenue.* Various departments, such as police and sanitation, collect money for various services.

3. *Trade licenses.* Any one who carries on a business is required to pay an annual fee to the municipality.

4. *Road tax.* Any one who does not pay taxes on real property may be required to pay a road tax, usually not more than $2 a year.

5. *Poll tax.* Any male resident of a municipality who does not pay other taxes may be required to pay a poll tax, usually $5 a year.

6. *Government grant for schools.* The provincial Government contributes annually to the cost of education.

7. *Other Government grants.* Some provincial governments divide part of the revenue from various sources among the municipalities. These sources are: *motor licenses,* profits from the *sale of liquor,* and *pari-mutuel,* or horse-racing, taxes.

8. *Public utilities.* Many municipalities operate their own electric power, water, or gas systems. These utilities produce revenue.

Municipalities often find it necessary to borrow money. This is usually done by issuing *bonds.* Later on we shall study bonds more fully, but in the meantime it is enough to know that these bonds are promises to pay *certain sums at certain times,* together with interest at stated intervals.

The combined receipts of the municipalities in a western province for the year 1937 were as follows:

Land-taxes	$18,065,047.00
Trade licenses	728,210.00
Road taxes	27,991.00
Poll taxes	167,392.00
Dog taxes	45,733.00
Fines	210,052.00
Government grants for schools	1,729,498.00
Government grants from liquor profits	834,126.00
Government grants from motor licenses	570,007.00
Government grants from *pari-mutuel*	160,555.00
Receipts from public utilities	4,045,570.00
Sundry receipts	2,451,817.00

(*a*) What is the total amount of the receipts?

(*b*) What is the main source of receipts?

(*c*) What percentage of the total receipts is obtained from this main source?

(*d*) What is the total amount received from Government grants?

(*e*) What percentage of the total receipts is received from the Government?

(*f*) What percentage of the total receipts is obtained from licenses and fines? (Do not include the Government grants from motor licenses.)

(*g*) Arrange the following items in order of the receipts obtained from them, putting the largest first: sundry receipts, Government grants, fines and licenses, land taxes, receipts from public utilities.

Project:

Get a copy of the Provincial Year Book, and make a list of the six main sources of revenue and the six main items of expenditure.

Taxes on Real Property

(1) GENERAL TAXES

The council of the village of Farwest meets to determine its rate of taxation or *mill rate* for the current year. After examining the village expenses for the past few years, it decides that $1,444 must be obtained from taxation on *real property,* that is, on *land and improvements.*

The *assessor* has estimated or *assessed* the value of all the land and of all the improvements, that is, of buildings, fences, and anything that increases the value of the property. He found the value of land to be $46,000, and the value of the improvements to be $50,000. The total *assessed valuation* therefore was $96,000.

In past years the village has found that the cost of collecting taxes has come to 5% of the amount collected. The city treasurer therefore says:

'We want 95% of the amount collected to be $1,444'.

100% of the amount collected must be $\frac{100}{95}$ of $1,444, or $1,520.

The *total amount* of tax is called the *tax levied.*

The *net amount* available after expenses are taken out is called the *tax raised.*

Taxes are levied on 100% of the value of land, but according to law, improvements may not be taxed for more than 75% of their assessed value.

Farwest taxes 60% of the assessed value of improvements. The amount on which tax is levied is found as follows:

100% of assessed value of land	$46,000
60% of assessed value of improvements	30,000
Valuation on which tax is based	$76,000

The tax on $76,000 =$1,520.

The tax on $\$1=\$\frac{1{,}520}{76{,}000}=\$.020=2\%$ of $\$1=20$ mills.

The tax rate is therefore 20 *mills on the dollar.*

Note: 10 mills=1 cent; 1000 mills=$1. The per cent on a dollar multiplied by 10 gives mills.

If any property owner in Farwest thinks that too high a value has been placed on his property, he complains to the *Court of Revision,* which considers his case and makes any necessary adjustment.

The *city collector* sends a statement to each property owner, giving the amount of tax he must pay.

(2) SPECIAL TAXES

The statement sent to each property owner usually includes the amount of certain other taxes called *special taxes,* which are not included in the mill rate.

These special taxes are charges made which are closely concerned with the property, and they consist of such items as Boulevard Tax, Sewer Rental, etc.

These special taxes are not included in the mill rate because they are not *general expenses.* Some properties have better boulevards, lights, water system, or roads than others, and, therefore, must pay more.

Exercises (*oral*):

Call the answer that should go in each blank:

Valuation	*Rate*	*Tax*	*Valuation*	*Rate*	*Tax*
1. $5,000	10 mills	?	7. $75,000	?	$300
2. 10,000	30 mills	?	8. 80,000	?	2,000
3. 23,000	20 mills	?	9. ?	10 mills	30
4. 600	25 mills	?	10. ?	20 mills	200
5. 1,000	?	10	11. ?	25 mills	150
6. 8,000	?	160	12. ?	30 mills	270

Exercises (*written*):

1. The tax rate in Mildford is 27½ mills. What is the tax on properties the taxable values of which are: (*a*) $4,567.80; (*b*) $1,253.90; (*c*) $9,263.56; (*d*) $52,827?

2. The town of Holtum wishes to levy a tax of $52,827. The assessed valuation of property in the town is $1,320,675. What mill rate will be struck?

3. A municipality wishes to *raise a* tax of $350,000. If the cost of collecting the tax is 4%, what tax must be *levied*?

4. To meet all expenses for the year, the town of Amity requires $83,245.70. From various sources, such as Government grants, poll tax, and so on, the estimated revenue is $26,972. What will the mill rate be on a property assessment of $2,679,700?

5. Mr. Rain pays a tax of $118.90 on land assessed at $1,460 and on improvements assessed at $2,400. If the improvements are taxed on 60% of their assessed value, what is the mill rate?

6. The town of Owenville has to raise by taxation on property $1,523.77. If the assessed value of the property is $72,560.30, what mill rate must be struck?

7. What is the taxable value of property in the town of Arnford, if it levies a tax of $8,575 and uses a mill rate of 12½ mills?

*8. The town of Gretna levies a tax of $10,155. Of this amount, $3,855 is on land and the remainder is on improvements which are taxed at 35% of their assessed value. If the tax rate is 30 mills, what is the total value of the property in the town?

Customs and Duties

A large part of the revenue of the Dominion Government is secured by collecting a tax on goods brought into the country. This is called a *tax on imports,* or a *duty.*

This tax is imposed on goods entering the country for the following reasons:

(*a*) To provide revenue for the Dominion Government.

(*b*) To bring the price of cheap foreign goods above the price at which the same goods are manufactured in Canada. This helps to keep our factories operating.

(*c*) To give the Canadian manufacturer his home market, and so allow him to produce goods in sufficiently large quantities to bring his price down to the point where he can sell in foreign markets.

The law requires that all goods entering Canada shall be landed at certain places where *customs houses* are maintained. These places are called *ports of entry,* and the duties charged are called *customs duties.*

(*a*) *Ad valorem.* An *ad valorem* duty is a tax on the value of the goods. It is calculated as a per cent fraction of the imported article.

(*b*) *Specific.* A *specific duty* is calculated on the number or weight or volume of the imported goods. For example: an import duty of $.95 per pound on tobacco is a *specific* duty.

The set of rules, telling the rate of duty on various kinds of goods is called the *tariff.* The rules change very frequently.

To encourage trade within the Empire, Canada charges a lower rate on goods imported from Britain. The tariff covering these goods is called the British Preferential Tariff.

Problems without Numbers

1. Explain the difference between an *ad valorem* duty and a *specific* duty.

2. Why are duties charged on goods entering a country?

3. Who really pays the duty charges?

4. Are some goods shipped into a country duty free? Why?

5. Why does Canada charge a lower rate of duty on goods coming from the British Commonwealth of Nations?

Finding the Amount of Duty

Exercises (oral):

Find the duty on the following:

Article	*Value*	*Quantity*	*Ad valorem Duty*	*Specific Duty*
1. Silk	$2,000		25%	
2. Paint	12,000		12½%	
3. Wheat		15,000 bu.		2¢ per bu.
4. Oil		50,000 gal.		4¢ per gal.
5. Soap		40,000 lb.		$1.10 per cwt.
6. Rubber	75,000		33⅓%	
7. Tobacco		1,000 lb.		$1.10 per lb.
8. Gloves	1,500	1,500 pr.	20%	
9. Suits	500	25 suits	30%	
10. Wine	2,400	600 gal.	40%	5¢ per gal.

Exercises (written):

1. Find the specific duty on 4 tons of sugar at 1⅜¢ per lb.

2. A Calgary merchant imported 8,400 lb. of confectionery from London, England, at 7½¢ per lb. The specific duty was ½¢ per lb., and the *ad valorem* duty 35%. Find the total amount of duty on the shipment.

3. The duty on sweater coats was 45¢ per lb., and 50% *ad valorem.* Find the duty on a sweater coat weighing 1¼ lb. and valued at $3.80.

4. What is the duty on 760 lb. of sulphuric acid, at ½¢ per lb.?

5. How much duty would be paid on 7,635 bushels of wheat, valued at $4,500, at 12¢ per bushel?

Exercises (*oral*):

1. On a shipment of carpets worth $400, the duty was $100. Find the rate per cent of duty.

2. If $36 duty was paid on 1,800 lb. of cherries, what was the rate charged? What kind of duty was this?

3. A Winnipeg musician imported from Vienna a violin valued at $900. He had to pay $300 for duty charges. Find the rate per cent charged. What kind of duty was this?

4. The Hudson's Bay Company imported 300 yds. of linoleum, valued at $1.50 per yd. The duty paid was $45. Find the rate.

5. Find the rate of the following:

	Value	*Quantity*	*Duty*	*Rate*
(*a*)	$75		$25	——
(*b*)	$6		$1	——
(*c*)		200 tons	$200	——
(*d*)		80 gallons	$32	——
(*e*)	$2,500		$500	——
(*f*)	$360		$60	——
(*g*)		1,800 yds.	$540	——
(*h*)	$6,000		$600	——
(*i*)	90¢		30¢	——
(*j*)	$14		$2	——

Review Problems on Duty

1. Sewing-silk is subject to a duty of \$1.50 per lb., and 40%. Find the duty on 100 lb. of silk valued at \$960.

2. A dealer buys a binder for \$160, paying 15% duty and \$36 freight. For how much must he sell it to gain 40% on the total cost?

3. A certain shipment of tea cost \$35 per cwt., and the rate of duty was 20%. A man buys 5 tons and pays freight of \$50. Find his gain if he sells the tea at 65¢ per lb.

4. W. Morris imported 600 boxes of cigars from Havana. Each box contained 50 cigars, and weighed ½ lb. The price per box was \$4.50. If the specific duty on cigars is \$1.75 per lb., and the *ad valorem* duty is 25%, find the amount of duty paid. What was the total cost of the goods?

5. If goods invoiced at \$875 cost \$1,265 delivered at the warehouse, the insurance and freight amounting to \$83.75 being included, find the rate per cent of duty.

6. An art dealer imported for me a collection of photographs valued at \$5.60. He paid 40¢ postage, 25% *ad valorem* duty, and made a profit of 20% on the total cost. What did he charge me for the photographs?

7. The duty on 150 gallons of preserved fruit amounted to \$414. The fruit was valued at \$1.20 per gallon. If the specific duty was \$2.40 per gallon, find the amount of *ad valorem* duty.

8. The duty on 40 gallons of wine imported from France was \$159. The duty rates on wines are \$2.40 per gallon and 30% on the value. Find the value of the wine.

9. The Hudson's Bay Company imported some crystal-glass tableware, on which they paid \$1,300 in duty. The rate of duty on this class of goods was 32½% *ad valorem.* Find the value of the glassware .

10. A jeweller imported 150 clocks from Switzerland, on which he paid a duty of $281.25. If the rate of duty was 25%, what was the invoice price of each clock?

The Dominion Income Tax

You have learned that cities, towns, and municipalities carry on their work by collecting moneys in various ways.

The Dominion Government also must secure money to carry on its work. This money is secured in many different ways, but one very important source of revenue is the Dominion Income Tax, because approximately 18% of the total revenue is collected by this means.

Before April 30 of each year every citizen is required by law to make a statement of his income for the previous year. The statement is called an *income-tax return.*

After deducting a certain amount, called an *Exemption* with which to meet the necessities of life, the Government collects a certain percentage of the *net taxable income.*

A large net taxable income is taxed a greater percentage than a small net taxable income.

Dependents are children under 21 years of age, or older persons who are physically or mentally unable to earn their own living .

Exemptions for the year 1938 were as follows:

(1) Single person	$1,000
(2) Married person	$2,000
(3) Widow or widower with 1 dependent	$2,000
(4) Each dependent (except in 3)	$400

The rates on net taxable income for 1938 were:

Net Taxable Income	*Tax Payable on Even* $1,000's	*Rates Applicable on Excess over even* $1,000's
Under $1,000	——	3%
1,000	$30	4%
2,000	70	5%
3,000	120	6%
4,000	180	7%
5,000	250	8%
6,000	330	9%
7,000	420	10%
8,000	520	11%
9,000	630	12%
10,000	750	13%

Example. What tax is payable by a married man with two children, if his annual income is $6055?

Exemption for married man is $2,000.
Exemption for 2 dependents is 800.
Total exemption is $2,800.
Net taxable income=$6,055—$2,800=$3,255.

First find the tax on $3,000, and then the tax on $255.
Tax on $3,000=$120.
Rate on excess over $3,000 is 6%.
Tax on $255 at 6%=$15.30.
Total tax=$120+$15.30=$135.30.

Exercises (*oral*):

1. What exemptions will be allowed in the following cases:
   (*a*) A single man.
   (*b*) Married man, no children.
   (*c*) Widow, 3 children.
   (*d*) Married, 6 children.
   (*e*) Single man, 2 dependents.
   (*f*) Married man, 3 children.
   (*g*) Widower with 1 son, and supports aged mother.

2. What is the tax on a net taxable income of:
   (*a*) $5,000 (*b*) $4,000 (*c*) $6,000
   (*d*) $9,000 (*e*) $7,000 (*f*) $10,000

3. What is the rate of tax:
   (*a*) On the $450 of a net taxable income of $3,450?
   (*b*) On the $263 of a net taxable income of $1,263?
   (*c*) On the $78 of a net taxable income of $5,078?
   (*d*) On the $989 of a net taxable income of $9,989?

Exercises (*written*):

Find the tax payable by the following:

1. Mr. Holmes, earning $3,100 a year, is married and has 1 child.

2. John Brown earns $2,800 a year. He is single, but supports his invalid mother.

3. Mrs. Donald, a widow with two children, has an income of $300.

Compute the income tax for the following cases:

Add 20% extra for *War Measures Act*.

1. Henry Yates is a single man with no dependents, earning $4,000 per annum.

2. Mr. Post is a married man with 2 children. His income is $2,400 a year.

3. Mr. Thompson earns $408 a month. He is married and has 5 children.

4. Mr. Terry earns $375 a month. He is a widower with 3 children.

5. Mr. Roberts is married, but has no children. He earns $275 a month.

6. Mr. Seely, a single man, supports his mother and father, neither of whom are able to work. His income amounts to $520 a month.

Simple Interest Extended

You will recall from your work in Grade VIII that:

(1) Money paid for the use of money is *interest.*

(2) The sum of money loaned or borrowed is the *principal.*

(3) The interest for 1 year expressed as a per cent fraction of the principal is the *rate of interest.*

(4) The sum of principal and interest is the *amount.*

Finding Exact Interest

In working all interest problems for any number of days, the denominator of the fraction of a year is *always* 365 and the numerator is the number of days.

Exercises:

1. Find the interest on $200 at 5% for:

(*a*) 73 days
(*b*) 146 days
(*c*) 292 days
(*d*) 219 days
(*e*) 95 days
(*f*) 260 days

2. Find the number of days between:

(*a*) March 3 and June 16
(*b*) April 10 and July 6
(*c*) Jan. 24 and May 4
(*d*) Oct. 19 and Dec. 30
(*e*) June 25 and Sept. 10
(*f*) Aug. 2 and Nov. 21

3. Find the interest on $500 from March 11 to August 15 at 6% per annum.

4. What is the interest on $2,000 at 8% per annum from May 7 to December 20?

5. A man borrowed $250 on March 2 at 7% . He paid the principal and interest on May 14. How much did he have to pay?

6. A city borrowed, at 5¼% on August 7, enough money to build 7.6 miles of street at $17,500 per mile. Find the amount owing as interest on December 31 .

7. A person borrowed $870 on June 3 at 7%. He paid the principal and interest on February 8 of the following year. How much did he have to pay?

8. Mr. Jones borrowed $2,500 on January 6, and promised to pay it back on October 4 of the same year, with interest at the rate of 4½% per year. What amount was paid back?

9. On June 29, 1933, a man borrowed $50, with interest at the rate of 6½% a year, and repaid it on April 30, 1934. What amount was necessary to pay his obligation?

10. A man lived on the income from $50,000. If a quarter of it was invested at 7% per year, and the rest at 5% per year, find his yearly income.

11. Find the interest on $7,750 from February 17 to July 9 of the same year at 5½%.

Using Tables for Finding Simple Interest

Banks and other business firms find interest by using interest tables like the one shown below:

INTEREST TABLE, 6%

Days	$100	$200	$300	$400	$500	$600	$700	$800	$900	$1,000
1	.016	.033	.049	.066	.082	.099	.115	.132	.148	.164
2	.033	.066	.099	.132	.164	.197	.230	.263	.296	.329
3	.049	.099	.148	.197	.249	.296	.345	.395	.444	.493
4	.066	.132	.197	.263	.329	.395	.460	.526	.592	.658
5	.082	.164	.247	.329	.411	.493	.575	.658	.740	.822
6	.099	.197	.296	.395	.493	.592	.690	.789	.888	.986
7	.115	.230	.345	.460	.575	.690	.805	.921	1.036	1.151
8	.132	.263	.395	.526	.658	.789	.921	1.052	1.184	1.315
9	.148	.296	.444	.592	.740	.888	1.036	1.184	1.332	1.479
10	.164	.329	.493	.658	.822	.986	1.151	1.315	1.479	1.644
20	.329	.658	.986	1.315	1.644	1.973	2.301	2.630	2.959	3.288
30	.493	.986	1.479	1.973	2.466	2.959	3.452	3.945	4.438	4.932
40	.658	1.315	1.973	2.630	3.288	3.945	4.603	5.260	5.918	6.575
60	.986	1.973	2.959	3.945	4.932	5.918	6.904	7.890	8.877	9.863
70	1.151	2.301	3.452	4.603	5.753	6.904	8.055	9.205	10.356	11.507
80	1.315	2.630	3.945	5.260	6.575	7.890	9.205	10.521	11.836	13.151
90	1.497	2.959	4.438	5.918	7.397	8.877	10.356	11.836	13.315	14.795

In lending money, Canadian banks consider a year as 365 days.

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Example 1. Find the interest on $1,750 for 40 days at 6%.

$1,750=$1,000+$700+$50.

Find 40 days in the left-hand column of the table. Lay your ruler on the line under 40, and follow horizontally to the number below $1,000. It is 6.575. Write it down. Then follow back to the number under $700. It is 4.603. Write it under the 6.575. Now follow back to the number under the $100. It is .658. Take half of .658. It is .329. Write it under the 4.603. Add:
6.575+4.603+.329=11.507=11.51.

$6.575
4.603
.329
$11.507 or
$11.51 nearly .

Using Tables for finding Simple Interest

Example 2. Find the interest on $1,830 for 74 days at 6%.

Since $1,830=$1,000+$800+$30, find the interest on these amounts for 70 days, then for 4 days.

Int. on $1,000 for 70 days	$11.507	
Int. on $800 for 70 days	9.205	
Int. on $30 for 70 days	.34	($30=$\frac{1}{10}$ of $300)
Int. on $1,000 for 4 days	.658	
Int. on $800 for 4 days	.526	
Int. on $30 for 4 days	.02	($30=$\frac{1}{10}$ of $300)
	$22.256	

Using the table, find the interest at 6% on:

3. $820 for 30 days
4. $1,240 for 50 days
5. $2,160 for 90 days
6. $8,240 for 70 days
7. $7,237 for 10 days
8. $330 for 34 days
9. $1,360 for 27 days
10. $1,500 for 12 days
11. $950 for 170 days
12. $1,540 for 133 days

Drill Work in Percentage

Copy and complete the following:

1. 1 is —— % of 10
2. 1 is ——% of 4
3. 33⅓% of —— is 6
4. 4 is ——% of 1
5. 25 is ——% of 75
6. 16 is 50% of ——
7. —— is 66⅔% of 18
8. 8 is 12½% of ——
9. 16 is 80% of ——
10. 6 is ——% of 9
11. 6 is ——% of 16
12. 3 is ——% of 9
13. 9 is ——% of 6
14. —— is 62½% of 24
15. 15 is 20% of ——
16. 12½% of —— is 7
17. 87½% of —— is 21
18. 125 is 100% of ——
19. —— is 25% of 120
20. —— is 75% of 60
21. 1.5 is ——% if 150
22. 75 is ——% of 125
23. 15 is ——% of 20
24. 50% of 50 is ——
25. 1.5 is ——% of 1
26. 22 is 33⅓% of ——
27. 2.2 is ——% of 3.3
28. 7 is 8⅓% of ——
29. 125 is ——% of 200
30. 3 is 30% of ——

Banks and Banking

In Canada most people do business through a bank. In return for the use of its depositor's money the bank offers several important services, such as the following:

(1) It provides a safe place to deposit money.

(2) It permits depositors to pay bills by cheque.

(3) It cashes cheques on other banks.

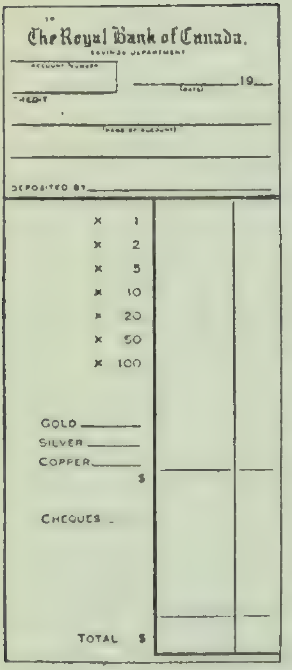
(4) It pays interest on savings accounts.

(5) It issues travellers' cheques and letters of credit.

(6) It issues bank-notes.

Canadian banks have two types of accounts: *current* and *savings* accounts. The current account is usually kept for business purposes, but is useful to any one wishing to pay bills by cheque. It provides a record of business transacted and receipts for bills paid. The savings account is an interest-bearing account, and is used for small savings which are not immediately needed.

Opening a Bank Account



TO

The Royal Bank of Canada.

SAVINGS DEPARTMENT

ACCOUNT NUMBER

(DATE) 19

CREDIT

(NAME OF ACCOUNT)

DEPOSITED BY

× 1

× 2

× 5

× 10

× 20

× 50

× 100

GOLD

SILVER

COPPER

$

CHEQUES

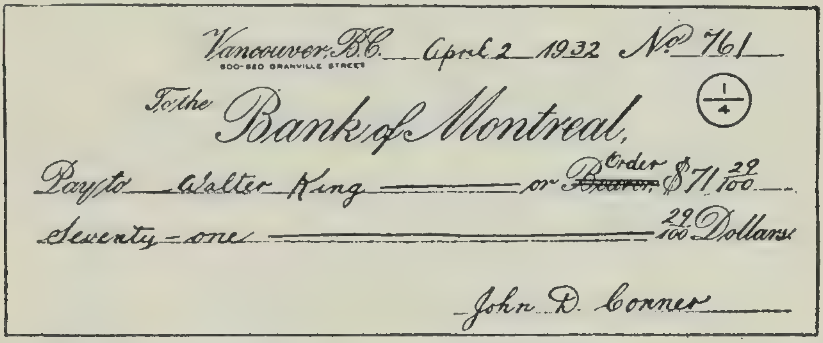
TOTAL $

Suppose that you are ready to open an account; you will find on a desk in the bank a printed *deposit slip,* like the one shown at the left of the page. After filling it out to show the money or cheques you wish to deposit, you sign it and hand it with the money to the *teller.* The teller will ask you your full name, occupation, and address, and will take a specimen of your signature. You will need to sign your name in the exact form in which you intend to sign cheques.

After making your deposit you will be asked to go to the ledger-keeper's wicket, where you will be given a bank book, in which is shown the amount of your deposit, and in which will be entered all future withdrawals or deposits.

Writing and Endorsing Cheques

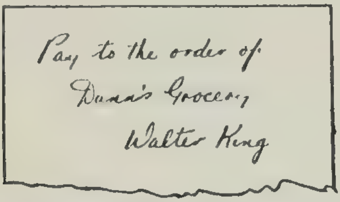
John D. Conner, of Vancouver, owes a bill of $71.29 to Walter King of Calgary. Here is Mr. Conner's cheque which he sent to Mr. King by mail:



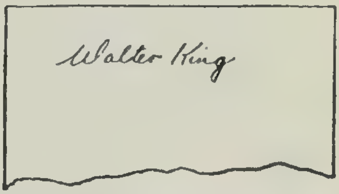
Vancouver, B.C. April 2 1932 No 761
600-620 GRANVILLE STREET
To the Bank of Montreal,
1/4
Pay to Walter King —— or ~~Bearer~~ Order $71 29/100
Seventy-one —— 29/100 Dollars
John D. Conner

In the cheque above, Mr. Conner is called the *maker,* and Mr. King the *payee.*

When Mr. King receives the cheque through the mail he takes it to his own bank, the Canadian Bank of Commerce, Calgary. On presenting it, he is asked to *endorse it in blank,* which he does by signing his name across the back, or he may use the cheque to pay his grocer, in which case he endorses it in *full,* thus:



Pay to the order of
Dunn's Grocery
Walter King



Walter King

in which case the Mr. Dunn who operates the grocery will turn it in to his bank, endorsing it in blank as he does so.

The bank will probably charge a small fee, called *exchange,* for cashing the cheque, because the bank has to return it to the place where it was made in order to obtain the money for it.

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VICTORIA, B. C.

In any case, the cheque will soon be forwarded to Mr. Conner's bank in Vancouver and charged against his account. At the end of the month the *cancelled cheque* will be sent to Mr. Conner as a receipt for his paid bill.

A cheque may be made payable to *bearer* instead of to *order.* Why is it not customary to make cheques payable to bearer?

A person who wishes to draw money from his own account may draw a cheque payable to 'self', or 'cash', or 'bearer'.

A record of all cheques issued should be kept on the *stub* of the cheque-book.

Practice in making Cheques

1. Why do banks have their cheque forms printed on *safety paper*?

2. Why is it not wise to cash a cheque for a stranger?

3. Give reasons why you should use pen and ink rather than a pencil in writing cheques.

4. Why do we write the amount of money on a cheque both in words and figures?

5. Give reasons why there should be no blank spaces left unfilled on a cheque. How would you fill up the spaces not covered by words and figures?

6. What is meant by *overdrawing* a bank account? Can you find the meaning of the letters: N.S.F.?

7. Why do we number cheques and stubs?

8. Write a cheque payable to L. E. Davis, with yourself as the maker.

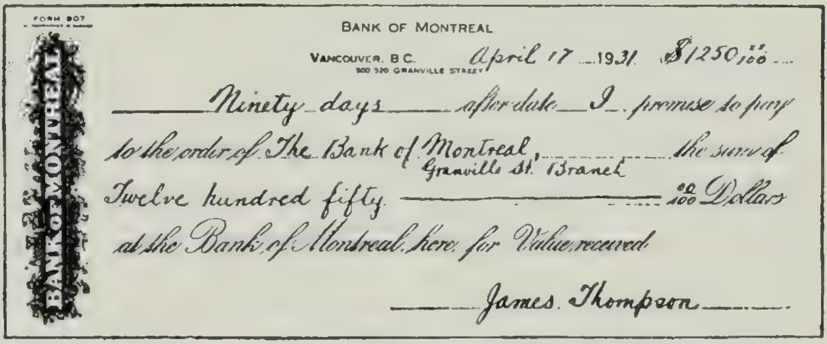
9. Write a cheque to pay your grocer an account of $36.45. Make a notation on the cheque to show that the grocer's account is now paid in full.

10. Make out a deposit slip to show a deposit of $32.65 in silver, $93 in bills, and three cheques totalling $172.90.

11. Draw a cheque in the name of David Clark on the Bank of Nova Scotia, to John Mills for $69.37. Endorse the cheque in full to William Black.

Bank Discount

You have learned that money may be borrowed from a bank by means of a *promissory note,* which is *a promise to pay a certain sum of money* in a *certain time,* sometimes with, and sometimes without, interest.



FORM 907

BANK OF MONTREAL

BANK OF MONTREAL

VANCOUVER, B.C. April 17 1931 $1250 $\frac{00}{100}$
300 320 GRANVILLE STREET

Ninety days after date I promise to pay
to the order of The Bank of Montreal, Granville St. Branch the sum of
Twelve hundred fifty $\frac{00}{100}$ Dollars
at the Bank of Montreal here, for Value received

James Thompson

What is the *face,* the *time,* and the *date of maturity* of the above note? What is the *face* of the note?

It is possible to collect the money promised by a note before the date of maturity by discounting it.

A note is *discounted* by taking from the face value of the note the amount of the simple interest at a stated rate. The *term of interest* is the number of days from the date of maturity, and is called the *term of discount.*

The *proceeds* of the note is the amount left after the discount has been subtracted from the face value of the note.

Remember, estimating the term or term of discount of a note from one date to another, the first date is not counted but the last one is.

Exercises without Number

1. What is the difference between the face and the face value of a note? Are they ever the same amount?

2. How do we find the proceeds of a note?

3. What is the difference between the term and the term of discount of a note? Are they ever the same?

4. Is the date of maturity taken as the date before or after the three days of grace are added?

Note that 3 days of grace are added to the time of a note.

Example: William Jones had a 90-day note for $730, dated March 2, with interest at 6%. He discounted it on April 10 at 7%. What were the proceeds?

Date of maturity is 93 days from March 2 = June 3
Interest = $\frac{93}{365}$ of $\frac{6}{100}$ of $730 = $11.16
Face value of note = $730 + $11.16 = $741.16
Term of discount April 10 to June 3 = 54 days
Discount = $\frac{54}{365}$ of $\frac{7}{100}$ of $741.16 = $7.68
Proceeds = $741.16 — $7.68 = $733.48

Exercises (oral):

1. What is the term of a 60-day note; a 90-day note; a 30-day note; a 120-day note?

2. What is the term of discount of a 60-day note which is discounted 40 days before its date of maturity?

3. What is the term of discount of a 90-day note discounted the same day as it was drawn; 12 days after it was drawn; 33 days after it was drawn?

4. What is the face value of a note if the face is $500 and the interest is $8.60; if the face is $900 and the interest is $11.80?

5. The bank discount on a note is $7.24. If the face value is $427.30, what are the proceeds?

6. The face of a note is $300, the interest is $10, and the bank discount is $6. What are the proceeds?

Exercises (written):

1. Find the date of maturity, bank discount, and proceeds of each of the following notes, discounted the day they are drawn:

Date	*Face of Note*	*Time*	*Rate*
June 14	$400	60 da.	8%
Mar. 3	$690	90 da.	6%
Aug. 24	$960	120 da.	7%
May 7	$5,600	15 da.	8%
April 13	$4,500	30 da.	6½%
May 23	$1,260	2 mo.	7%
July 9	$2,450	3 mo.	7½%
Sept. 15	$9,000	75 da.	8%

Ratio

If George is 12 years old and Peter is 6, George is $\frac{12}{6}$ or 2 times as old as Peter. We compare the ages of the two boys by seeing how many times one age is contained in the other. We call this finding the ratio of their ages.

By *ratio* we mean the relation between two quantities *of the same kind* as expressed by the fraction obtained by dividing the one number by the other.

The ratio of a to b is sometimes written $a:b$, but more usually $\frac{a}{b}$.

The ratio of 3 inches to 1 foot is $3:12$ or $\frac{3}{12}$, which equals $\frac{1}{4}$. The ratio of 35 cents to \$10 is $\frac{35}{1000}$ or $\frac{7}{200}$.

Exercises:

Express the following ratios as fractions:

1. 10 in. to 15 in.
2. \$6 to \$12
3. 150 lb. to 80 lb.
4. \$8 to 25¢
5. 60 oz. to 20 lb.
6. $3\frac{1}{2}$ to 7
7. 1 in. to 1 ft.
8. 3 pt. to 2 qt.
9. 4 in. to 1 yd.
10. 2 mi. to 1,000 yds.
11. $\frac{1}{2}$ to $\frac{1}{4}$
12. $\frac{3}{8}$ to $\frac{3}{16}$

Exercises (written):

1. If the rent of a room for 12 days is \$15, how much would the rent be for 7 days at the same rate?

2. If 16 ft. of wire weighs $2\frac{1}{2}$ lb., how much will 20 ft. of the same wire weigh?

3. If, during a rainstorm, $5\frac{1}{4}$ tons of water fall on 3 acres, how many tons would fall on 7 acres?

4. If an electric current costs \$17.60 for 27 hours, how much would the current cost for 16 hours?

5. If a seeder plants 3 bushels of wheat in $1\frac{1}{2}$ hours, in how many hours would it plant 20 bushels?

6. If 6 dozen pencils are used by 40 children in 2 months, how many pencils would they require for 9 months?

7. A man received \$6.30 for 9 hours' work. How much does he receive for 15 hours' work at the same rate?

8. Mr. Ford raises 650 bushels of wheat on a 25-acre field. How many bushels could he raise, at that rate, on a 35-acre field?

9. At a certain time a post 8 ft. high casts a shadow 5 ft. long. What will be the height of a tree which, at the above-mentioned time, casts a shadow 35 ft. in length?

Sharing

1. Divide $12.60 between George and Alex in the proportion of 7 to 5. (George has $\frac{7}{12}$ and Alex $\frac{5}{12}$).

If George has 7 parts and Alex has 5, how many parts will they have altogether? Divide $12.60 into 12 parts. 7 of these parts belong to George and 5 to Alex.

2. A football team won 11 out of 16 games played. The number of games won is what ratio to the number played? Express this ratio as a decimal of three places. The number of games lost is what ratio to the number won?

3. Fred and Andrew made $2.40 by selling papers. If Fred sold 75 and Andrew 45, how should the profits be divided?

4. A chick-feed mixture consists of 3 parts cracked wheat and 5 parts cracked corn. How much of each would there be in 500 lb. of the mixture?

5. Three men invest in a business. Mr. A invests $400, Mr. B $500, and Mr. C $600. How should they divide a profit of $120?

6. A and B buy a business for $15,000, A furnishing $8,000 of the money, and B the rest. How should they divide:

   (*a*) A profit of $1,950?
   (*b*) A loss of $1,650?

7. Mr. Barnes failed in business, owing $4,000 to A, $5,000 to B, and $7,000 to C . How should his assets of $5,440 be divided?

8. A cement sidewalk costing $875 is built along one side of a block. The cost is charged to the property owners in proportion to the frontage of their properties, which are: A's lot, 120 ft.; B's lot, 100 ft.; C's lot, 80 ft.; and D's lot, 50 ft. How much of the cost will be charged to each?

9. An anti-freeze solution for a car radiator in winter consists of 2 parts alcohol to 3 parts of water. If the alcohol costs 90¢ a quart, find the cost of an anti-freeze solution for a radiator which holds $3\frac{3}{4}$ gallons.

***10.** A certain recipe for a cake calls for $2\frac{1}{2}$ cups of flour, $\frac{3}{4}$ cup of butter, and 4 eggs. With other ingredients, the cake is supposed to form a dessert for 12 persons. How much flour and butter, and how many eggs, would be required for a similar cake large enough to serve 50 persons?

***11.** In mixing concrete to build a wall, 5 sacks of cement were used to 12 cu. ft. of sand and 1 cu. yd. of gravel. If a sack of cement weighs 90 lb., how many pounds of cement would be needed for a cubic yard of sand to continue the wall with the same strength of concrete?

***12.** A man died, leaving $50,000 in life insurance. His wife was willed $20,000 of this money, and the remainder was divided between his son and daughter in the proportion of 3 parts to the son and 2 parts to the daughter. The man also left $20,000 in real estate. What would be the son's share of this property if all the estate was divided among the heirs in the same proportion?

Investments—Stocks and Shares

As money is saved in small amounts, it is usually placed in a savings bank to accumulate.

If saving is carried on carefully, there comes a time when there is enough money to *invest*. This means that the money is used directly by businesses instead of indirectly through a bank.

We are going to study investments and try to find out the difference between a good investment and a poor one.

Companies or Corporations. When men organize a business which requires more money than they possess, they usually organize as a *corporation or joint-stock company*.

The total amount of money required by the company is called the *capital,* which is divided into small equal portions called *shares of stock.* For example, if $10,000 is required it may be divided into 1,000 shares of $10 each. A share may have a stated *nominal* or *par value,* but nowadays it is usual for shares to have no stated par value.

Any person holding one or more shares is called a *shareholder* or *stockholder,* and thus becomes part owner of the business.

When stocks are bought or sold the transactions are carried on through *stock exchanges.* In Canada the three main exchanges are situated in Montreal, Toronto, and Vancouver.

Only certain men or firms that are *members* of each exchange are permitted to buy and sell in that exchange. Membership in each exchange is limited to a certain number of *seats.* The men who arrange the sales are called *brokers.* For their services they charge a fee called brokerage. The fees charged for Western Canada are shown below:

Stock Exchange – Commissions

On Stocks	selling				under	.02	$ .50	per M.
“	“	at	.02	and	“	.03	1.00	per M.
“	“	“	.03	“	“	.05	⅛¢	per Sh.
“	“	“	.05	“	“	.10	¼¢	“
“	“	“	.10	“	“	.30	½¢	“
“	“	“	.30	“	“	.75	$ .01	“
“	“	“	.75	“	“	1.00	.01½	“
“	“	“	1.00	“	“	2.00	2.00	per C.
“	“	“	2.00	“	“	3.00	3.00	“
“	“	“	3.00	“	“	4.00	4.00	“
“	“	“	4.00	“	“	5.00	5.00	“
“	“	“	5.00	“	“	10.00	7.50	“
“	“	“	10.00	“	“	20.00	15.00	“
“	“	“	20.00	“	“	30.00	20.00	“
“	“	“	30.00	“	“	50.00	25.00	“
“	“	“	50.00	“	upwards–Half of 1% of amount of money involved.			

If stock can be bought only by paying more than the par value, it is said to be at a *premium,* while if it can be bought for less than the par value it is at a *discount.* The price at which stock can be bought is called the *market value.*

Exercises (*oral*):

1. Market quotations are given in dollars per share. Below is part of a list which appeared in a newspaper.

Mining Shares	
Hollinger	14.75
Home Oil	1.18
Howey	.27
Hudson Bay	29.50
Jackson Manion	$.14\frac{1}{2}$
Kirkland Lake	1.14
Lakeshore	51.00
Little Long Lac	3.50
Mocassa	4.55
Manitoba and Eastern	$.01\frac{3}{4}$
McIntyre	43.75
Bralorne	9.50
Cariboo	2.52
Mining Corporation	2.00

Industrials	
Canadian Pacific	$7\frac{1}{8}$
Consolidated Smelters	59
Dominion Bridge	$32\frac{1}{2}$
Ford 'A'	$19\frac{5}{8}$
General Steel Wares	$7\frac{5}{8}$
Gypsum	$7\frac{1}{2}$
Goodyear Tire and Rubber	64
Hamilton Bridge	$8\frac{1}{2}$
Imperial Oil	17
Imperial Tobacco	$15\frac{3}{4}$
International Nickel	$49\frac{1}{4}$

What is the cost of 10 shares of each stock?

2. What is the cost of brokerage for (*a*) 1 share of each industrial, (*b*) 1,000 shares of each mining stock?

3. Lakeshore declares a dividend of $1.245 a share. What would be the total dividend on 100 shares?

Buying and Selling Stocks

Exercise:

Mr. James makes the following investments. What is the total cost including brokerage?

15 shares Baring Cement at	106.00
200 shares Pioneer at	2.95
76 shares Steel Manufacturing at	13.60

Investing in Bonds

When countries, provinces, municipalities, or corporations require money they may borrow it by issuing *bonds.*

A bond is a promise to pay a certain sum of money at the end of a stated period, together with interest at regular intervals. They are usually issued in amounts from $100 to $1,000.

The date on which the bond is to be paid is called the *date of maturity.* In the case of a city bond, all the property of the city is *security* that the debt will be paid.

While bonds are a safer form of investment than stocks, not all bonds are equally secure. People are willing to buy Government bonds at a very low rate of interest because they are the safest possible type of investment. The bonds of a strong organization usually bear interest of from 4 to 5 per cent; weaker corporations offer higher rates of interest.

The Interest on Bonds

The quotations on 10 bonds were as follows:

(1)	Dominion of Canada 1949, 3½%	97.50
(2)	Prov. of Quebec 4½%, 1963	109.75
(3)	B. C. Telephone 5%, 1950	103.75
(4)	Gatineau Power 5%, 1956	95.50
(5)	Australia 5%, 1957	96.87
(6)	General Steel Wares 6%, 1952	83.50
(7)	Simpson's Limited 6%, 1949	101.00
(8)	City of Edmonton 5½%, 1945	92.00
(9)	Sweden 5½%, 1954	100.50
(10)	Calgary Power 5%, 1960	98.25

Find the cost, including brokerage, of:

(1) 6 Sweden 5½'s of $1,000 each.
(2) 3 Dom. of Can. 3½'s of $500 each.
(3) 8 B. C. Telephone 5's of $500 each.
(4) 2 Simpson's 6's of $200 each.
(5) 12 City of Edmonton 5½'s of $500 each.
(6) 17 Calgary Power 5's of $750 each.
(7) 9 Australia 5's of $500 each.
(8) 13 Prov. of Quebec 4½'s of $1,000 each.
(9) 7 Gatineau Power 5's of $250 each.
(10) 56 General Steel Wares 6's of $1,000 each.

Mortgages

Very often a man borrows money by putting a *mortgage* on his property. Property is either *real* or *personal*. Real property is property such as land and buildings and is commonly called *real estate*. Personal property is property such as automobiles, jewels, and clothes.

For the most part mortgages are raised on real estate as security because, unlike personal property, it cannot be moved or destroyed, and is therefore better security.

When Mr. Smith borrows money by means of a mortgage from Mr. Jones, a *contract* is drawn up between Mr. Smith, who is called the *mortgagor,* and Mr. Jones, who is the *mortgagee.*

The contract states, among other things, the amount of the loan, the date of repayment, the rate of interest and when it is to be paid, and a description of the property.

If either principal or interest become due and is unpaid, the mortgagee may *foreclose;* that is, he may appeal to the law courts and take possession of the property.

It is usually necessary to have a mortgage drawn up by a lawyer. In order to be legal, it must be registered with the proper Government authorities.

A mortgagee does not usually lend money amounting to more than 60% of the value of the property. When one mortgage is well below the value of the property, it is possible to raise a second, third, or even a fourth mortgage. In the event of a foreclosure, the first mortgage gets first claim, and so on.

The rate of interest on money lent by mortgages is usually higher than on other good investments. The usual rate is from 6% to 7%. The rate is probably higher because it is not easy to convert a mortgage into cash immediately.

1. Walter Craig bought his home for $5,000, paying for the balance at 7% interest. The mortgage, with interest, was to be paid off like rent at $80 a month. The principal was reduced as follows:

	Principal	*Total Payment*	*Interest*	*Reduction of Principal*
First month	$2,000	$80	$11.67	$68.33
Second month	1,931.67	80	11.27	68.73
Third month	1,862.94	80	10.87	69.13
Fourth month	1,793.81	80	10.46	69.54
Fifth month	1,724.27	80	10.06	69.94

(*a*) On what amount is the interest computed for the first month? The second month? The third month?

(*b*) Continue the table for the first 8 months.

2. Roy Kent raised a mortgage of $4,000 on his house. He paid it back at $90 a month plus the interest at 6%. Find his monthly payments for the first 4 months.

3. George Holt bought a house for $9,000, paying $5,000 down and $120 a month, which included the interest at 6%. Show how the principal was reduced for the first 6 months.

4. Now suppose that the $120 in Question 3 was all used in reducing the principal and that the interest was paid in addition. Show the amounts paid for the first 6 months.

Project

Keep graphs showing the daily rise or fall in the prices of 3 mining stocks, 3 industrial stocks, and 3 bonds. See which group changes most over a month's time.

Compound Interest

Frequently money is left *on loan* or on deposit for long periods. The person who loans the money may not need to use the interest, so it is arranged that the interest shall be compounded or added to the principal every 3 months, 6 months, or 1 year. When so added, the interest becomes a part of a new principal, and interest is paid on it.

Interest on a sum of money which is composed of the original sum and added interest is called *compound interest.*

Banks, investment firms, and loan companies pay compound interest twice a year.

1. Find the compound interest on $2,500 for 2 years at 6% if the interest is added to the principal each year.

Original principal = $2,500
Interest for 1 year at 6%=6% of $2,500 = 150
Amount which becomes new principal = $2,650
Interest on $2,650 for 1 year at 6%=6% of $2,5650 = 159
Amount at the end of second year = $2,809
Compound interest=$2,809—$2,500=$309, or
Compound interest=$150+$159=$309

Notice that had the $150 interest been withdrawn at the end of the first year, and not added to the principal, the problem would have been one of simple interest.

2. Donald Fraser placed the sum of $600 for 1½ years with an investment company which paid 4% interest, compounded every 6 months. Find the compound interest.

Original deposit = $600.00
Interest for first 6 months=½ of 4% of $600 = 12.00
Amount on deposit for second 6 months = $612.00
Interest for second 6 months=½ of 4% of $612 = 12.24
Amount on deposit for third 6 months = $624.24
Interest for third 6 months=½ of 4% of $624 = 12.48
Amount on deposit at end of 1½-year period = $636.72
Compound interest=$636.72—$600=$36.72

NOTE: (*a*) That compound interest is always estimated on an even number of dollars; e.g., for the third period interest is paid on $624, not on $624.24. (Problem 2).

(*b*) Instead of finding interest for 6 months in this way, ½ of 4% of $600, we might have written 2% of $600. For a 3-month period (¼ year), we could write 1% of $600.

3. Find the compound interest, compounded semi-annually, at 8% on $700 for 1 year and 2 months.

Original deposit = $700.00
Interest for first 6 months=½ of 8% of $700 = 28.00
Amount on deposit for second 6 months = $728.00
Interest for second 6 months=½ of 8% of $728 = 29.12
Amount on deposit for last 2 months = $757.12
Interest for final 2 months=⅙ of 8% of $757 = 10.09
Amount on deposit at end of period = $767.21
Compound interest=$767.21—$700=$67.21

Exercises:

1. Find the compound interest on $200 at 6% for 1½ years if the interest is added every 6 months.

2. What is the amount and compound interest on $350 for 1 year at 4%, interest being compounded semi-annually?

3. On January 1, 1932, John Brown deposited $50 in a savings bank which allowed interest at 2%. The interest was added to the principal half-yearly. How much money had he in the bank on July 1, 1934?

4. If $126 was the simple interest on a certain sum of money for 3 years at 5%, what would be the compound interest on the same sum for the same time and at the same rate, interest being added every 6 months?

5. Find the compound interest on $675.75 for 2 years at 6% per annum, interest being compounded semi-annually.

USING COMPOUND INTEREST TABLES

In business, it is usual to estimate compound interest from tables. These tables enable you to write down quickly the compound interest of various sums of money. The following is a part of such tables. It gives the amount of $1.

No. of Years	1%	2%	3%	4%	5%
1	1.0100	1.0200	1.0300	1.0400	1.0500
2	1.0201	1.0404	1.0609	1.0816	1.1025
3	1.0303	1.0612	1.0927	1.1249	1.1576
4	1.0406	1.0824	1.1255	1.1699	1.2155
5	1.0510	1.1041	1.1593	1.2167	1.2763

Using the above table, work the exercises given below:

Exercises:

Find the amount at compound interest of:

1. $300 for 2 years at 5%.
2. $254 for 3 years at 2%.
3. $1,200 for 2 years at 3 %.
4. $230 for 1½ years at 4%, compounded half-yearly.
5. $150 for 2 years at 6%, compounded half-yearly.

Instalment Buying

Problems for Discussion:

1. Why should merchants ask 30% down on clothing bought on the instalment plan and only 5% down on electric refrigerators sold on time?

2. Why do you think that people have less money in the bank since instalment buying has come into practice?

3. Why are groceries not sold on the instalment plan?

4. Why are goods sometimes sold on the instalment plan for the same price as for cash?

5. How do you explain the fact that merchants lose only about 1% of the money contracted for under the instalment plan?

1. A teacher bought a new car for $1,080. He paid $250 cash and traded in a car at $175. He contracted to pay the balance in monthly payments over a period of 18 months. The registration of the new car was $10, the insurance for a year was $50.80, and the license was $20. These three items were added to his indebtedness to the company.

After he had added up the total amount due, the bookkeeper figured the interest on it for 18 months at 7% and added it to the principal. Remember that in instalment buying the purchaser pays interest on the total amount contracted for until the last payment is made. What was the amount of the monthly payments?

2. An exhaustive research has been made on living conditions in Canada and the results tabled in the House of Commons. The average Canadian family earns $1,500 a year and spends 18% for shelter, 30% for food, 9% for household operations as fuel and light, 17% for clothing, 5% for

medical care, 9% for transportation, 5% for amusement, 2% for personal care, 3% for church and charities, and 1% for education.

(*a*) Name any incidentals, such as newspapers, that are not included.

(*b*) What per cent is left?

(*c*) Does the average Canadian save any money?

(*d*) According to the scale, a family earning $5,000 a year would spend how much for food, clothing, and shelter?

(*e*) In the scale, where would you place taxes and the upkeep of schools?

(*f*) Consider the earnings of a family in the $5,000 class and explain how they would spend three times as much on each of the above items as a family in the $1,500 class would spend.

Review Exercises on Social Arithmetic

A. (*Oral*). Which one or more of the following are correct:

1. Goods sold by agents on commission save their firms: (*a*) taxes; (*b*) commission; (*c*) fire insurance; (*d*) cost of buildings; (*e*) delivery.

2. Goods sold at a discount are: (*a*) out-moded; (*b*) of poor quality; (*c*) unsaleable at regular prices; (*d*) marked down from an advanced price; (*e*) used as an advertisement; (*f*) put on sale at a reduction to raise cash.

3. Goods should sell more cheaply in a town than in a city because: (*a*) merchants pay smaller taxes in towns; (*b*) rents are cheaper in towns; (*c*) there is a smaller turnover with fewer customers in towns; (*d*) there is more competition in the city; (*e*) there is a smaller range of goods to select from in a town store.

4. A city should enter the business of automobile insurance because: (*a*) it could compel every citizen who is a car owner to insure; (*b*) it could reduce automobile hazards by proper police control; (*c*) it would be likely to make out a good case for insurance damages when one is involved in an accident; (*d*) this business could produce a large revenue.

5. Taxes on city land are collected by: (*a*) the Provincial Government; (*b*) the Municipal Government; (*c*) the Dominion Government.

6. The burden of taxation should be lifted from real estate, because: (*a*) real estate has dropped in price; (*b*) with excessive taxation it is cheaper to rent than to own property; (*c*) many large salaried men do not own real estate; (*d*) taxation makes it very difficult for a labouring man to own his home; (*e*) a new house depreciates rapidly.

7. When a young man first obtains regular employment, he should insure his life: (*a*) so that he need not worry about accidents; (*b*) in order to have money when he is too old to work; (*c*) so that those dependent on him later will be cared for after his death; (*d*) to establish credit for borrowing money.

B. (*Written*):

1. A house was listed for $2,800. If the commission was 5%, how much did the owner receive when the house was sold at a reduction of 10%?

2. A coat was bought for $30 after a discount of $16\frac{2}{3}$%. How much was the discount?

3. What single discount is equivalent to discounts of 5%, 10%, and 15%?

4. What per cent of the cost price is gained in selling shoes for $6.75 which cost $4.50?

5. A refrigerator sells for $225. If the cost to the retailer is $160 and his overhead expense is $30, what per cent of the selling price is profit?

6. A house is insured for 80% of its valuation of $5,000. The premium for 3 years is $48. At what rate per cent is the insurance?

7. A car insured on February 8 at a yearly premium of $36.80 was sold on May 1. If the insurance is adjusted, how much of the premium should the buyer pay?

8. William Jones was insured at the age of 25 with a policy of $1,000 at a yearly premium of $21.50. At the age of 40 he received a cash surrender value of $186. How much did it cost him per year for protection?

9. What mill rate must be levied on lands assessed for $920,000 to raise $23,000 for school purposes?

10. A house is assessed for $4,600 and the lot for $1,250. If the tax rate is 36 mills and the lot is taxed for full and the house for half assessment, how much will the taxes on the property be?

11. Find the *ad valorem* duty at 12½% on 600 cans of paint at $1.25 a can.

12. Find the specific duty on 2½ tons of tobacco at $1.10 a pound.

13. Find the Dominion income tax on a yearly salary of $3,360 for a married man without children. Salary in such a case is taxed 3% between $2,000 and $3,000 and 4% between $3,000 and $4,000.

14. Find the simple interest on $7,180 for 2½ years at 6¼%.

15. A 60-day note for $900 without interest, dated August 1, is discounted on August 15 at 8%. Find the net proceeds that the holder of the note will receive from the bank.

16. What is the ratio of $3\frac{1}{4}$ to $16\frac{1}{4}$?

17. If $21\frac{3}{4}$ lb. of wire have a length of 86 ft., how many feet will 16 lb. 10 oz. of the same wire reach?

18. Divide $1.26 between two boys so that their shares are as 2 to 5.

19. Find the compound interest on $400 for 2 years at 6%, compounded half-yearly.

20. A radio was sold for $120, with quarter cash and the balance in 10 monthly payments. Interest on the balance was reckoned on the full amount until the time when the last payment was made. What was the amount of each monthly payment?

Oral Review

1. In a problem on commission, what must be given in order to find the net proceeds?

2. When the per cent of discount and the selling price are given, how can you determine the marked price which was subject to discount?

3. If successive discounts are added together as per cents and taken as a single discount, will the result be larger or smaller than the discounts taken successively? Why?

4. Is the rate per cent larger when the profit is computed on the selling price rather than on the buying price?

5. When the premium is given both in per cent and dollars, how do you find the amount of the insurance policy?

6. If the assessed value and the amount of taxes are given, how is the rate found in mills?

7. What rule is followed in counting for interest the number of days between two dates?

8. When a note is discounted at the bank, what must be known in order to find the net proceeds?

9. If John is to have 3 shares and Fred 7 shares of \$2.60, what is the first step in the solution?

10. What is the method of finding compound interest on a sum of money for one year with the interest compounded half-yearly?

Review Test

1. How long will it take a sum of money to double itself at 6% simple interest?

2. The price of a car which cost \$780 is advanced 20% and discounted 20%. What is the sale price?

3. What profit is made in selling for \$90 and gaining 33⅓% on the cost price?

4. If 12 mills on the dollar raises a tax of \$18,000, under this assessment what will the taxes be on a property assessed for \$4,800?

5. Divide \$480 in the proportion of 7, 8, and 9.

6. If the principal is \$600, the time 3 years, and the interest \$108, find the rate per cent.

7. What single discount is equivalent to successive discounts of 5%, 10%, and 15%?

8. If the commission is \$80 and the rate of commission 4%, what are the net proceeds?

9. A note for \$400 is drawn on May 1 for 3 months at 6% interest. It is discounted at the bank on June 10 at 8%. Find the net proceeds.

10. Two radios are sold for \$63 each, one at a loss of 12½% and the other at a gain of 12½% on the cost price of each. How much is lost or gained on the double transaction?

Problem Methods

*1. The cost of painting the four walls of a room is found as follows:

$$\frac{10\times 2(18+16)}{9}\times 30¢$$

How high is the ceiling? How long is the room? How wide is the room? In what unit are the walls measured? What is the cost of painting the walls? What is the cost of painting the ceiling at the same rate?

*2. The following is the statement for finding the interest on a sum of money:

$$\$680\times .06\times \tfrac{67}{365}$$

What is the principal, rate, time, and interest?

*3. $\tfrac{21}{4}\times\tfrac{21}{4}\times\tfrac{22}{7}\times 16\times\tfrac{25}{4}$

The statement above gives the capacity of a cylindrical tank in gallons. What is the diameter of the tank? What is its height? In what unit is it measured? Find the capacity of the tank in gallons. State a rule for finding the capacity of a cylindrical tank in gallons?

*4. $18\times 16\times\tfrac{10}{12}\times 2\times\tfrac{\$26}{1000}$

The above fraction expresses the cost of 18 pieces of lumber. How long are the pieces? How wide and how thick are they? What is the amount of the bill?

*5. The following indicates the number of tons of water that will fall on a certain number of acres in a rainfall of 5 inches:

$$2\tfrac{1}{2}\times\tfrac{5}{12}\times 43560\times\tfrac{125}{2}\times\tfrac{1}{2000}$$

How many acres are considered? Why is 43560 used? Explain the use of $\tfrac{125}{2}$. What part of the statement represents cubic feet?

CHAPTER V—(OPTIONAL)

SOCIAL ASPECTS OF ARITHMETIC, PART TWO

NOTE TO TEACHER: This chapter may be used as enriched material for those pupils who complete class assignments in less than the allotted time. It should not be considered as essential work for all pupils.

The Cost of Owning a Home

Discussion

Compare owning a home and renting under the headings of depreciation, excessive taxation, desirability of changing location, pride in ownership, security, thrift, and independence.

1. John Griffin bought a house in the city for $4,200. He was able to pay one-third cash, and assumed a mortgage for $1,500 at 7% interest. The balance that he owed was arranged on an agreement of sale and was to be repaid in 6 annual payments with interest at 7%. The taxes on the property were 36 mills on the dollar on a valuation of $500 for the lot and $2,850 for the house. Improvements were assessed for 50% of the valuation. The water rate was $8.75 half-yearly. At the end of 5 years he sold his house for $3,500. During his occupancy, he had the house painted outside for $95 and the interior decorated for $180. Over the period of 5 years ,how much was the average cost annually to maintain his house property?

2. George Allan decided to give up renting and own his home. He bought a city lot for $650 and paid $80 for having it cleared and graded. He took out a building permit, and employed a good carpenter at the union wage of 90¢ an hour for a 40-hour week to build the house. An architect drew the plans for $115. The carpenter worked 110 days. The cement work for the foundation, 9 in. thick, an average of $3\frac{1}{2}$ ft. deep, was laid out 32 ft. by 40 ft. at 55¢ a cu. ft. (allow for double corners). The rough lumber amounted to 24 M

ft. at $22.50 per M. The contract for wiring was $125, and for plumbing $625. The cost of lathing and plastering was 55¢ per sq. yd. for 980 sq. yds. Stucco cost an additional $225. The finish lumber amounted to 4,500 ft. at $62.50 per M. Hardwood flooring cost 50¢ a sq. ft. for 860 sq. ft. Two additional carpenters helped with the finishing for 16 days each. Painting cost $175, and extras $320. Shingling and shingles, two chimneys, hardware, and furnace cost $610. What was the total cost of the house?

3. In 1938, under the Dominion Housing Act, the Government set aside $40,000,000 for financing the building of new homes in Canada. The owner, under the Act, must supply 20% of the cost in cash or a deed to a lot as part of the amount. Money for the balance is loaned through the banks at 5% interest. To repay a loan of $2,400 in 20 years, including interest, requires a monthly payment of $15.69. A home valued at $4,000 would cost $20.92 a month, and one for $5,000 would cost $26.15 a month. The Government also pays the taxes for the first year, 50% of the taxes for the second year, and 25% of the taxes for the third year. Building companies, under Government supervision, secure the loans and build the houses.

Robert Greer wishes to secure a home under the plan for $4,500. What will be his initial payment, and how much a month must he pay on the balance, calculated on the same basis as the above?

Cost of Owning and Operating an Automobile

Discussion

1. Which Government collects the revenue from automobile licenses?

2. How much tax on every gallon of gasoline does the Provincial Government collect?

3. What extra expense does the Government have because of the great number of automobiles?

4. What benefit are automobile licenses apart from the revenue they yield?

5. In what way do city police benefit the automobile insurance companies?

6. What are the advantages and disadvantages of owning and operating a car?

7. Why should a city not take over automobile insurance, make it compulsory, and gain a large revenue?

Problems

1. James Hardy, an automobile agent, sold eleven cars in one month and only two during another month. His commission was 5% of the price of the cars, less the trade-in value of the cars taken as part payment. What were his earnings for the two months, if the cars were priced at $1,180 each and four of the buyers traded in cars at an average price of $285? Why do you think the two months mentioned were not successive months?

2. Walter Borden bought a new car on March 1 for $1,080. His insurance for a year was $62.80 and his license $20. He bought a greasing contract for twelve times at $5 and had his car greased every 1,000 miles. The first two times the oil was changed at 500 miles and after that at 1,000 miles. A gallon of oil for a change cost 40¢ a quart. During the year he drove his car 12,800 miles. He estimated that for the year his automobile averaged 21 miles to a gallon of gasoline, for which he paid 27¢. How much did it cost him to operate his car during the year, if he considered the fact that the company valued his car at the end of the year at $900?

3. Counting both operating expenses and depreciation, how much did it cost Mr. Borden a day on the average to keep up his car?

4. During the second year, the cost of insurance, gasoline, and oil was the same as before. The mileage on the gauge advanced to 27,200 miles. But Mr. Borden had additional

expenses for the second year. He bought two new tires with tubes at $18.75 each, a new set of chains at $12.50, a rug at $6.75, eight spark plugs at 75¢ each, and a new muffler for $6.80. He had the engine tuned up for $4.80, and bought a driver's license for himself, his wife, and two daughters. One of the girls had a collision and damaged a car to the extent of $96, covered by insurance. Counting the depreciation at $200, find what it cost Mr. Borden to operate his car during the second year?

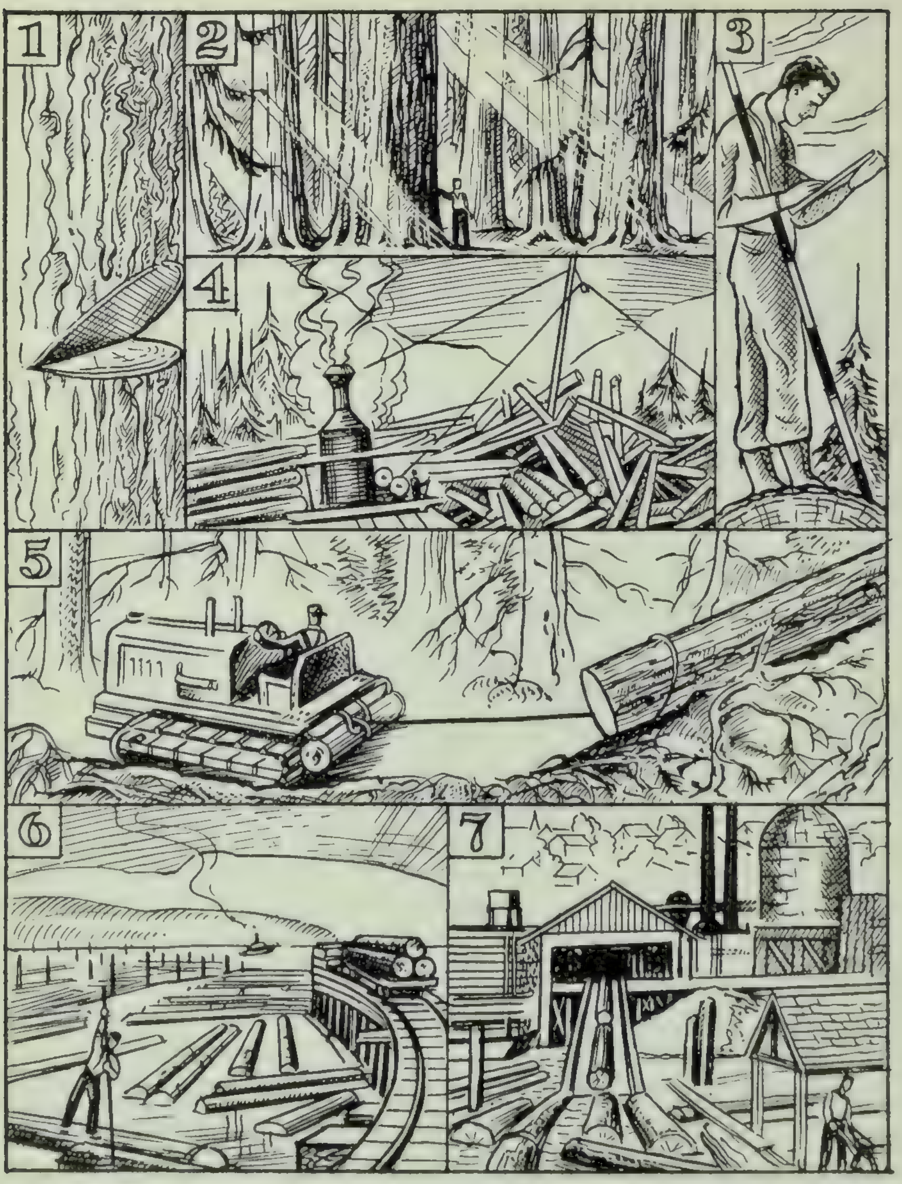
Research Work

1. How many automobiles are in operation in British Columbia?

2. When are new licenses issued?

3. Where are the license plates made?

4. Are bicycle licenses imperative for the whole province? How much do they cost?



(1) Cutting a Douglas fir. (2) A good stand of Douglas fir. (3) A log scaler. (4) The old 'high lead' system of logging. (5) Logging with a caterpillar tractor. (6) Making up the log booms. (7) The sawmill.

Lumbering

1. The Western Logging Company logged on Vancouver Island a timber limit of 10 sq. miles. They paid $1 per M. for western hemlock, $1.50 per M. for red cedar, and $2 per M. for Douglas fir. Find the cost of each of three trees which scaled the following:

(*a*)	Douglas fir	16,500 board feet
(*b*)	Red cedar	8,750 " "
(*c*)	Western hemlock	800 " "

2. The Government scaler charges 6¢ per M. for scaling or measuring the logs in the forest in order to find the number of board feet they contain. Half of the fee is paid by the loggers and half by the millowners. How many trees, averaging each 4,000 bd. ft. of lumber, must be scaled each day in order to earn $4.20 a day?

3. The scaler measures the diameter of the log at the small end in inches, subtracts $1\frac{1}{2}$ in., squares the number, multiplies by .7854, then by the length of the log in feet, divides by 12, and takes off $\frac{3}{11}$ of the product for loss in sawdust. The result is the estimated number of board feet in the log.

Example: A log 80 ft. long is 18 in. in diameter at the top. The scaler finds the number of board feet as follows:

$$18 \text{ in.} - 1\tfrac{1}{2} \text{ in.} = 16\tfrac{1}{2} \text{ in.}$$

$$\frac{8}{11} \times \frac{16\frac{1}{2} \times 16\frac{1}{2} \times .7854 \times 80}{12} = 1036.728 \text{ bd. ft.}$$

$$= 1.036 \text{ M.}$$

Scale the following logs:

(*a*) 120 ft. long and 24 in. in diameter at the top.

(*b*) 50 ft. long and 18 in. in diameter at the top.

*Can you explain how the above method is an application of $A = \pi r^2$ and 1 *cu. ft.* = 12 *bd. ft.*?

4. The Company estimated that fir logs cost $4.70 per M. to put in the water ready for transferring to the mill. If the royalty paid to the Government was 90¢ per M., how much per thousand was allowed for the cost of logging the fir? (*See* Problem 1).

5. They estimated that the cost of 'high-rigging' was $5,000 per set-up for each spar tree. How many feet of logs must be logged in a set-up to spread the cost of high-rigging at the rate of 75¢ per 1,000 ft.?

6. If the cost of towing a boom of logs to the mill, a distance of 180 miles, was 90¢ per M., what was the cost per mile of towing 240 M. ft. of logs?

***7.** The following prices were for logs delivered at the mill: fir, $10.50; cedar, $8; hemlock, $8.25. If the scaling exceeded the lumber sawn by 16%, what was the actual price to the millowners of a boom of fir and hemlock scaling 280 M. ft. and containing 60% fir?

***8.** In England, lumber is measured by the *standard*, which contains 1,980 bd. ft. When fir is quoted in Liverpool at £16 a standard, and the English pound is worth $4.88, what is the price in Liverpool in Canadian money per 1,000 bd. ft.?

***9.** If the freight on lumber from Vancouver to London is 45s. per M., how much does the lumberman in Vancouver receive for a shipment of fir per M. which sells in Liverpool for £17 per standard, when the value of the English pound is $4.88?

***10.** If the freight on lumber from Vancouver to Australia is $7.25 per M., what price can a Vancouver lumberman quote to an Australian firm for delivery of $56\frac{3}{4}$ M. of cedar which cost him $8 per M. at the mill and allow for $11.35 per M. as profit and cost of milling operations?

***11.** If a cubic foot of green fir weighs 47 lb., how many pounds will 1,000 bd. ft. of green fir weigh?

The Budget of a Fisherman on the Pacific Coast

Heikki Lahtinen is a fisherman at Rivers Inlet. He and his wife and four children live on a 20-acre bush farm on Malcolm Island. The farm helps to feed the family, but their main living comes from fishing.

1. Last year Heikki decided to provide and outfit his own boat. He paid $100 down of the $250 needed for material for building the hull, and did the work himself. He also paid $100 down on an Easthope gasoline engine, which cost $250. He bought a new net for $250, on which he paid one-fifth cash. He paid $48.75 for groceries to last him for five weeks of Sockeye fishing. He also bought the following gear: hip boots, $5; slicker and rubber pants, $10; 5 pairs gloves at 20¢ a pair; lantern, dishes, axe, oil, etc., $6.35; and a primestone, $3. How much did it cost him to outfit for fishing?

2. The fishing regulations allowed him to fish from Sunday at 6 p.m. until Friday at 6 a.m., when he took his fish to the cannery. How many hours during the week was he actually on the fishing grounds?

3. On Monday, while his net was out, he found that he had overlooked the following items in outfitting: 32 gallons of gasoline at 25¢ a gallon for his first week's fishing; $4.85 for hanging his net; $1.90 for oiling it; and $2 for a license. How much did his outfit then total?

4. In reaching Rivers Inlet, his engine consumed 1 gallon of gas in $\frac{3}{4}$ of an hour, and made 6 miles an hour. If the distance from his home to Rivers Inlet was about 84 miles, how much did it cost him for gasoline to make the trip?

5. On Sunday, June 24, he started fishing, and took in 120 Sockeye salmon during the first week. The price for the season was 50¢ a fish. How much money did he average for each day of his first week's fishing?

6. If he took in 2,240 fish during the season of five weeks, and used on the average 36 gallons of gasoline a week, how

much money did he clear in the season, not counting his boat, net, and outfit?

7. Deducting the amounts spent in Questions 1 and 3 how much money did he have clear at the end of the season?

8. He first put $300 in the savings bank to carry him over if the next year's fishing should be poor. He then cleared $250 in the fall fishing for Cohoe and Humpback salmon. How much money had he then for budgeting his expenses until the next Sockeye season?

9. Heikki first set aside sufficient money for meeting fixed expenses, which were: insurance, $38.50; taxes, $48.80; a new web for his net, $180; paint for boat, $4.75; and one-quarter of what he still owed on his boat, engine, and net. How much did he allow for fixed expenses?

10. The children wanted a radio, the wife wanted a cow, but Heikki had in mind an orchard. He made the following purchases:

1 box of stumping powder at $3.75
24 apple-trees at 90¢ each
100 lb. of sugar at 7.75 a pound.
1 pair of shoes for each of the family at an average price of $4.10.
6 yards of cloth at $1.40 a yard to make dresses for the two girls.
3 yards of cloth at $1.25 a yard to make trousers for the boys.
12 lb. of yarn at $1.10 a lb. for sweaters, stockings, and mittens.
How much did these articles total?

11. Heikki then said that if $400 was left he would buy a cow, and if $500 remained he would also buy a radio. The figuring was left to Olavi, the older boy, who was in Grade IX at school. With the help of the teacher, the right balance was found. Did Mrs. Lahtinen get her cow, and did the children have a radio for Christmas?

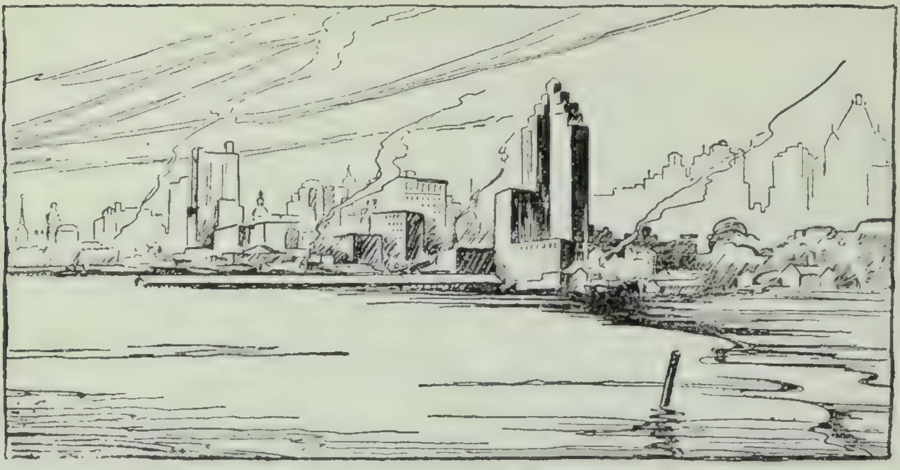
Halibut Fishing

A crew of from five to twelve men operate halibut boats out of Prince Rupert. They are at sea from four to twenty-one days, and take ice for keeping the fish fresh. A fair catch is 20,000 lb., which sells at an average price of about $5\frac{1}{2}$¢ a pound. The big fishing companies at Prince Rupert freeze about 50% of their halibut and salmon, salt 1%, and dry cure 5%. These companies make 20,000 tons of ice annually. Frozen halibut is shipped to New York in refrigerator cars, and sells for about 20¢ a pound. The freight from Prince Rupert to New York is 6¢ a pound and the commission for selling is 10%. From 20,000 to 30,000 lb. of halibut make a car-load. One million pounds of red spring salmon are dry cured annually and marketed in Europe. A halibut fisherman pays a yearly license of $1. The season opens on February 15.

1. If 8 men in a crew average 15 days and 12,000 lb. of halibut to a trip, how much does each man earn from the opening of the season to May 1?

2. A shipper at Prince Rupert sent to New York a full carload of halibut, which sold for 18¢ a pound. If he paid $4\frac{3}{4}$¢ a pound for the fish, what was his profit on the car-load, not counting overhead expenses?

3. If an average crew out an average time from Prince Rupert makes a fair catch and receives an average price, what daily wage would a man make, counting operating expenses at $6 a day? Will the answer found be exact or approximate?



The Port of Vancouver

1. A total of 15,777 vessels of all classes, with a net tonnage of 10,897,302 tons entered the port during one year.

(*a*) What was the average number of vessels per day entering the port?

(*b*) What was their average tonnage?

2. If the total water-borne exports amounted to 3,387,534 tons, what was the average tonnage that each steamship line took on at the port if there were 106 deep-sea steamship lines making Vancouver a port of call?

3. If the grain elevators in the harbour have a capacity of 17,843,000 bushels, how many cars of wheat, each containing 1,500 bushels, would be required to fill the elevators?

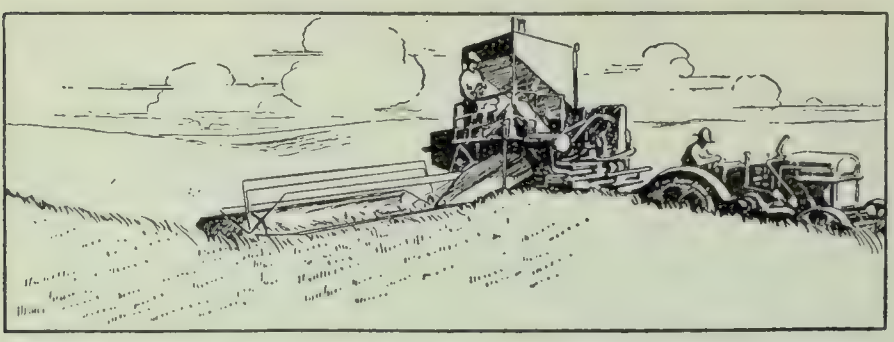
4. If there are 1,666,964 sq. ft. of shed area on the piers, find, to the nearest acre, the ground space occupied by the sheds.

5. Grain cars are unloaded at the rate of 21 cars per hour. By this means, how many minutes and seconds does it take to unload a car?

6. A total of 1,271,127 barrels of flour were exported in 1933, an increase of 18% above the figures for 1932. Find the increase in the number of barrels of flour.

Cost of Producing Wheat

One-section Farms operated with	*2 men, 6 horses, 15 h.p. tractor thresher owned*	*1 man, 15 h.p. tractor, combine and motor truck owned*
Cultivated Area:	*Acres*	*Acres*
Summer-fallow	200	200
Wheat after fallow	200	200
Wheat after wheat	180	200
Feed	20	—
Annual operating Cost:	$	$
Taxes, $58 per quarter section	232.00	232.00
Seed wheat, $1.00 per acre, oats 60¢ per acre	392.00	400.00
Twine, 37¢ per acre	148.00	—
Depreciation, interest, upkeep on equipment, 15% cost	860.25	843.00
Labour cost, men 8 months: wages $50, board $15	1,040.00	520.00
Day labour, stooking, 32 days, $5.00 per day	160.00	—
Day labour, threshing, 8 days, 6 stook teams, $5.00 per day	240.00	—
Day labour, combining, 12 days, 1 operator, $5.00; 1 truck driver, $5.00	—	120.00
65 days fuel for tractor, 24 gal. per day, 25¢; oil, 1 gal. per day, $1.00	455.00	—
79 days' fuel for tractor, 24 gal. per day, 25¢; oil, 1 gal. per day, $1.00	—	553.00
Fuel for combine, $\frac{3}{4}$ gal. per acre, 25¢; oils, 1 gal. per day, $1.00	—	87.00
Fuel for truck 954 miles, 14 miles per gal.; oil, 4 gal. per season	—	21.00
Truck license	—	15.00



The following problems are on the above table published by the Department of Agriculture at Ottawa on the cost of wheat farming in Western Canada:

1. What is the total number of acres in crop on each farm?

2. What is the total operating cost on each farm? (Interest on land investment not included).

3. What was the cost per acre for the crop on each farm?

4. If the total wheat yield on Farm A was 6,840 bushels and on Farm B 7,200 bushels, what was the average yield per acre?

5. What was the cost per bushel of growing the wheat on each farm?

6. How many acres a day did the combine harvest?

7. How was the cost of $87 for combine fuel arrived at?

8. Why is there a difference of $8 in the cost of seed for the two farms?

9. How many acres a day did the men average in stooking?

10. What was the average number of acres of crop a day threshed by the threshing-machine?

11. Spring wheat is graded into No. 1 Hard, No. 1 Northern, No. 2 Northern, and No. 3 Northern. How much did Mr. Jacobson receive for his crop of wheat, consisting of 680 bushels of No. 1 Hard at 78¢ a bushel; 945 bushels of No. 1 Northern at 72¢ a bushel; 1,264 bushels of No. 2 Northern at 65¢ a bushel; and 1,580 bushels of No. 3 Northern at 52¢ a bushel?

12. If a bushel of No. 1 Hard makes a 49-lb. sack of flour, how many bushels of wheat will make one ton of flour?

13. If the New Welland Canal saves 4¢ a bushel in the freight rate from Fort William to Montreal, how much will be saved on a season's shipment of 180,000 bushels through the St. Lawrence route?

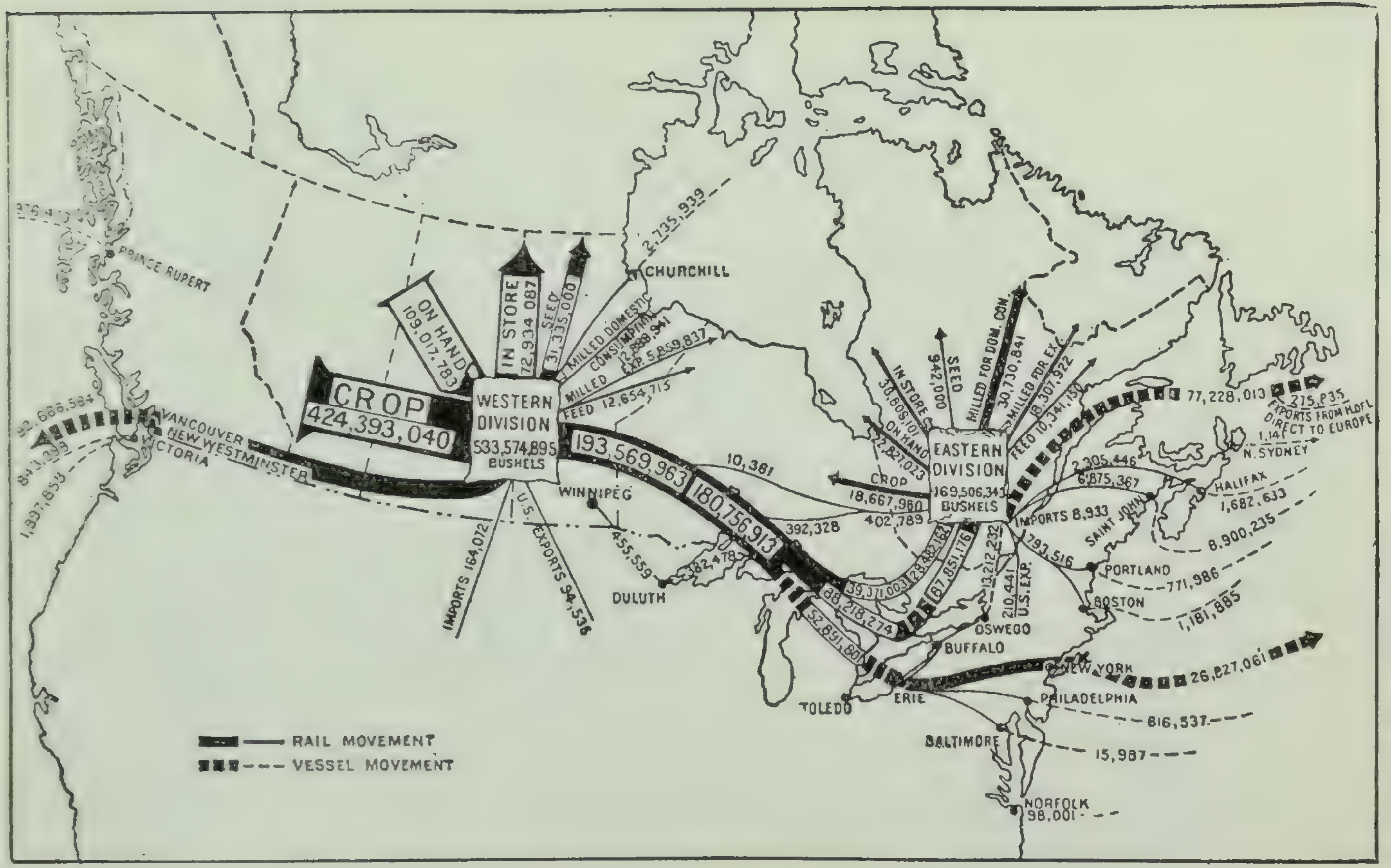
14. Wheat may be shipped from Saskatoon to London by way of Montreal, Vancouver, or Fort Churchill. If the cost per cwt. is as follows: via Montreal, $.2315; via Vancouver, $.2794; via Fort Churchill, $.224, find the cost of shipping 20 tons of wheat by each of the different routes.

15. In one year Western Canada produced 452,260,000 bushels of wheat from 21,665,200 acres. What was the average return per acre at $.96 a bushel?

Cost of Shipping Wheat

Approximate average charges between the producer in Western Canada and the arrival of steamer in Liverpool docks per bushel of wheat.

	¢
1. Receiving at country elevator, weighing, elevating, fire insurance, storing for fifteen days, loading into cars, cost of inspection and weighing, registration fees, and selling on Winnipeg market	5
2. Railway freight from average western point to Fort William–Port Arthur terminal elevators	13½
3. Unloading at terminal elevator, elevating, weighing, cleaning, insurance, storing for fifteen days, and loading into vessel or cars (including inspection and weighing fees, and marine insurance)	2
4. Lake freight from Fort William to Montreal	8½
5. Elevation of grain from steamer at Montreal, weighing, storage, and insurance against loss by fire or explosion, for 10 days and loading into ocean steamers (including brokers' fees)	1


CROP 424,393,040
WESTERN DIVISION 533,574,895 BUSHELS
ON HAND 109,017,783
IN STORE 72,934,087
SEED 31,335,000
MILLED DOMESTIC CONSUMPTION 12,888,941
MILLED EXP. 5,859,837
FEED 12,654,715
CHURCHILL
2,735,939
U.S. EXPORTS 94,536
IMPORTS 164,072
WINNIPEG
455,559
DULUTH
382,478
193,569,963
180,756,913
10,381
392,328
PRINCE RUPERT
VANCOUVER
NEW WESTMINSTER
VICTORIA
92,666,584
843,339
1,397,853
EASTERN DIVISION 169,506,343 BUSHELS
CROP 18,667,960
402,789
ON HAND 22,827,023
IN STORE 38,808,101
SEED 942,000
MILLED FOR DOM. CON. 30,730,841
MILLED FOR EX. 18,307,922
FEED 10,341,150
77,228,013
275,835 EXPORTS FROM NDFL DIRECT TO EUROPE
1,141
N. SYDNEY
2,305,446
6,875,367
HALIFAX
1,682,633
SAINT JOHN
8,900,235
IMPORTS 8,933
793,516
PORTLAND
771,986
BOSTON
1,181,885
13,212,232
210,441 U.S. EXP.
87,851,176
28,482,163
39,371,003
88,218,274
52,891,801
OSWEGO
BUFFALO
NEW YORK
26,827,061
ERIE
TOLEDO
PHILADELPHIA
816,537
BALTIMORE
15,987
NORFOLK
98,001
RAIL MOVEMENT
VESSEL MOVEMENT

Total charges for all services between producer and f.o.b. steamer at Montreal, per bushel of wheat	30
6. Approximate cost of Atlantic freight and insurance	4½
Total costs between producer and Liverpool per bushel	34½

The map on the preceding page shows the movement of the Canadian wheat crop in 1935-6.

1. How many bushels were shipped from Montreal and Vancouver together?

2. What six Canadian ports handled more than one million bushels each?

3. How many bushels were shipped through American ports?

4. How much did the railways earn in hauling the wheat to Fort William?

5. How much is the freight from Montreal to Liverpool on a ton of wheat?

6. How much of the cost of 34½¢ from producer to Liverpool is for actual transportation?

7. If a farmer received $780 for 1,000 bushels of wheat, what was the net amount in Liverpool for the grain?

8. What does the map indicate as the total amount of wheat produced that year in Canada?

9. If it costs a certain farmer 58¢ a bushel to produce his crop of wheat, how much does he lose in receiving 78¢ a bushel for 1,200 bushels in Liverpool?

The Apple Industry of the Okanagan Valley and Mainline of the C. P. R.

The Okanagan Valley and Mainline of the C. P. R., which refers to the Okanagan and Similkameen Valleys and the district from Lytton to Salmon Arm, produces in commercial quantities many different varieties of fruits and vegetables. These include peaches, apricots, prunes, cherries, crabapples, onions and tomatoes; but the district is especially noted for its production of apples.

In the 1939 crop-season approximately 5,280,000 boxes of apples were produced for commercial purposes, and about 2,600,000 were exported.

Apple trees are planted about 30 feet apart or about 48 to 50 trees to the acre. Some orchards market more than 20 boxes to the tree, but the average yield in a commercial orchard of good varieties over this area is approximately 5.5 boxes to the tree. This refers to apples that are actually sent in by the farmer to be marketed or processed.

The grower picks the fruit at the proper stage of maturity and delivers it to a central packing house where it is graded according to color and defects. The apples are sized by machinery which weighs them and drops them into a bin with all the other apples of the same grade, size, and variety. They are then wrapped in individual oiled-paper wrapping and packed into boxes. Boxes weigh 42 pounds net, 8 pounds tare, and 50 pounds gross. The boxes are lidded, labelled, and marked with the name of the variety, the number of apples in the box, and a code number indicating the lot and the date the apples were packed. They are usually placed in cold storage until required for shipment.

Some packing houses look after the fruit they pack, while others leave this work to Centralized Selling Organizations, who may sell for as many as twenty packing houses. These selling organizations employ agents (commonly known as brokers) in all the large cities throughout Canada, United States, and all foreign countries where Okanagan apples are sold. The selling of apples on the Domestic Market is done entirely through one Central Selling Organization. Any profits realized by the brokers acting as agents for this organization are returned to it for distribution to the farmers on a tonnage basis.

The selling organizations usually charge three cents per box for their own services, and three cents per box for the broker who is paid a brokerage of approximately $15.00 on the Western Markets to $20.00 per car on the Eastern Markets.

The usual load for Export is 756 boxes and for Domestic Markets, 700 boxes. Almost 100 per cent of Okanagan apples sold in Canada are sold at a definite price, f.o.b. (free-on-board) railway cars at shipping point. The buyer pays the transportation costs and adds his margin of profit to arrive at his selling price to the retailer. Some Okanagan apples are sold at auction in the United States and in the British Isles. When apples are sold at auction or on what is termed "consignment or commission basis" the wholesaler makes an accounting to the shipper showing the price at which he sold them and then takes his commission ranging from 5% to 15% of the selling price. He also deducts transportation, storage, and special costs and remits the balance to the shipper.

The proceeds of all sales are placed by the shipper in what is called a "pool" and distributed to the grower according to variety, grade, and size. Before making payments to the growers the shipper deducts his charges including assessments made for such organizations as the British Columbia Fruit Board. This Board is the organization which operates as a legal controlling factor under the Provincial Marketing Act.

Charges are about as follows, but are subject to change from time to time as conditions warrant:

Cost of production, including picking (exclusive of overhead)	Per box $.55
Cost of grading, packing, paper, boxes, warehousing, labels and labelling	.45
Cold storage	.10
Cost of selling, including brokerage	.06
Wiring and marking for export	.05

Transportation costs (normal) per box of 50 lb.

Domestic

To Vancouver	$.225
" Calgary	.355
" Edmonton	.40
" Regina	.565
" Winnipeg	.565
" Toronto and Montreal	.75

Export	*Rail*	*Ocean*
To United Kingdom (via Vancouver)	$.20	$.825
" South Africa (via Vancouver)	.20	1.25
" South Africa (via Montreal)	.60	1.00
" Egypt (via New York)	.60	.60
" Brazil and Argentine (via New York)	.60	.85

(Ocean charges extra for refrigerator storage)

Comprehension Test

1. What is the average yield per tree in boxes of apples? To what does the average yield actually refer?

2. In the Kelowna District the average yield in boxes of apples per tree (for trees 10 years old and over) is 6.7. Make a problem using this and other information given in the unit.

3. What does *sized* mean?

4. Explain clearly what *F.O.B.* means?

5. What is meant by selling apples at *auctions*? Do you know of any other commodities sold in this way?

6. What is the meaning of *tare*?

7. Could you calculate the approximate number of trees in the Okanagan and Mainline of the C. P. R.? What is the result?

8. The cost of producing a box of apples, exclusive of overhead, is given as 55¢ per box. What would this cost include?

Problems

1. On the basis of 5.5 boxes to the tree and 50 trees to the acre, what will it cost a grower to produce and harvest the crop from 15 acres?

2. A 12-acre orchard of McIntosh apples with 48 trees to the acre averaged 10.2 boxes to the tree of which 42% was exportable sizes and grades. How many carloads Export and how many carloads Domestic were produced?

3. What would be the value, delivered at Liverpool, of a carload of apples valued at $1.10 per box shipping point, and shipped via the Panama Canal?

4. If a Buyer in Montreal buys a carload of McIntosh Fancy Grade at 95¢ shipping point, and pays $45.00 for protective service on the car, what would the apples cost per box delivered at Montreal?

5. If the same buyer adds 25¢ per box to his cost price for his margin of profit what would his percentage of profit be, based on his selling price?

6. If a Winnipeg jobber receives a car of 700 boxes of apples on consignment basis and sells 500 boxes at $1.60 box, and the balance at $1.50 per box, what amount would the shipper receive per box f.o.b. shipping point? The jobber's commission is 15%.

7. If a packing house handles the crop from 500 acres with an average of 330 boxes to the acre what would their charges amount to including cold storage and selling, and providing that 40% of the apples were shipped export and the balance was sold domestic?

8. What would the total freight charges be on a carload of apples to London via the Panama Canal?

9. What would the freight charges be on a carload of apples shipped to Buenos Aires?

10. What per cent of the apples produced in the 1939 crop-season was exported?

11. 50% of the apples exported go to the United Kingdom via Vancouver. What would the freight charges on the apples amount to?

***12.** If a carload of Extra Fancy Delicious sells in Glasgow at 15s. 6d., and the commission is 5% with Port charges 11d. per box, what price would the sale net f.o.b. the car shipping point in the Okanagan. The rate of exchange at the time is $4.43 to the pound Sterling and the shipment moves by the Panama Canal. The marine insurance is 1.6¢ per box and special risks amount to 6.4¢ per box.

Fruit Farming

From the table which follows, obtain the cost of, receipts from, and profits on the growing of an acre of strawberries in British Columbia. The plants are set in the spring and no crop is taken the first year. The ground is ploughed up after two crops are picked. There is no straw used the first year and no blossoms or runners are cut the second year. Items 20 and 21 are not effective for the third year.

1. What is the average profit per acre for the three years?

EXPENDITURES

Particulars	*First Year*	*Second Year*	*Third Year*
1. Rent of land, 8 per cent on $250			
2. Preparation of land, man and team, 20 hr. at 75¢			
3. Manure or fertilizers	$25.00	$12.50	$12.50
4. Plants, 10,000 at $4			
5. Planting, 2 men, 20 hr. each at 20¢			
6. Cultivation, man and horse, 30 hr. at 50¢			
7. Hoeing, 100 hr. at 20¢			
8. Cutting blossoms, 20 hr. at 20¢			
9. Cutting runners, 50 hr. at 20¢			
10. Weevil-bait, 100 lb. at $7.50 per cwt.			
11. Straw, 2 tons at $11.85 per ton			
12. Scattering and setting straw, 60 hr. at 20¢			
13. Crates, 250 at 19¢			
14. Picking, 250 crates at 30¢			
15. Nailing and hauling, 250 crates at 2¢			
16. Association charges, 250 crates at 14¢			
17. Picking, 3,000 lb. fruit for manufacturing at 1¢			
18. Handling and hauling, 140 packages at 2¢			
19. Association charges, 140 packages at 10¢			
20. Cutting off tops after crop, man, 20 hr. at 20¢			
21. Hauling off straw, 2 men and team, 3 hr. at 95¢			

Total expenditure, first year = $
Total expenditure, second year =
Total expenditure, third year =
Grand total =

RECEIPTS

First crop: 250 crates at $1.25
3,000 lb. fruit for manufacturing at 5¢ = $

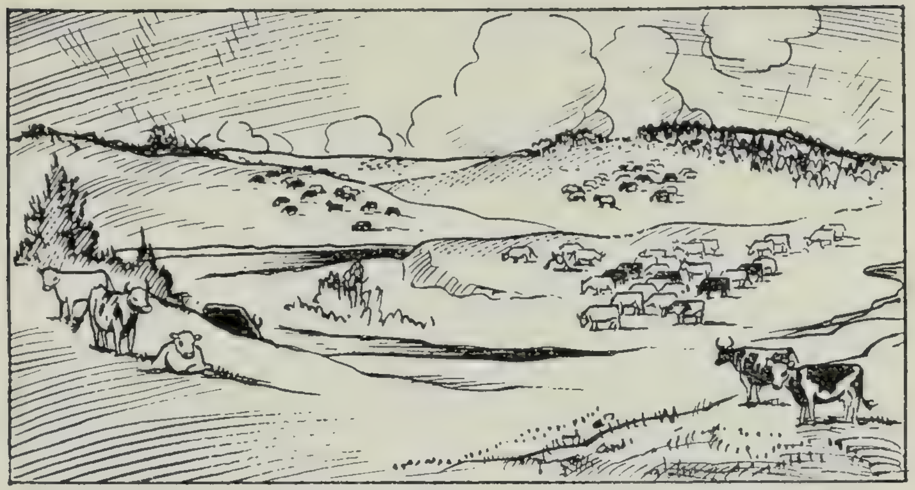
Second crop: 250 crates at $1.25
3,000 lb. fruit for manufacturing at 5¢ = $

Total receipts =
Total expenditures =
Profits =

2. The express on a car-load of berries from Vancouver Island to Winnipeg is $2.65 per 100 lbs. A car-load holds 800 crates which weigh 22 lb. each. How much is the freight per car-load?

3. The express on strawberries from Vancouver Island to Calgary is $2.25 per cwt., and the commission is $70 per car lot. If the berries sell for $2.50 a crate of 24 boxes: (*a*) What is the commission in per cent? (*b*) How much a box does the shipper net?

4. How many strawberry plants, 18 in. apart in rows 3 ft. apart are needed to set an acre.



Cattle Ranching

1. If, in a certain part of the Chilcotin ranching district, 8 acres are needed for each head of stock, how many cattle could a rancher keep if his ranch is 12 miles long and $6\frac{1}{2}$ miles wide?

2. A rancher paid $1.80 an acre for an area 6 miles long and $3\frac{1}{4}$ miles wide. He put on improvements in the form of buildings and fences which cost him $5,600. How much had his ranch then cost him per acre?

3. A rancher with 640 head of stock put up $1\frac{1}{2}$ tons of alfalfa per head for carrying his cattle through the winter. The feed was grown on irrigated land and produced during the season three crops, averaging $2\frac{3}{4}$ tons per acre for each cutting. How many acres did he have in alfalfa?

4. The beef produced by an animal is one-half of its live weight. A rancher has four three-year-old steers weighing 1,120 lb., 1,310 lb., 1,280 lb., and 1,260 lb., respectively. Will it pay him better to sell the two smallest for $4\frac{1}{2}$¢ a pound live weight and the other two for $9\frac{1}{2}$¢ a pound dressed, or to sell the lot for $4.65 per cwt. live weight?

5. A rancher paid his cowboys $45 a month, and estimated the cost of their board each at $18 a month. He needed two

men for each 300 head of stock. What would it cost him yearly for help if he ran 7,200 head of stock?

6. The Bar X ranch sold 26 head of Hereford yearlings, weighing on the average 712 lb. each, at $5.35 per cwt. on the hoof, commission being 15%, and freight $78 for the car-load lot. How much a pound did the beef cost the meat packers if they added 2¢ a pound for dressing?

The Selling of Cattle

1. A rancher in Northern Alberta shipped to Vancouver a carload of steers consigned to a packing company to be sold on commission. The 20 steers in the carload had an average weight of 1,175 lb. each.

- (*a*) How many hundredweight did the steers weigh over the 20,000 lb. necessary for the minimum carload rate?
- (*b*) If the carload rate was 56¢ per cwt., how much did it cost to ship the steers, allowing 80¢ per animal for unloading, feeding, and watering charges at Kamloops?
- (*c*) How much was the average freight charge per steer?
- (*d*) The packing company dressed the beef and sold it at $9.80 per cwt., retaining for their work and commission 15% of the selling price. If the beef was 50% of the weight of the live animals, how much did the rancher receive for his steers?

2. A company in Alberta shipped a consignment of 120 steers to a firm in Liverpool. The average weight of the stock was 1,350 lb. each. The freight was $1.14½ per cwt. to Montreal, and $40 a steer ocean freight. The cost of feeding and care on the journey was $3.20 a steer. The price at Liverpool was £1 18*s.* per cwt. live weight and the shrinkage in weight due to the voyage was 5%.

Find:

- (*a*) The total freight per head, including feed and care.
- (*b*) The total weight of the stock landed in Liverpool.
- (*c*) The amount of money received for the stock in England, allowing £1=$4.88.
- (*d*) The net profit per steer after charges were all paid.

3. One year Canada exported to all countries 204,668 head of beef cattle for which $11,571,796 was received. The same year she exported 1,222 head to England, for which she received $130,140. By how much was the English price per head better than the average price received per head for export cattle that year?

Dairying in the Fraser Valley

1. A three-unit milking machine, costing $500 installed, milks 45 gallons an hour. If a dairyman has a milking herd of 48 cows averaging 15 quarts of milk daily from two milkings:
   (*a*) How many hours a day is his milking machine used?
   (*b*) If a man milks a cow in 10 minutes and his wages amount to 20¢ an hour, in how many days would the milking machine pay its cost in the saving of labour?

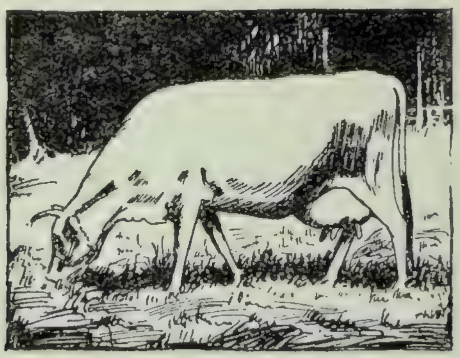
2. A dairyman feeds each of his cows daily the following ration: 8 lb. of shorts, 4 lb. of oat-chop, 4 lb. of bran, and 2 lb. of cotton-seed meal. The cost of the grain is: cotton-seed meal, $46 a ton; bran, $30 a ton; shorts, $30 a ton; and oat-chop, $40 a ton. If he has a herd of 25 cows, how much a day is his bill for grain?

3. A gallon of milk weighs nearly 10 lb. If a dairyman sold, on June 2, 1,600 lb. of milk, testing 4.5% butterfat, at the pool price of 14½¢ a gallon for 4% f.o.b. Vancouver, how much did he receive for his milk when the cost of delivering was $3.50?

Dairying

1. Rainton Rosalind V, an Ayrshire cow eleven years old, imported from Scotland, produced more than 100 lb. of milk daily. Her record for one day was 109.5 lb. If a gallon of milk weighs 10 lb., how many quarts of milk did this famous cow produce on her record day?

2. If in one year she gave 14,232 lb. of milk containing 654 lb. of butterfat, what per cent of butterfat did her milk average?



3. When six years old she produced in one year 15,238 lb. of milk. What would have been its value at 10¢ a quart?

4. Rosalind was fed 32 lb. of such grain as oats, bran, and oilcake-meal a day. How many tons of grain did she consume a year?

5. A milch cow may be fed 20 lb. of mangels a day. How many tons of mangels would she eat in a year?

6. A dairy farmer puts green feed through a chopper and stores it in a tight, circular bin called a silo, where the heat caused by fermentation cooks the mixture, which is called ensilage. A farmer puts in a silo, 10 tons of clover hay, 3 tons of sunflowers, 2 tons of beet-tops, and 4 tons of green oats. If he fed each of his twelve cows daily an amount equal to 20 lb. of the original mixture, how many weeks would the ensilage last?

7. The new world's record for the milk production of a goat is 5,306 lb. in a year. The highest day's yield was 19½ lb. How many pounds was this yield in excess of the daily average for the year?

The Mining Industry of British Columbia

ANNUAL REPORT OF THE MINISTER OF MINES

The value of mine production in 1937 was $74,475,902, an increase of $20,393,935 over 1936. The increase is largely due to abnormally high base-metal prices prevailing throughout the greater part of the year. This is clearly shown in the case of lead, where the volume increase amounted to 10.9%, and the value increase 44.8%. All phases of the mining industry have shown increases in both volume and value.

Lead production has again stepped into the lead, with an all-time record in volume of 419,118,371 lb., valued at $21,416,949, this being the greatest value attained in the history of mining in the province for any one metal or material in a single year.

Both lode and placer gold have again shown appreciable increases, and once again a new record has been established with an output of 514,934 oz., valued at $17,680,972.

Zinc production, which in 1936 fell slightly below the 1935 volume record, has again shown an appreciable increase, and established a new all-time record with a production of 291,192,278 lb., valued at $14,274,245. This shows an increase of 14.4% in volume and 69.1% in value.

Coal, valued at $6,139,920, shows a smaller increase over 1936 than did the production in 1936 over 1935.

Copper, which in 1936 had decreased both in volume and in value to the lowest point since 1900, has, with the return of Britannia to full capacity and the reopening of Copper Mountain, shown an even better recovery during 1937 than was anticipated. Volume production increased 121.4%, and value production 205.5%. The latter, however, was due to abnormally high prices for copper.

Silver production in volume was the greatest in the history of the province, with an output of 11,308,685 oz., valued at $5,075,451. The value is considerably below that of the peak year of 1926, when 10,748,556 oz. was valued at $6,675,606. The record volume production clearly shows that silver in British Columbia today is a by-product, and largely dependent upon the production of base metals.

Non-metallic minerals and structural materials as groups show substantial increases in value of 31% to 37%, and only in a few individual items in each group has there been any decrease. The steady increase in the value of these materials is a healthy sign.

The total number of shipping-mines increased from 168 to 185, those shipping over 100 tons increasing from 70 to 113.

The number of men employed increased from 14,180 to 16,129, the greatest number employed in any year; and wages and salaries increased from $17,917,221 to $21,349,690, the greatest amount ever paid out in any year.

Dividends increased from $10,513,705 in 1936 to an all-time record of $15,085,293 in 1937.

The figures quoted above should be checked against the latest annual report.

Problems

1. The coal production in 1937 was 1,346,471 tons. What was the average value per ton at the mine?

2. How many tons of zinc were produced in 1936?

3. What per cent was the price of silver in 1937 of that in 1936?

4. Compare the average wage of miners in 1936 and 1937.

5. What per cent of the value of minerals in 1937 was paid in wages and salaries?

6. In 1936, 20,806,672 lb. of copper were produced, which had a value of $1,971,848. What was copper worth a pound in 1937?

7. What price was gold an ounce in 1937?

8. The royalty on coal is 10¢ a ton. How much did the Government collect in 1937?

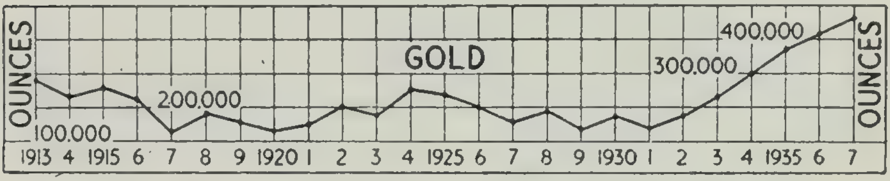
Problems for Discussion and Investigation

1. Why does coal cost the consumer nearly double the price at the mine?

2. In what ways does the mining industry benefit the province?

3. Which kind of mining is most dangerous, and why?

4. Why is so much capital required now for gold-mining?



1. What per cent was the increase of the gold output in 1936 over that of 1934?

2. In what two years was the output the same?

3. For what year is the graph inaccurate? How can this inaccuracy be explained?

Making your own Problems

The following questionnaire on gold-mining was answered by the manager of one of the best organized and most important gold-mines in the province. This material, in its present form, is well worth studying, and should form the basis of a number of practical problems made by a group of the class.

Questionnaire on Gold-mining

1. What is the greatest depth at your mine?
   Answer: 3,050 ft., collar to sump.

2. What is the total length of your underground tunnels?
   Answer: 18 miles.

3. What is the cost per foot of making tunnels?
   Answer: $12.

4. How many men do you employ?

Answer: 325.

5. How many hours a week do they work?

Answer: 48.

6. How many shifts in a 24-hour day?

Answer: Mine, 2; mill, 3.

7. What per cent (cost) of your machinery was made in Vancouver?

Answer: Negligible.

8. How are your men proportioned as to kinds of work?

Answer: Underground, 200; mill, 24; surface, shop, etc., 73; staff, 30.

9. Are the men in the mine paid by output or a regular hourly wage?

Answer: Regular wage, plus bonus on various types of work.

10. What is the annual pay roll?

Answer: Approximately $540,000.

11. What is the annual output of gold?

Answer: 1937-8, 60,654.491 oz.; $2,122,891.07 value.

12. From a ton of ore, how many ounces of gold do you extract on the average?

Answer: 0.480 oz. (1937-38).

13. What is gold worth an ounce?

Answer: Varies. Approximately $35 per oz.; last return $35.36 per oz.

*14. What royalty does the British Columbia Government take?

*15. What royalty does the Federal Government take?

*Investigate from any reliable source the current answer.

16. How is the ore taken to the mine shaft?

    Answer: By mine cars of about 20 cu. ft. capacity in trains of four or five cars hauled by electric motors, or singly by hand.

17. How many tons of coal do you use a year?

    Answer: None, except blacksmith coal.

18. How long and thick is the cable that hoists the cage?

    Answer: For deepest shaft—1 in. diameter, 3,750 ft. long.

19. How thick is the drum?

    Answer: 6 ft. in diameter.

20. What are the dimensions of your tunnels?

    Answer: 5 ft. by 7 ft.

21. What amount of air is pumped daily into the mine?

    Answer: 33,750 cu. ft. of air per minute—recent estimate.

Problems for Discussion

1. What would be the main items of expenditure for a crew engaged in halibut fishing?

2. What kind of farmers are the least dependent on weather conditions?

3. What is the busiest time of the year for the ranchers?

4. What are the advantages of a two-way freight, and how does it operate against shipping wheat via Vancouver?

5. What raw materials does British Columbia possess for manufacturing?

6. What are Canada's chief competitors in wheat, salmon, lumber, and gold?

7. What are British Columbia's best markets for apples, lumber, halibut, salmon, and herring?

Research Work

1. From the latest Canada Year Book find what the figures are for the value of the salmon and halibut fisheries of British Columbia.

2. Find the price at which wheat is selling, and the number of bushels harvested in the last crop.

3. Check the information given in this section against the latest figures.

4. Does the Dominion Government collect a tax on gold mined in the country?

5. Is book paper manufactured in British Columbia?

The Trail Smelter

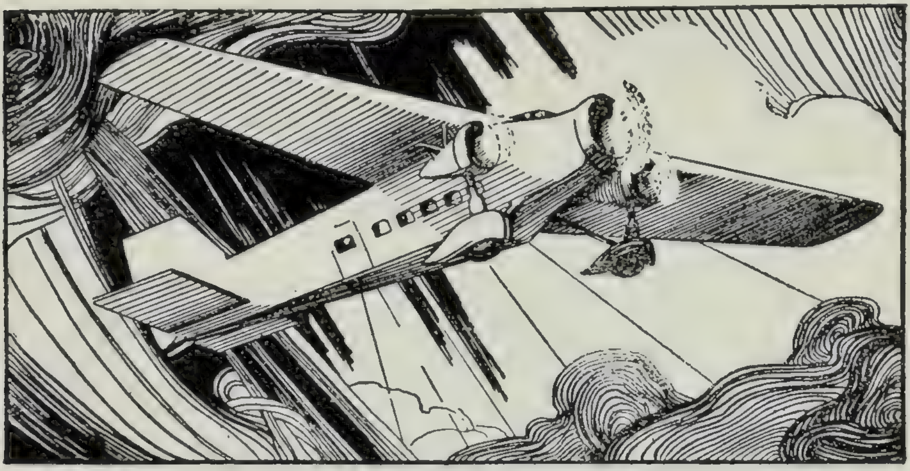
1. 6,650 men are employed at the Trail Smelter, which reduces more ore than any other in the British Empire. Their wages amount to $8,110,000 annually. If the men work 40 hours a week, what is their average wage per hour?

2. The output in lead runs to 575 short tons daily, which sold in London in 1937 at $115.23 a long ton. Lead is principally used for storage batteries, paint, and cable covering. If the Trail Smelter in 1937 produced 419,118,371 lb. of lead:
   (*a*) How many days during the year would the plant have to operate at capacity to produce that amount?
   (*b*) What was the price per pound at which lead sold in London?

3. If 176 lb. of lead are obtained from a ton of Kimberley ore, how many tons of ore were mined at Kimberley during 1937?

4. If a ton of Kimberley ore also produces 180 lb. of zinc, what was the daily output of zinc at Trail for the same year?

5. The two big chimneys are 409 ft. high. The lead stack is 25 ft. in diameter at the top and 29 ft. at the base, and the zinc stack is 21 ft. in diameter at the top and 28 ft. at the base. How many cubic feet does each chimney occupy?



Travelling across Canada by Aeroplane

The Trans-Canada Air Lines are using two types of aeroplanes, Lockheed Electra or 10-A type for short runs, and the Lockheed 14-H for transcontinental flights. These liners cost $90,000 and $120,000, respectively. The Electra makes 170 miles per hour and the 14-H 200 miles per hour. The planes use 40 gallons of gasoline an hour costing 30¢ a gallon. The Electra carries nine passengers, and the 14-H ten passengers. The lapsed time from Vancouver to Montreal is approximately 17 hours for the larger plane, the distance being 2,508 miles. The fare is 6¢ a mile.

1. What is the cost of gasoline in flying the larger plane from Vancouver to Montreal?

2. How many hours and minutes are the passengers actually in the air during the flight?

3. Why is the fare by air less than twice the 3¢ fare by rail?

4. If the distance by air from Vancouver to Winnipeg is 1,180 miles, when should a passenger who leaves Vancouver at 7 a.m. arrive at Winnipeg via a 14-H plane if there is a stop at Lethbridge and Regina of 35 minutes each?

Transportation

Discussion (oral)

1. Why is it cheaper to transport goods by water than by rail?

2. What kind of goods cannot be transported by truck, but must be shipped by train?

3. What is a 'tramp' steamer?

4. What is meant by ships 'going in ballast'?

5. What kind of ballast is now used?

6. Why do so few persons travel from British Columbia to England by way of the Panama Canal?

Problems

1. The cost of passing through the Panama Canal is 90¢ per net vessel ton (long ton). What would be the cost to a ship going through the Panama Canal if its cargo consists of 60,000 bushels of wheat?

2. The freight on wheat from Vancouver to Liverpool via the canal is 15*s.* per long ton. Allowing $5 to the pound, what is the freight per bushel?

3. The freight on lumber from Vancouver to Liverpool through the canal is 55*s.* per 1,000 bd. ft. What is the freight in Canadian money on 20 sticks of timber 40 ft. long and 8 in. square?

4. The freight on automobiles from Vancouver to Shanghai is $5 per 40 cu. ft. space. What is the freight on a car 15 ft. long, 8½ ft. wide, and 7½ ft. high?

5. The freight on paper newsprint from Vancouver to Yokohama is $6.75 for 40 cu. ft. of space. If a ton of paper required 36 cu. ft., what would be the freight on 2½ tons of newsprint?

6. A bushel of oats weighs 34 lb. If the freight on oats to the Orient is $8.50 per 2,000 lb., what will be the freight on 1,000 bushels of oats?

7. The following passenger rates prevail from Vancouver to Hong Kong:

First class	*Empress of Canada*	$415
	Empress of Russia	375
	Empress of Japan	425
Tourist	*Empress of Russia*	215
	Empress of Canada	245
	Empress of Japan	255

Which steamship allows the greatest per cent of reduction for travelling tourist?

8. The speed record on the Pacific is held by the *Empress of Japan,* the flagship of the Canadian Pacific Steamships' fleet. The time from Yokohama to Race Rocks was 7 days 20 hours. If the distance is 4,360 miles, how many *knots* did the *Empress of Japan* average?

9. A freight locomotive uses about 160 lb. of coal per mile and a passenger locomotive about 65% of that amount. The coal for 1937 cost the railways $4.28 a ton. At these figures what were:

(*a*) The cost of coal per mile for freight locomotives?

(*b*) The cost of coal per mile for passenger locomotives?

10. A 40-ton steel box-car is 40 ft. 6 in. long, 9 ft. 2 in. wide, and 10 ft. high. If a bushel of wheat occupies $\frac{32}{25}$ cu. ft., how many bushels of wheat will this car carry?

CHAPTER VI

THE FORMULA

The formula is a *shorthand* way of writing a statement. It may also be called a *symbolic* method of writing, since we make use of symbols, that is, letters or signs representing words or quantities or operations. The formula has been used in your geometry, and also in your generalized arithmetic or algebra.

The formula is perhaps the most widely used application of algebra, and its use is not confined solely to the engineer or surveyor or scientist, but is being extended to many other occupations.

Writing Formulas from Statements

Exercises:

Express the following statements by formulas:

1. The loss is equal to the cost diminished by the selling price. (Use l, c, s as symbols).

2. The price (p) of a certain weight of sugar is the product of the number of pounds (n) and the cost per pound (c).

3. The perimeter of a triangle is found by adding together the lengths of the three sides (a, b, c).

4. The area (A) of a square is found by multiplying the length of a side (s) by itself.

5. The surface (S) of a cube is equal to six times the square of an edge (e).

6. The time (t) taken to travel a certain distance (d) is obtained by dividing the distance by the rate (r).

7. The amount of commission (c) is the product of the amount of the sales (s) and the rate of commission (r).

8. The simple interest on a sum of money for one year is the product of the principal and the rate. (Use i, p, r).

9. The area of a circle is π times the square of its radius.

10. The circumference of a circle is twice the product of π and the radius.

11. The curved surface (s) of a cylinder is found by multiplying the height (h) by the circumference (c) of the base.

12. The number of tickets (n) that may be bought for a sum of money (s) is found by dividing the sum by the cost (c) of one ticket.

Exercises:

Change the following formulas into statements:

1. $c=2\pi r$, c being the circumference, r the radius of a circle.

2. $A=s^2$, A being area, s a side of a square.

3. $V=e^3$, V being volume, e the edge of a cube.

4. $d=rt$, d being distance, r rate, t time.

5. $t=3c$, t being total cost, c the cost of one article.

6. $w=\frac{70n}{100}$, w being wages in dollars, n the hours worked.

7. $c=s-p$, c being the cost, s the selling price, p the gain.

8. $n=i+12f$, i being the number of inches, f the number of feet.

9. $c=lwy$. l is the length and w the width of a floor in yards, y is the cost of flooring per square yard.

Formulas Used in Arithmetic

Commission:

If we represent the selling-price by s, the rate of commission by r, and the commission by c, then we may write the formula:

$$c = rs.$$

By substituting values for r and s we obtain an equation which, on solving, gives c, the commission.

Example 1. Find the commission at 2% on an article sold for $350.

Since $c = rs$

we have $c = .02 \times 350 = 7.$

Thus the commission is $7.

Example 2. Find the rate of commission when an agent received $2.50 for selling goods for $200.

Since $c = rs$

we have $2.5 = r \times 200$

that is $200r = 2.5$

Then $r = \frac{2.5}{200} = .0125 = 1\frac{1}{4}\%.$

We know that the rate of commission is found by dividing the commission by the selling price. Thus the formula changes to

$$\frac{c}{s} = r.$$

Again, the commission divided by the rate gives the selling-price. So we have a third form,

$$\frac{c}{r} = s.$$

Exercises:

Using the formulas just given, work the following:

1. Find the commission at 3% on sales amounting to $650.

2. What is the commission for selling goods for $824 if the rate of commission is $2\frac{1}{2}$%?

3. Find the rate of commission if an agent received $210 for selling goods for $3,500.

4. An agent was paid $54 at a rate of commission of $2\frac{1}{4}$%. What was the selling-price of the goods?

5. In the formula $r = \frac{c}{s}$, find r if $c = 25.6$ and $s = 640$.

Trade Discount:

If p equals the list price, d the discount, and r the rate of discount, then

$$d = rp.$$

This formula may be used to estimate discounts.

Example 1. The list price is $35, and the rate of discount is 8%. Find the discount.

Using the formula $d = rp$

$$d = .08 \times 35 = 2.8.$$

Thus the discount is $2.80.

Example 2. If $66 is the discount, and 12% is the rate of discount, find the list price.

The formula is

$$\begin{aligned} d &= rp \\ 66 &= .12p \\ \frac{66}{.12} &= \frac{.12p}{.12} \\ 550 &= p \end{aligned}$$

Thus the list price is $550.

Exercises:

Use the formula in solving the following problems:

1. If the discount is \$135 and the list price is \$900, find the rate of discount.

2. If the net price is \$574 and the list price is \$820, find the rate of discount.

3. Complete the table given below by using the formula:

	List Price	*Rate of Discount*	*Discount*	*Net Price*
(1)	\$440	20%	——	——
(2)	\$40	$33\frac{1}{3}\%$	——	——
(3)	\$480	——	\$19.20	——
(4)	\$9	——	\$0.90	——
(5)	——	25%	\$15.50	——
(6)	——	30%	\$60.00	——

Simple Interest:

You have learned that, in order to obtain the simple interest on a sum of money, you multiply the principal by the rate by the time. We shall let i represent the number of dollars of interest, p the number of dollars of principal, r the rate of interest, and t the time in years.

Then, applying our rule for finding simple interest we have

$$i=prt.$$

This is the 'interest formula'. Its use is shown by the examples:

(1) Find the simple interest on \$352 for 6 years at 2%.

Solution: $p=352$ (we do not write $p=\$352$, because p is the *number* of dollars), $r=.02$, $t=6$.

Then since $i=prt$

$$i=352\times.02\times6=42.24.$$

Thus the simple interest is \$42.24.

(2) Find the simple interest on \$86.25 from March 10 to July 15 at $3\frac{1}{2}\%$.

Solution: The time is 127 days$=\frac{127}{365}$ years, $p=86.25$, $r=.035$.

Then since $i=prt$

we have $i=86.25\times.035\times\frac{127}{365}=1.05$.

Thus the simple interest is \$1.05.

NOTE: We do not put the \$ sign or the word 'years' in our formula. The letters i, p, r, t represent numbers only.

Exercises:

Using the interest formula, work the following questions:

1. Find the interest on \$300 at 4% for 2 years.
2. Calculate the interest on \$256 at $3\frac{1}{2}\%$ for 4 years.
3. Find the amount of \$1,290 at $2\frac{1}{4}\%$ simple interest for $3\frac{1}{2}$ years.
4. Find the interest on \$120 at 5% for 146 days.
5. Find the interest on \$2,000 at 6%, from June 1 to September 15.
6. What will \$1,000 amount to at $4\frac{1}{2}\%$ simple interest, from January 1, 1937, to August 4, 1938?

Problem: At what rate will the interest on \$2,500 for 4 years be \$300.

Two methods of solution are given:

1. Using the formula

$$i=prt$$

we have $300=2{,}500\times r\times 4$

that is $300=10{,}000r$

or $10{,}000r=300$

$$r=\frac{300}{10{,}000}=\frac{3}{100}=.03 \text{ or } 3\%.$$

2. We use the formula

$$r=\frac{i}{pt}$$

Filling in the values for i, p, and t we obtain

$$r=\frac{300}{2{,}500\times 4}=.03 \text{ or } 3\%.$$

Exercises:

1. Write formulas for finding p and t.

Use these formulas to solve the remaining problems of this exercise.

2. What sum of money at 5% for 2 years will produce an interest of $25?

3. In how many years will the interest on $520 at 4% be $72.80?

4. At what rate will the simple interest on $350 for 6 years be $73.50?

5. What principal will produce $16.20 in 3¼ years at 2½%?

6. At what rate will the interest on $1,000 from July 10 to December 31 be $21?

7. How long will it take $100 to produce simple interest of $4 at 5½%?

Evaluation of Formulas

Example 1. By using the formula $A=\pi r^2$ find the area of a circle whose radius is 2 in. (Use $\pi=3\frac{1}{7}$).

$$A=\pi r^2=\frac{22}{7}\times 2^2=\frac{22}{7}\times 4=\frac{88}{7}=12\tfrac{4}{7}.$$

Since r represents inches, r^2 will represent square inches. Hence area$=12\frac{4}{7}$ sq. in.

Example 2. By using the distance formula, find the distance travelled in 5½ hours at a rate of 15 miles per hour.

The formula is $d=rt$, $t=5\frac{1}{2}$, $r=15$.

Then $d=rt=15\times 5\frac{1}{2}=15\times\frac{11}{2}=\frac{165}{2}=82\frac{1}{2}$.

Hence distance$=82\frac{1}{2}$ miles.

Exercises:

1. Using $A=lw$, find the area of a floor 30 ft. long and 25 ft. wide.

2. Find the perimeter of a triangle whose sides are 6.2 cm., 3.5 cm., and 4.7 cm.

3. Using $c=\pi d$, calculate the circumference of a circle whose diameter is $3\frac{3}{4}$ in.

4. Find the area of a parallelogram with a base of 16.3 yd. and altitude 5.6 yd., using $A=ba$.

5. Find the area of a square whose side is $18\frac{1}{2}$ cm. ($A=s^2$).

6. Using $S=6e^2$, find the total surface of a cube whose edge is three-quarters of an inch.

7. Using the interest formula, find the simple interest on $320 for $6\frac{1}{2}$ years at 4%.

8. Given $V=\pi r^2 h$, find V if $r=6$ in. and $h=3\frac{1}{2}$ ft.

9. Find the area of a triangle with base=2.3 in. and altitude=5.4 in.

10. Using $s=\sqrt{A}$, compare the sides of two squares whose areas are 16 sq. in. and 64 sq. in. respectively.

11. The commission on sales being given by $c=sr$, find c when $s=\$1,250$ and $r=1\frac{1}{2}\%$.

12. If the commission paid at the rate of 3% is $54, find the amount of the sales. ($s=c\div r$).

13. If $V=\frac{1}{3}\pi r^2 h$, find V: (i) when $r=3.2$ cm., $h=8$ cm.; (ii) when $r=6.4$ cm., $h=8$ cm.; (iii) when $r=3.2$ cm., $h=16$ cm.

14. Using $F=\frac{9}{5}C+32$, the temperature formula, find F when $C=90°$ and also when $C=180°$.

***15.** Using $A=P\ (1+rt)$, find A: (i) when $P=\$325$, $r=4\%$, $t=10$ years; (ii) when $p=\$325$, $r=2\%$, $t=10$ years. Compare the two results.

Review Exercises

1. Write algebraic expressions for the following:
   (*a*) Twice the sum of two numbers (x and y).
   (*b*) The square of the sum of two numbers (c and d).
   (*c*) Four times the cube of a number x.
   (*d*) Three-quarters the square root of a quantity w.

2. Using the formula $d=rt$, find the distance in yards when the rate is 10 ft. per second and the time is 5 minutes.

3. Using the formula, find the height of a triangle 32 sq. cm. in area, having a base of 3.6 cm.

4. Evaluate $V=\frac{1}{3}\pi r^2h$ when $r=4$ in., and $h=14$ in .

5. If $2n$ represents any even number, what would $2n+1$ represent?

6. Given $y=3.2$, $z=1.9$, evaluate x to one decimal place in the formula $x^2=y^2+z^2$.

7. A ship sails due east for 20 miles, then due south for 48 miles. How far is it then from its starting-point?

8. Using $s=\sqrt{A}$, find the side of a square whose area is 1681 square feet.

9. Find the length of the diagonal of a square each side of which is 2 in. long.

10. If $a=\frac{1}{2}b-5$, find the value of a when $b=13.7$.

11. In the same equation if $a=0$, what is the value of b?

12. If $y=mx^2$, find the value of y when $m=1\frac{1}{2}$ and $x=\frac{3}{4}$.

13. A rectangular tank contains 63 cu. ft. It is $3\frac{1}{2}$ ft. long and $2\frac{1}{4}$ ft. wide. What is its depth?

14. If $6\frac{1}{4}$ gallons occupy 1 cu. ft. of space, how many gallons of water would be contained in the tank used in Question 13?

15. Calculate, by formula, the simple interest on \$1200 at $3\frac{1}{2}\%$ for 2 years.

Achievement Test

1. Use algebraic expressions to indicate the following:
   (*a*) The number of ounces in x pounds.
   (*b*) The number of days in y weeks.
   (*c*) The number of cents in f dollars.

2. Give the formula required to find:
   (*a*) The curved surface of a cylinder.
   (*b*) The space enclosed by a conical tent.
   (*c*) The flat surfaces of a 25¢ piece.
   (*d*) The diagonal of a rectangle whose sides are a and b.

3. Evaluate (find the value of the remaining letter):
   (*a*) $A=\frac{1}{2}bh$, when $b=4\frac{1}{2}$ in., $h=2\frac{1}{2}$ in.
   (*b*) $c=2\pi r$, when $\pi=3\frac{1}{7}$, $r=20$ cm.
   (*c*) $r=\frac{i}{pt}$, when $i=\$58.50$, $p=\$225$, $t=6\frac{1}{2}$ years.

4. Using the formula $d=rt$ find the rate when the distance is 660 miles and the time $5\frac{1}{2}$ hours.

5. From the formula for the area find the base of a triangle of area 150 sq. cm. and height 15 cm.

6. An agent was paid \$8.00 at a rate of commission of $2\frac{1}{2}\%$. Apply a formula to find the selling price.

7. If $y=mx+b$, find y when $m=\frac{2}{3}$, $x=15$, $b=3$.

8. If $c=8$, $d=3$, find the value of c^2d-cd^2.

9. When $t=.5$, evaluate $3t^2+\frac{1}{t}$

10. (*a*) If x lb. tea cost y cents, what would 1 lb. cost?
    (*b*) If c dollars buys d books, how many cents will one book cost?
    (*c*) If I drive y miles at z miles per hour, how many hours do I take?

CHAPTER VII

DIRECTED NUMBER

If you examine a thermometer, you will find that the numbers on it range upward from 0° and also downward. On some thermometers, the numbers below 0° have a minus sign before them, as—20°. If you look at a weather report in the newspaper during the winter months, you will also find temperatures with this minus sign attached, as in the following:

	Highest Temp.	*Lowest Temp.*
Regina	0°	—8°
Calgary	—5°	—15°
Montreal	10°	—2°
Vancouver	36°	30°

This shows you the use of two kinds of numbers. The minus sign in these cases does not indicate subtraction.

There are numerous other examples of the use of these two kinds of numbers. If you are on the main or ground floor of a large building, the elevator may take you up two floors or down two floors. We may say that we travel in the elevator +2 floors or —2 floors. The sign + may mean up or down, that is, if we take +2 to mean up two floors, then we must take —2 to mean down two floors. It is important, however, for you to understand that you may allow +2 to mean two floors down, but in that case —2 means two floors up.

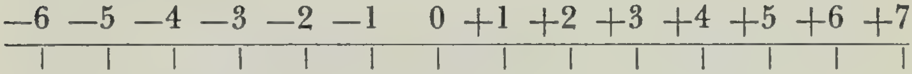
In the weather report shown above, it is generally recognized that a number with a minus sign attached to it means a temperature *below* zero (0°). In this way, all who read the report interpret it in the same way.

In algebra, which is simply another name for generalized arithmetic, these two kinds of numbers are known as **positive numbers** and **negative numbers.** Those with a plus sign (+) before them are positive numbers. If no sign is in front it is understood that they also are positive. The negative numbers are indicated by the minus sign (—).

One of the many ways in which algebra is superior to arithmetic comes from the use of positive and negative numbers. In ordinary arithmetic, numbers begin at 0 and increase without limit. On the other hand, in generalized arithmetic, numbers have neither a beginning nor an end. The integer (whole number) coming before 0 is denoted by —1, while —2 is the next one smaller than —1, and so on. The number 0 should not be considered as a starting-point, but rather as a convenient point from which to reckon any other number.

This scale of numbers may be represented graphically on a line drawn across the page from left to right. The negative numbers may be ranged from 0 to the left and the positive numbers from 0 to the right.

The diagram below is the one commonly used, but we may place the negative numbers at the right-hand side and the positive ones at the left. We may even place the line up and down.



This scale may be extended in both directions as far as you please. To the right of zero, we note that any number of the scale is greater than the number on its left, and smaller than the one on its right. We make a similar statement regarding negative numbers. We know, for example, that —5° represents a higher temperature than —6°. We say that —6 is *less* than —5, or —5 *greater* than —6. Thus, with reference to the scale, we say that any number, whether negative or positive, is greater than the numbers to its left and smaller than the numbers to its right. For example, —1 is greater than —4, 0 is greater than —1, —5 is less than —2.

Exercises (oral)

1. Is -4 greater or less than $+3$?

2. Which is greater, 0 or -2, -6 or -3, $+5$ or -6?

3. Which is the least of these numbers: 8, -2, -5, 2, -1?

4. On the scale, where are the numbers placed which are greater than $+3$, those less than $+4$, those greater than -2, those less than -5?

5. What numbers in general are greater than 0, and which are less than 0?

Positive and Negative Signs

The signs $+$ and $-$ now have two meanings:

(1) They indicate addition and subtraction as formerly, and, as such, they are *symbols of operation.*

(2) They are also *symbols of quality* or *direction.* In this sense they distinguish positive numbers from negative numbers. $+6$ is read 'plus six', and -5 is read 'minus five'.

The following exercises illustrate this second use.

1. By stating, first, which represents the positive direction, use $+$ and $-$ signs to illustrate the following latitudes and longitudes:

15° N. lat. 85° S. lat. 8° S. lat.
45° E. long. 80° W. long. 161° W. long.

2. Indicating which is to be the positive direction, illustrate the following movements of an elevator: (*a*) six floors up; (*b*) four floors down; (*c*) three floors down.

3. Taking sea-level as zero or 0, use symbols for the following:

(*a*) The summit of Mount Everest is nearly 29,000 ft. high.

(*b*) The level of the Dead Sea is 1,280 ft. below sea-level.

4. If a bank deposit is considered $+$ and a withdrawal $-$, use algebra to indicate the following transactions made by T. H. Smith:

September	1	Deposited	\$180.25
"	8	Withdrew	50.00
"	12	Withdrew	28.50
"	15	Deposited	73.20
"	20	Withdrew	45.80

Indicate his balance by positive or negative numbers after the last withdrawal.

5. Use $+$ and $-$ signs to indicate the dates:
55 B.C.; A.D. 1603; A.D. 352; 532 B.C.

6. The shares of a certain company were selling at \$3.20 each at the beginning of the week. The following changes occurred during the week: $+5¢$, $+2¢$, $-10¢$, $-2¢$, $+3¢$. What was its price after the last rise?

7. The following hourly readings on a thermometer were registered: 54°, 56°, 55°, 62°, 67°, 61°. Indicate the changes of temperature by positive and negative numbers.

8. The pressure of the atmosphere was shown by the following daily barometric readings: 30.25, 30.28, 30.27, 30.29, 30.15, 29.90. Indicate algebraically each change of pressure.

Directed Numbers

A number is said to be *in the positive direction* from a given number on the scale when it lies *to the right* of that number.

In this sense, the following numbers, 5, 0, -1, $+10$, -3, are in the positive direction from -6.

A number is said to be *in the negative direction* from a given number when on the scale it lies *to the left* of that number.

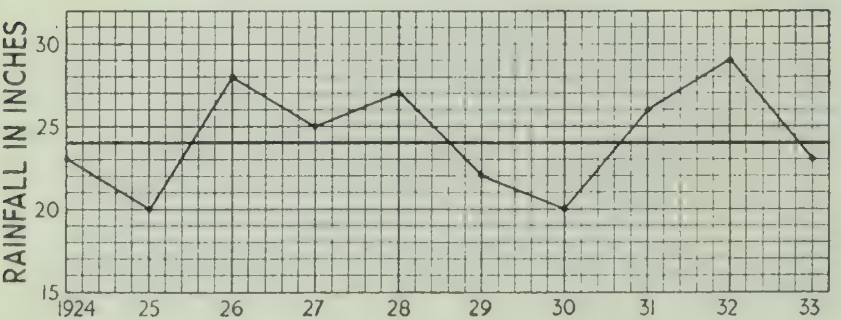
The direction from $+2$ to -6 is negative, while the direction from -6 to $+2$ is positive.

The following table will perhaps make clearer to you how a number has a positive or a negative direction with respect to a given number. It shows the annual rainfall in inches of a certain place over a period of ten years. The fractional parts have been omitted for the sake of clearness.

Year	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933
Rainfall	23	20	28	25	27	22	20	26	29	23
Deviation from Average	−1	−4	+4	+1	+3	−2	−4	+2	+5	−1

The average rainfall is 24 in. The third row in our table shows whether a number is greater or less than the average; in other words, whether its direction from the average number is positive or negative.

A graph of the same figure is shown below .



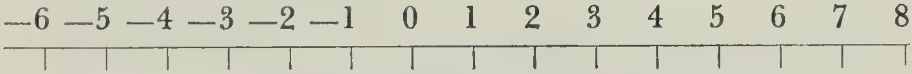
Exercises:

1. The room temperature being considered average, or normal, at 68°, indicate deviations from normal by a table like that used in the above example on rainfall. Beginning at 8 a.m., and at every hour until 5 p.m., the temperatures were 69°, 68°, 70°, 71°, 68°, 67°, 66°, 67°, 68°, 69°.

2. The average outside temperature during a week in January was −3°. Show deviations from the average in the case of these temperatures taken during that week: Sunday, −4°; Monday, −8°; Tuesday, −3°; Wednesday, −1°; Thursday, 0°; Friday, 2°; Saturday, 3°.

Addition

The application of addition to negative as well as to positive numbers is most easily understood by the use of a number scale. However, your teacher may have other ways of explaining the process.



You must understand that +6 has two meanings:

(1) It may represent a point on the number scale 6 units to the right of 0.

(2) It may mean a distance of 6 units measured in a positive direction, *beginning at any given point* on the scale. So also, —3 may mean a point on the scale 3 units to the left of 0, or it may indicate a measurement of 3 units in a negative direction, which, on the scale, is to the left, *beginning at any given point.*

The operation of addition will be understood much more clearly if these facts are kept in mind.

Example 1. Add (+2) to (+5).

We begin at a point on the scale represented by +5, that is, 5 units to the right of 0, and, with that as a starting-point, we count 2 units in a positive direction, which, on the scale, is to the right. We reach the point 7, which, being on the right of 0, is represented by +7.

Thus $\begin{array}{r} +5 \\ \underline{+2} \\ +7 \end{array}$ or $(+5)+(+2)=+7$

In like manner
$(+2)+(+5)=+7$

Example 2. Add (—2) to (+5).

We again begin at the point +5, but we count 2 units in a negative direction, which is to the left, reaching the point +3.

So $\begin{array}{r} +5 \\ \underline{-2} \\ +3 \end{array}$ or $(+5)+(-2)=+3.$

Now consider the problem: Add $(+5)$ to (-2).

We begin at the point (-2), and count 5 units in a positive direction, reaching the same point as above, namely, $+3$.

Thus $$(+5)+(-2)=(-2)+(+5).$$

Example 3. Add $(+2)$ to (-5).

Here we begin at -5, which is a point at the left of 0, and, with that as the starting-point, we count 2 units in a positive direction, or to the right, reaching -3.

Hence $\begin{array}{r} -5 \\ \underline{+2} \\ -3 \end{array}$ or $(-5)+(+2)=-3$

So also $(+2)+(-5)=-3.$

Example 4. Add (-2) to (-5).

Again, beginning at -5 and counting 2 units in a negative direction, we reach the point represented by -7.

So $\begin{array}{r} -5 \\ \underline{-2} \\ -7 \end{array}$ or $(-5)+(-2)=-7$

Again, from the scale, $(-2)+(-5)=-7$

Exercises:

1. Using the number scale, add the following pairs of numbers:

$$\begin{array}{rrrrrrrrrr} +2 & -6 & +2 & +7 & -1 & -5 & +1 & 0 & -5 & +4 \\ \underline{-3} & \underline{+4} & \underline{-8} & \underline{-5} & \underline{-4} & \underline{-3} & \underline{-4} & \underline{-6} & \underline{0} & \underline{-2} \end{array}$$

2. In the next group the positive numbers are written without the sign $+$.

$$\begin{array}{rrrrrrrrrr} 5 & -2 & -3 & 4 & -3 & -2 & -5 & 6 & 0 & -2 \\ \underline{-1} & \underline{0} & \underline{-3} & \underline{-2} & \underline{3} & \underline{8} & \underline{-3} & \underline{-6} & \underline{-7} & \underline{-6} \end{array}$$

3. $(-2)+(+3)=$
$(+3)+(-2)=$
$(+6)+(-4)=$
$(-6)+(+1)=$
$(-3)+(-3)=$
$(-4)+(+2)=$
$(+3)+(+4)=$
$(0)+(-8)=$

$(-4)+(0)=$
$(-4)+(-3)=$
$(+1)+(-6)=$
$(-3)+(-4)=$
$(4)+(-1)=$
$(-2)+(6)=$
$(3)+(-5)=$
$(-6)+(6)=$

$(+5)+(+3)+(-2)=$
$(+6)+(-1)+(-4)=$
$(-2)+(+8)+(-5)=$
$(-3)+(-2)+(-1)=$

4. Make exercises of your own and practise adding.

The rule for addition follows. To understand it you must know the meaning of *absolute value.* By that we mean the value of a number without considering its sign. Thus the absolute value of $+5$ or of -5 is simply 5.

RULE FOR ADDITION

To add numbers with like signs, find the sum of their absolute values and keep the same sign. To add numbers with unlike signs, find the difference of their absolute values and give the sign of the number with the greater absolute value.

The unit in our scale may represent inches, which we can express by the letter i. Then $(-3i)+(+7i)=+4i$. It may represent any sort of quantity, and the various units may then be expressed by literal numbers, $c, d, x, y, r, a,$ and so on.

Thus
$$\begin{array}{r} -2ab \\ \underline{+6ab} \\ +4ab \end{array} \qquad \begin{array}{r} +3x^2 \\ \underline{-2x^2} \\ +x^2 \end{array} \qquad \begin{array}{l} (-5c)+(+2c)=-3c \\ \\ (-8d^3)+(-2d^3)=-10d^3 \end{array}$$

Exercises (oral)

Add:

(1)
$$\begin{array}{r} -6a \\ \underline{+2a} \end{array} \quad \begin{array}{r} -5x \\ \underline{-2x} \end{array} \quad \begin{array}{r} +8a^2 \\ \underline{-3a^2} \end{array} \quad \begin{array}{r} -12bc \\ \underline{-3bc} \end{array} \quad \begin{array}{r} +2c^2d \\ \underline{3c^2d} \end{array} \quad \begin{array}{r} -rt \\ \underline{-rt} \end{array} \quad \begin{array}{r} \pi d \\ \underline{2\pi d} \end{array}$$

$$(2)\quad \begin{array}{r} 3a \\ 0 \\ \hline \end{array} \qquad \begin{array}{r} 0 \\ 2c \\ \hline \end{array} \qquad \begin{array}{r} -5x \\ 0 \\ \hline \end{array} \qquad \begin{array}{r} -5\pi r^2 \\ -6\pi r^2 \\ \hline \end{array} \qquad \begin{array}{r} -7lu \\ 3lu \\ \hline \end{array} \qquad \begin{array}{r} 8cd \\ cd \\ \hline \end{array} \qquad \begin{array}{r} x^2 \\ -x^2 \\ \hline \end{array}$$

$$(3)\quad \begin{array}{r} -3\,(a+b) \\ -2\,(a+b) \\ \hline \end{array} \qquad \begin{array}{r} 7\,(x-2) \\ -3\,(x-2) \\ \hline \end{array} \qquad \begin{array}{r} -4\,(x+3) \\ +6\,(x+3) \\ \hline \end{array} \qquad \begin{array}{r} -2\,(a+3b) \\ -5\,(a+3b) \\ \hline \end{array}$$

Ans. $= -5\,(a+b)$.

4. $(3x)+(-5x)=$
5. $(-6y)+(-10y)=$
6. $(2c^2)+(5c^2)=$
7. $(-2xy)+(-3xy)=$
8. $(8a^2)+(-10a^2)=$
9. $(a^2)+(5a^2)=$
10. $(-bc)+(bc)=$
11. $(xy)+(-2xy)=$
12. $(4x^3)+(-3x^3)=$
13. $(-8x^2y)+(-2x^2y)=$
14. $(3c)+(0)=$
15. $(2\sqrt{x})+(5\sqrt{x})=$
16. $(0)+(+2ab)=$
17. $(-5\sqrt{a})+(2\sqrt{a})+(-\sqrt{a})=$
18. $(-ab^2)+(7ab^2)+(-5ab^2)=$
19. $(\frac{1}{2}xy)+(-\frac{1}{2}xy)+(xy)=$

Addition of Polynomials

Observe the following:

3 feet 4 inches	3 ft. 4 in.	$3f+4i$	Addends
6 feet 2 inches	6 ft. 2 in.	$6f+2i$	
9 feet 6 inches	9 ft. 6 in.	$9f+6i$	Sum

Example **1.** Find the sum of $2a+3b$ and $4a-8b$.

Arrange like terms in the same column and add the columns separately:

$$\begin{array}{r} 2a+3b \\ 4a-8b \\ \hline 6a-5b \\ \hline \end{array}$$

NOTE: If the first term of a polynomial is positive the sign is generally omitted.

Example 2. $(3a-2x)+(-5a+3x)+(5x-6a)$.

In this case, the addition is indicated by the signs that separate the three groups.

We may arrange our work as below:

$$\begin{array}{r} 3a-2x \\ -5a+3x \\ -6a+5x \\ \hline -8a+6x \\ \hline \end{array}$$

Or we may set down the individual terms in a row and add horizontally, thus:

$$\begin{aligned} &(3a-2x)+(-5a+3x)+(5x-6a) \\ &=3a-2x-5a+3x+5x-6a \\ &=3a-5a-6a-2x+3x+5x \\ &=-8a+6x \end{aligned}$$

NOTE: A *polynomial* has more than one *term*. Terms are separated by + or − signs. In $2a+3b^2$, the terms are $2a$ and $3b^2$.

Adding Polynomials

Exercises. Add:

(1) $\begin{array}{r} 3x+2y \\ 2x+5y \\ \hline \end{array}$ (2) $\begin{array}{r} +5c-2d \\ +3c+6d \\ \hline \end{array}$ (3) $\begin{array}{r} -4x-8a \\ +3x+2a \\ \hline \end{array}$

(4) $\begin{array}{r} a+b-c \\ a+3b-2c \\ \hline \end{array}$ (5) $\begin{array}{r} 3x+a+2y \\ 2x-a+5y \\ \hline \end{array}$ (6) $\begin{array}{rr} -5a & +2b \\ +3a & \\ \hline \end{array}$

(7) $\begin{array}{rr} 6c & \\ -2c & -d \\ \hline \end{array}$ (8) $\begin{array}{r} 5x^2-y^2 \\ -3x^2+7y^2 \\ \hline \end{array}$ (9) $\begin{array}{r} -4c+x \\ 6c-3x \\ \hline \end{array}$

(10) $\begin{array}{r} 3ab-2cd \\ -6ab+8cd \\ \hline \end{array}$ (11) $\begin{array}{rr} -3a & -5x \\ & +8x \\ \hline \end{array}$ (12) $\begin{array}{r} -2a-2b \\ -2a+2b \\ \hline \end{array}$

(13) $\begin{array}{r} 3a-2x \\ -2a-5x \\ -6a+8x \\ \hline \end{array}$ (14) $\begin{array}{rr} -2l & +3k \\ 5l & -6k \\ 4l & \\ \hline \end{array}$ (15) $\begin{array}{r} 3x^2-2x \\ -5x^2+8x \\ -x^2-9x \\ \hline \end{array}$

(16) $\begin{array}{r} .2a+.3b \\ 1.4a+.8b \\ \hline \end{array}$ (17) $\begin{array}{r} 3.5c-2.5d \\ -1.2c+.6d \\ \hline \end{array}$ (18) $\begin{array}{r} .05x+.25y \\ .35x-y \\ \hline \end{array}$

(19) $\begin{array}{rrr} -3a & +2b & -c \\ -a & -b & -c \\ a & +5b & +c \\ \hline \end{array}$ (20) $\begin{array}{rrr} 4x^2 & -2xy & -y^2 \\ -3x^2 & & +5y^2 \\ & -6xy & -y^2 \\ \hline \end{array}$ (21) $\begin{array}{rr} 14c^3 & -12c^2 \\ -8c^3 & \\ -6c^3 & +12c^2 \\ \hline \end{array}$

(22) $\begin{array}{r} 5-6x-3x^2 \\ -3+2x-8x^2 \\ -4-3x+9x^2 \\ \hline \end{array}$ (23) $\begin{array}{rrr} 9a & -2b & +5 \\ 4a & & -6 \\ & -5b & -2 \\ \hline \end{array}$ (24) $\begin{array}{rr} 2.1a & -b \\ -1.6a & +.3b \\ -.8a & -b \\ \hline \end{array}$

Exercises (written):

Arrange the expressions in vertical columns with like terms in the same column, and add:

(1) $3a+2b;\ 2a-5b;\ 6a-8b.$
(2) $+7x-3y;\ -7x-5y.$
(3) $3a+2b+4c;\ 2a-3b-5c.$
(4) $-5x+y;\ -x+2y;\ 4x-3y.$
(5) $a-b+c;\ b+c-a;\ a-c-b.$
(6) $3a-2x+5y;\ 7y-4a;\ -2y-a+x.$
(7) $2a^2-3b^2;\ 6b^2-5a^2;-8a^2-7b^2.$
(8) $3ab-2bc+ca;\ 3bc-2ca+ab.$
(9) $5x^2y-3xy^2;\ 8xy^2-2x^2y.$
(10) $.5a^2-.2x^2;\ a^2+1.5x^2;\ x^2-1.7a^2.$

Addition by the 'Horizontal Method'

One example was shown to you, but perhaps further explanation would help.

Instead of writing $(+5)+(+2)$ we may write $+5+2$ by dropping the brackets. Also, of course, we may write $5+2$. Thus $(+5)+(-2)$ may be written as $5-2$ where we may think of it as the sum of $+5$ and -2, which equals 3, or we may think of it as 5 with 2 subtracted, giving 3 also. Again $(-4)+(-5)$ may be written as $-4-5$, meaning that -5 is being added to -4.

To add a series of numbers such as -3, 2, -5, -6, 7, -9, we might place them in a vertical column, or we might place each in brackets $(-3)+(+2)+(-5)$, etc., but generally we write:

Sum$=-3+2-5-6+7-9=-14.$

Example 1. Add: $-2a+3b;\ 5a-6b;\ -3a+5b;\ a-b.$

Sum$=-2a+3b+5a-6b-3a+5b+a-b$
$=a+b.$

Example 2. Add: $(-2x+y)+(-3x-2y)+(-2x)+5x.$

Sum$=-2x+y-3x-2y-2x+5x$
$=-2x-y.$

Exercises:

Add:

1. $(3a-2b)+(-2a-3b)+(-7a+8b)$.
2. $(2c-d)+(+5c-d)+(-2c-6d)$.
3. $4x+(3x-2y)-2y$.
4. $(-5a-3x+2y)+(a+x-y)$.
5. $(-6c+2a)+(4c-3a)+2c$.
6. $(2c-a)+(a-2c)+(4a+5c)$.
7. $(-5x-y)+(-2x+5y)+(7x-4y)$.
8. $(x+2)+(3x-5)+(2x-8)+(x-1)$.
9. $(4-2a)+(-3a+a)+(-1+5a)$.
10. $6-(2x-3y)+(5-2y)+(4x-8)$.
11. $(3a^2+2a+4)+(2a^2+6a+3)+(4a^2-9a-10)$.
12. $(-2a^2-7a+6)+(5a^2+8a-7)+(-3a^2-a+1)$.
13. $(8x^2-9)+(4x-6)+(-2x^2-5x)+(x^2-x+1)$.
14. $(6x^2+2xy-y^2)+(3x^2-5xy+2y^2)+(-10x^2-12xy-4y^2)$.
15. $(-5x^3-3x^2)+(2x^2-5x)+(6x-3)+(8-6x^3)$.
16. $(2ab-3bc-ca)+(5ab+2ca-bc)+(ac-cb)$.
17. $(6xya-2)+(4-3ayx)+(5+2yxa)$.
18. $(2x^2y+5xy^2)+(3xy^2-x^2y)+(2xy^2+5x^2y)$.
19. $(2xy^2+3)+(4-5y^2x)+(4y^2x-8)$.
20. $(1.5c-d)+(2.3c+5d)+(-c-1.6d)$.
21. $(.2x+.3a+.5)+(.8x-.3a-.7)$.
22. $(a-1.3)+(-a-2.5)+(-3a+1.8)$.

*23. $8+(2a^2b+3ab^2)+(5a^2b-7)+(10-b^2a)$.

*24. $(y^2-x^2)+(x^2-3xy)+(x^2+3y^2-4xy)$.

*25. $(4r^2-25s^2)+(r^2+13rs-40s^2)+(r^2-12rs+27s^2)$.

*26. $(m^4+8m^2n+27n^4)+(m^4-5n^4)+(2m^4-11m^2n-2n^4)$.

*27. $(\frac{4}{5}a+\frac{2}{3}b+\frac{3}{4}c)+(\frac{2}{3}a+\frac{1}{4}b+\frac{2}{3}c)+(\frac{3}{4}a-\frac{1}{5}b-\frac{4}{5}c)$.

Subtraction

Example 1. Take (+3) from (+5).

This means: How much must be added to (+3) to give (+5)? Since (+2) added to (+3) gives (+5), the required answer is (+2).

Thus (+5)−(+3)=(+2).

Example 2. Take (+3) from (−5).

We ask: How much must be added to (+3) to give (−5)? Since (−8) added to (+3) gives (−5), the required answer is (−8).

So (−5)−(+3)=(−8).

Example 3. Take (−3) from (+5).

Since (−3)+(+8)=(+5),
it follows that (+5)−(−3)=(+8).

Example 4. Take (−3) from (−5).

Since (−3)+(−2)=(−5),
then (−5)−(−3)=(−2).

Exercises:

1. What must be added to (+3) to give (+5)?
to (+8) to give (+6)?
to (−2) to give (+7)?
to (+4) to give (−4)?
to (−1) to give 0 ?

2. Subtract the lower number from the upper:

+6	+4	+3	−5	+2	+5	−2	+6	+2	+8
+4	+6	−2	−2	−3	−4	−5	−7	+9	−2

3	−6	−1	5	−3	−5	0	8	0	−2
−4	5	0	0	−3	−4	2	−5	−4	−6

3. Give answers to the following:

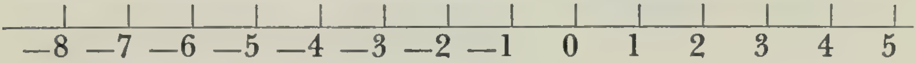
(+8)− (+3)=
(+8)− (−3)=
(−8)− (+3)=
(−8)− (−3)=

(−4)− (−4)=
(+5)− (+5)=
(0)− (−3)=
(−8)− (0)=

$(-3)-(+8)=$
$(+3)-(-8)=$
$(+3)-(+8)=$
$(-3)-(-8)=$

$(-8)-(+8)=$
$(-6)-(5)=$
$(5)-(-6)=$
$(3)-(7)=$

The examples given above may also be worked by means of the number scale. One example will be illustrated.



Example: Take $(+3)$ from (-5).

We ask the question: How much must be added to $(+3)$ to give (-5)?

Beginning at $(+3)$, a point 3 units to the right of 0, we must go to (-5), which is 5 units to the left of 0. This requires 8 units measured to the left, that is, in a negative direction.

Thus $(-5)-(+3)=(-8)$.

Addition		*Subtraction*
$(+5)+(-2)=+3$	$\longleftrightarrow$	$(+5)-(+2)=+3$
$(-5)+(-2)=-7$	$\longleftrightarrow$	$(-5)-(+2)=-7$

By comparing these results we find that adding (-2) has the same effect as subtracting $(+2)$.

Now compare these:

$$(+5)+(+2)=7 \longleftrightarrow (+5)-(-2)=7$$
$$(-5)+(+2)=-3 \longleftrightarrow (-5)-(-2)=-3$$

Here we find that adding $(+2)$ has the same effect as subtracting (-2).

Using symbols, we have:

$$+(-2)=-(+2)$$
$$+(+2)=-(-2).$$

We thus have a simple *rule for subtraction*:

To subtract, change the sign of the quantity to be subtracted (the subtrahend), then add it to the other (the minuend).

In practice change the sign mentally, then add.

Example 1.

$$\begin{array}{lr} \text{From} & +9 \\ \text{take} & \underline{-2} \\ & 11 \end{array}$$

Mentally change (-2) into $(+2)$ than add $+2$ to $+9$.

We may also write $(+9)-(-2)=(+9)+(+2)=9+2=11$.

Example 2. Subtract $+3x$ from $-4x$.

$$\begin{array}{r} -4x \\ \underline{+3x} \\ -7x \end{array}$$

Mentally change $+3x$ into $-3x$ then add $-3x$ to $-4x$.

Or, $(-4x)-(+3x)=(-4x)+(-3x)=-4x-3x=-7x$.

Exercises:

1. Using the Rule for Subtraction, subtract the lower number from the upper:

5	6	−2	−11	−3	−10	−9	0	7	8
−4	−8	−9	−4	12	10	8	−6	−2	−13
2	−6	−7	−8	2	−15	−12	−7	11	0
−7	6	−8	−8	−9	3	0	7	−6	14

2. Give answers to these:

$(3)-(-4)=$ $(-5)-(4)=$
$(-6)-(1)=$ $(-12)-(10)=$
$(-2)-(0)=$ $(14)-(-3)=$
$(8)-(-10)=$ $(16)-(20)=$
$(0)-(6)=$ $(-10)-(12)=$

3. Work the following:

$(+4)+(-3)-(+5)=$ $(-6)+(+9)-(-4)=$
$(+2)+(-6)-(+3)=$ $(-2)+(-8)-(-2)=$
$(-5)+(+8)-(+4)=$ $(8)-(6)+(-3)=$
$(-3)+(-4)-(6)=$ $(-3)-(2)+(5)=$
$(6)-(5)+(7)=$ $(7)-(9)-(4)=$
$(-3)-(4)+(-2)=$ $(2)-(-8)-(-6)=$

Exercises (oral):

Subtract the lower number from the upper:

$$(1)\ \begin{array}{r} +4a \\ \underline{+6a} \end{array} \qquad (2)\ \begin{array}{r} +4a \\ \underline{-6a} \end{array} \qquad (3)\ \begin{array}{r} -4x \\ \underline{-8x} \end{array} \qquad (4)\ \begin{array}{r} -3y \\ \underline{+7y} \end{array} \qquad (5)\ \begin{array}{r} -7y \\ \underline{+3y} \end{array}$$

$$(6)\ \begin{array}{r} 2ab \\ \underline{7ab} \end{array} \qquad (7)\ \begin{array}{r} -8a^3 \\ \underline{-3a^3} \end{array} \qquad (8)\ \begin{array}{r} 5x^2y \\ \underline{8x^2y} \end{array} \qquad (9)\ \begin{array}{r} -6ab \\ \underline{ab} \end{array} \qquad (10)\ \begin{array}{r} d \\ \underline{-3d} \end{array}$$

$$(11)\ \begin{array}{r} -2xa \\ \underline{-2xa} \end{array} \qquad (12)\ \begin{array}{r} +9a^2 \\ \underline{-9a^2} \end{array} \qquad (13)\ \begin{array}{r} -10cd \\ \underline{+10cd} \end{array} \qquad (14)\ \begin{array}{r} 5c \\ \underline{0} \end{array} \qquad (15)\ \begin{array}{r} 0 \\ \underline{3x} \end{array}$$

$$(16)\ \begin{array}{r} -2b \\ \underline{0} \end{array} \qquad (17)\ \begin{array}{r} 0 \\ \underline{-5y} \end{array} \qquad (18)\ \begin{array}{r} 6vx \\ \underline{-2vx} \end{array} \qquad (19)\ \begin{array}{r} 2(a+b) \\ \underline{5(a+b)} \\ ?\ (a+b) \end{array} \qquad (20)\ \begin{array}{r} 4(x+y) \\ \underline{-8(x+y)} \end{array}$$

The following may be written or done orally:

21. $(+2a)-(+5a)$
22. $(-6y)-(8y)$
23. $(7b)-(9b)$
24. $(-5xy)-(-2xy)$
25. $(8ab)-(3ab)$
26. $(-6a^2)-(7a^2)$
27. $(-2x^3)-(-2x^3)$
28. $(-bc)-(bc)$
29. $(-x)-(-x)$
30. $(4a^3)-(5a^3)$
31. $(-3x^3)-(-2x^3)$
32. $0-(2a)$
33. $(-6a)-0$
34. $0-(-8x)$
35. $7ab-0$
36. $(-6vx)-(-2vx)$
37. $8c^2-(-5c^2)$
38. $-7xyz-(5xyz)$
39. $r^2-(-6r^2)-(+2r^2)$
40. $(3a)-(-5a)-(2a)$

Subtraction of Polynomials

Example 1. Subtract $3a-2b+5c$ from $-2a+b-6c$.

First Method:

$$\begin{array}{r} -2a+\ b-\ 6c \\ \underline{3a-2b+\ 5c} \\ -5a+3b-11c \end{array}$$

Arrange in vertical columns with like terms in each column. Subtract as if you were dealing with three separate pairs of monomials. $3a$ is changed mentally to $-3a$, then added to $-2a$, and so with the others.

Second Method: We may set down the question horizontally, enclosing the expressions in brackets and placing a minus sign between to indicate subtraction:

$$\begin{aligned} &(-2a+b-6c)-(3a-2b+5c) \\ =&(-2a+b-6c)+(-3a+2b-5c) \\ =&-2a+b-6c-3a+2b-5c \\ =&-5a+3b-11c \end{aligned}$$

You will notice that in the second line we do two things: we change the $-$ between the expressions into $+$, and we change the signs of all three quantities in the expression which is being subtracted.

Example 2. From $-6a-8a^2+5$ take $-7a^2+8$.

First Method: In the second expression there is no term corresponding to $-6a$, so, in arranging in columns, we simply leave a space. You will observe also, that the expression has been arranged with the a^2 term coming first, followed by the term containing a:

$$\begin{array}{rrr} -8a^2 & -6a & +5 \\ \underline{-7a^2} & \underline{} & \underline{+8} \\ -\ a^2 & -6a & -3 \end{array}$$

Second Method $(-6a-8a^2+5)-(-7a^2+8)$

$$\begin{aligned} &=-6a-8a^2+5+7a^2-8 \\ &=-a^2-6a-3. \end{aligned}$$

In the second method it is not necessary to arrange the expressions in order, nor is it necessary to consider the missing term. You should notice, too, that the brackets are dropped as soon as the signs are altered.

Exercises (oral or written):

Subtract the lower expression from the upper:

(1) $\begin{array}{r} 5a+3b \\ \underline{2a+6b} \end{array}$ (2) $\begin{array}{r} 6c-3d \\ \underline{3c+7d} \end{array}$ (3) $\begin{array}{r} -2x+4y \\ \underline{+3x-2y} \end{array}$ (4) $\begin{array}{r} -8x-\ y \\ \underline{+3x-5y} \end{array}$

(5) $\begin{array}{r} 3c-2d \\ \underline{c-\ d} \end{array}$ (6) $\begin{array}{r} -c+5d \\ \underline{-c-6d} \end{array}$ (7) $\begin{array}{r} x-y \\ \underline{x-y} \end{array}$ (8) $\begin{array}{r} x+y \\ \underline{x-y} \end{array}$

$$(9)\ \begin{array}{r}3a-2b\\-2a\\\hline\end{array}\qquad(10)\ \begin{array}{r}-5x\\-6x+y\\\hline\end{array}\qquad(11)\ \begin{array}{r}7c-2d\\6d\\\hline\end{array}\qquad(12)\ \begin{array}{r}-2b\\a+b\\\hline\end{array}$$

$$(13)\ \begin{array}{r}-8x+9y-z\\-7x+12y+z\\\hline\end{array}\qquad(14)\ \begin{array}{r}-5a-6b\\9a+7b-c\\\hline\end{array}\qquad(15)\ \begin{array}{r}12a\qquad-10c\\13a-2b+c\\\hline\end{array}$$

$$(16)\ \begin{array}{r}1.5a-2.5b\\2.3a+.6b\\\hline\end{array}\qquad(17)\ \begin{array}{r}.25c+.75x\\-.30c+.04x\\\hline\end{array}\qquad(18)\ \begin{array}{r}2.35+1.8d\\5.14-2.9d\\\hline\end{array}$$

$$(19)\ \begin{array}{r}\frac{1}{2}a-2b+c\\\frac{1}{4}a+\frac{1}{2}b-3c\\\hline\end{array}\qquad(20)\ \begin{array}{r}\frac{5}{2}b-\frac{3}{4}h\\\frac{7}{2}b-\frac{5}{4}h\\\hline\end{array}\qquad(21)\ \begin{array}{r}\frac{4}{3}-y\\1+\frac{5}{8}y\\\hline\end{array}$$

Arrange the next five subtractions in vertical form:

(22) Subtract $-7c-5d-6$ from $8c-7d-5$.
(23) From $5a^2-2ab-3b^2$ take $8a^2-9b^2$.
(24) Take $10x^2-8x-7$ from x^2-x-1.
(25) Take $-2ab-3bc-2ac$ from $5bc-6ac$.
(26) Subtract $5x^3-2x-9$ from $6x-8x^3-2x^2$.

In the next group perform the operation indicated, and, after removing brackets, collect terms:

(27) $(4a-2b-c)-(2a-6b-4c)$.
(28) $(-6a-2c-b)-(5b-3a-c)$.
(29) $(-x^2-2x-3)-(4x^2-5x-6)$.
(30) $(4y^2-3x^2)-(7x^2-3xy)$.

Use the bracket form in the following examples:

(31) Subtract $-3x-12y$ from $-5x-6y$.
(32) Subtract $2a-3b$ from $7a-6b$.
(33) Take $-3x-2a-4b$ from $-3a-8b-5x$.
(34) From $3x-2y$ take $-8x-9y$.
(35) Take away $3a^2-2a-5$ from $9a^2-6-7a$.
(36) From $7a-9c$ take $-8a-4c-2b$.
(37) Subtract $11-a-5b$ from $8b-2a-7$.
(38) Take $9r^3-2r^2-7r$ from $6r-8r^2-12r^3$.
(39) From $6ab-2bc-3ca$ take $-9bc-8ca-11ab$.

**Harder Exercises* (*optional*):

Retain the brackets.

(40) Subtract $6(a-b)$ from $2(a-b)$.

(41) Take $8(y-2x)$ from $7(y-2x)$.

(42) From $3(a-b)-2(x-y)$ take $8(a-b)-7(x-y)$.

(43) From $-8(x-2y)-9(a-2b)$ take $7(x-2y)-9(a-2b)$.

(44) Subtract $5(a-b)^2-3(a-b)$ from $7(a-b)^2-9(a-b)-7$.

(45) From $6x^2-2(a-b)^2$ take $-5x^2-4(a-b)^2$.

Descending and Ascending Order

When doing much work with polynomials (that is, expressions with many terms) it is usual to arrange the terms in a definite order.

1. With only one letter present, arrange either in descending order, that is, with the highest power coming first, or in ascending order, with the lowest power at the beginning.

The following example will illustrate this for you:

Expression	$2x+2-5x^2-6x^3$
Descending order	$-6x^3-5x^2+2x+2$
Ascending order	$2+2x-5x^2-6x^3$

In descending order, note that the exponents diminish in size: 3, 2, 1 (because x means x^1), and then the term with no x.

2. Where there are two letters present, decide on one of the letters, and arrange in the order of that letter.

Expression	$2a^3-5b^3-6a^2b+ab^2$
Descending order in a	$2a^3-6a^2b+ab^2-5b^3$
Ascending order in a	$-5b^3+ab^2-6a^2b+2a^3$

Exercises:

Arrange each of the following expressions in ascending order:

1. $3c^2-2+5c^3-6c$.

2. $\frac{1}{2}b+\frac{5b^2}{2}-b^4-3b^3$.

3. x^2-1.

Arrange the next group in descending order:

4. $2y^2-3y+y^3$.
5. $1+x^3$.
6. $1-a^2-a^4-a$.

7. Place the following three expressions in columns in descending order of x and add together:

$$2x^3-x^2y+y^3;\ 2y^3-3xy^2+x^3;\ 5y^3-3xy^2.$$

8. Arrange the following two expressions in ascending order of a, and subtract the second from the first:

$$3a^2c+2c^3-a^3+4ac^2;\ 5ac^2+2a^3-c^3-2a^2c.$$

Removing Brackets

You have already used brackets in addition and subtraction, so what follows will be mainly a review.

Since $6+(+2)=6+2$ and $6+(-2)=6-2$, it follows that when a plus sign precedes a bracket, the bracket may be removed without altering the sign inside the brackets. But $6-(-2)=6+2$ and $6-(+2)=6-2$, from which it follows that the minus sign does alter the sign within the brackets when the brackets are removed. The following rule may be stated:

If brackets are preceded by a +, we may remove the brackets together with the +, leaving the signs within unchanged. If brackets are preceded by a —, we may remove the brackets together with the —, changing the signs of all the terms within.

When an expression begins with a bracket, it is understood that + *precedes* the bracket.

Example:

$$\begin{aligned}&(7x-2y)+(-2x+3y)-(-5x+y)-(2x-7y)\\&=7x-2y-2x+3y+5x-y-2x+7y\\&=8x+7y\end{aligned}$$

Note that $7x$ within the first bracket and $2x$ within the last stand for $+7x$ and $+2x$, respectively.

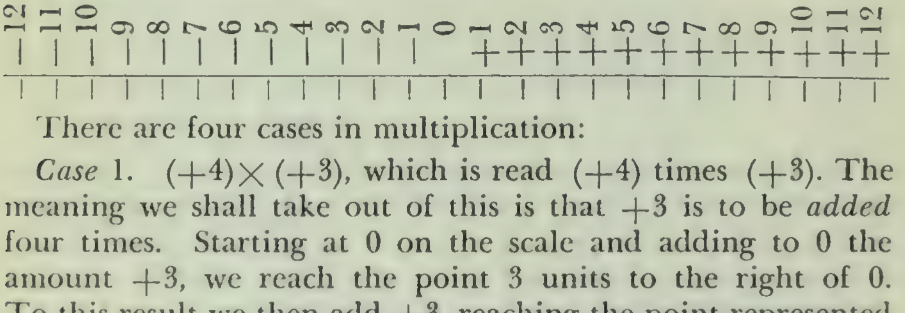
Exercises:

Remove brackets and collect terms:

1. $5a+(a+b)+2b$
2. $4a-b+(2a-b)$
3. $(3x+y)+(-2x+y)$
4. $6A-(A-2B)$
5. $3ab-5xy-(4xy+8ab)$
6. $a+b-(a-b)$
7. $c-(-c+d)$
8. $-3c+(-c+d)$
9. $-5a-(-a+3)$
10. $(-2b+1)+(1-2b)$
11. $1-(a-b-1)$
12. $4-c-(4+c)$
13. $-2c+d-(-2c+d)$
14. $-2c+d+(-2c+d)$
15. $-(-s+t)+(s-t)$
16. $5-(D-3)+(2-D)$
17. $0-(3-a)$
18. $v^2-3+(-v^2+2)-v^2+1$
19. $a^2+(-3a^2-6)-(2a^2+3)$
20. $AB-(-AB)-2AB$
21. $(\frac{1}{2}a-\frac{3}{2})-(1-a)$
22. $a-(\frac{2}{3}b+\frac{3}{2}a)$
23. $(1.5x+.5y)-(2x-y)$
24. $c-(1.2c-4)+3.4$

Multiplication

We shall again use our number scale.



There are four cases in multiplication:

Case 1. $(+4)\times(+3)$, which is read $(+4)$ times $(+3)$. The meaning we shall take out of this is that $+3$ is to be *added* four times. Starting at 0 on the scale and adding to 0 the amount $+3$, we reach the point 3 units to the right of 0. To this result we then add $+3$, reaching the point represented by $+6$. Again adding $+3$ we have $+9$, and doing this a fourth time gives $+12$:

So we have $\quad (+4)\times(+3)=+12.$

Case 2. $(+4)\times(-3)$, which is read $(+4)$ times (-3). This may be taken to mean that (-3) is to be *added* four times. To 0 we add (-3) which brings us to the point 3 units to the left of 0. To this result, we add -3 three times more, which carries us to the point represented by -12:

Then $\quad (+4)\times(-3)=-12.$

Case 3. $(-4)\times(+3)$, which is read (-4) times $(+3)$. This may be taken to mean that $+3$ is to be *subtracted* four times. Starting at 0 and subtracting $+3$ four times takes us to the point -12:

Thus $$(-4)\times(+3)=-12.$$

Case 4. $(-4)\times(-3)$, which is read (-4) times (-3). We may take this to mean that (-3) is to be *subtracted* four times. Beginning at 0 and subtracting (-3) four times brings us to the point 12 units to the *right* of 0. This is evident since $0-(-3)=0+(+3)$; that is, subtracting -3 is the same as adding $+3$.

Hence $$(-4)\times(-3)=+12.$$

Multiplication

Exercises:

Using the number scale, find these products:

1. $(+5)\times(+2)$
2. $(+3)\times(-2)$
3. $(-2)\times(+4)$
4. $(-3)\times(-5)$
5. $(+2)\times(-2)$
6. $(-3)\times(-3)$
7. $(+1)\times(-1)$
8. $(-1)\times(+1)$
9. $(-1)\times(-1)$
10. $(0)\times(-3)$
11. $(-2)\times(0)$
12. $(0)\times(0)$

From the last three examples you may conclude that if at least one of the factors of a product is 0, the whole product is 0. From the other examples you obtain the *rule of signs*: *In multiplication, like signs give plus and unlike signs give minus.*

Exercises (*Oral*):

Using the rule, read off these products:

1. $(+3)\times(+8)$
2. $(-2)\times(-4)$
3. $(-6)\times(-3)$
4. $(+5)\times(-2)$
5. $(-5)\times(+2)$
6. $(-1)\times(+8)$
7. $(-6)\times(+1)$
8. $(-1)\times(-1)$
9. $(-6)\times(0)$
10. $(+4)\times(0)$
11. $(-3)\times(+5)$
12. $(+7)\times(-7)$
13. $(-8)\times(+8)$
14. $(-1)\times(+1)$
15. $(+5)\times(-5)$
16. $(-3)\times(-1)$
17. $(0)\times(-3)$
18. $(-4)\times(-5)$
19. $(10)\times(-6)$
20. $(+\frac{1}{2})\times(12)$
21. $(\frac{3}{2})\times(-6)$
22. $(12)\times(-\frac{2}{3})$
23. $(-.5)\times(8)$
24. $(.6)\times(-.2)$

The Literal Parts of a Product

Early in your mathematics you found that $l \times w$ is written lw, as in the formula $A = lw$, and you found also that $a \times a$ is written as a^2, while the formula $V = e^3$ means V is equal to $e \times e \times e$. Hence, in a product, if the literal parts are unlike, the multiplication sign is omitted. If the letters are like, then we use indices or exponents for the shortened form.

$$a^2 \times a^3 = a \times a \times a \times a \times a = a^5$$
$$t \times t^5 = t \times t \times t \times t \times t \times t = t^6$$

This illustrates the *Law of Exponents in Multiplication. When powers of the same letter* (*or base*) *are multiplied together, add the exponents.*

The rule is conveniently expressed by the formula

$$a^m \times a^n = a^{m+n}$$

Meaning of coefficient. The expression $3a$ has two factors 3 and a. In elementary work we speak of 3 as the *coefficient*. In later work, we distinguish between numerical and literal coefficient. In $5a^2$, 5 is the coefficient and 2 the exponent. The coefficient of $6x^2y$ is 6.

Multiplication of Monomials

In multiplying monomials:

(1) The *sign* is determined by the rule of signs.
(2) The coefficient is obtained by the rules of arithmetic.
(3) The *literal part* is found by combining letters and by the law of exponents.

Examples:

$$(-2a) \times (+6b) = -12ab$$
$$(-5ax) \times (-8bx) = +40abx^2$$
$$(-2a)^2 = (-2a) \times (-2a) = 4a^2$$
$$(4v^3) \times (-2v^2t) = -8v^5t$$
$$0 \times (-6c^2) = 0$$
$$(-3a^2y^3) \times 1 = -3a^2y^3$$

NOTE: A monomial has only one term.

Exercises: Write or read off the products:

1. $(4a)\times(5b)$
2. $(-3a)\times(-2b)$
3. $(-5c)\times(4d)$
4. $(+2d)\times(-a)$
5. $(-8x)\times(-2y)$
6. $(-9a)\times(2x)$
7. $(\frac{1}{2}a)\times(4b)$
8. $(\frac{3}{2}x)\times(-x)$
9. $(\frac{1}{4}c)\times(\frac{1}{2}d)$
10. $(-\frac{1}{2}d)\times(\frac{3}{2}d)$
11. $(+c)\times(-c)$
12. $(2a)\times(2a)$
13. $(-3b)\times(-b)$
14. $(-5a)\times(5a)$
15. $(c^2)\times(c^3)$
16. $(2d)\times(3d^2)$
17. $(-a)\times(-a)$
18. $(a^3)\times(a^3)$
19. $(-2c^3)\times(-3c^2)$
20. $(\frac{1}{2}d^2)\times(\frac{1}{2}d^3)$
21. $(3)(2a^2)$
22. $(-4)(-c^2)$
23. $(.5a)(6b)$
24. $(-.1)(2a^2)$
25. $(1.5)(-3a)$
26. $(2a)^2$
27. $(-3)^2$
28. $(-4c)^2$
29. $(2x^2)^3$
30. $(-3a^2)^3$

In the exercises that follow, lay a strip of paper over the answers, and on the strip write the products. Compare your answers with those given. In case you make mistakes, try to find the cause. Try the exercises again at different times until you are able to write all the products correctly. After that, the exercises may be used as a timed test to see how fast you can multiply.

Exercises. Multiply:

	Example	*Answer*
1.	$(-6)\times(+4)$	$=-24$
2.	$(+5)\times(-7)$	$=-35$
3.	$6\times(-8)$	$=-48$
4.	$-4\times(-2)$	$=8$
5.	$0\times(-2)$	$=0$
6.	$(-2)\times(-10)$	$=20$
7.	$(-8)\times 0$	$=0$
8.	$(+6a)\times(-4b)$	$=-24ab$
9.	$(-2b)\times(-8b)$	$=16b^2$
10.	$-3c\times(-5d)$	$=15cd$
11.	$(-c)\times(+\frac{1}{2}d)$	$=-\frac{1}{2}cd$
12.	$(-c)\times(-c)$	$=c^2$
13.	$4x\times(-3y)$	$=-12xy$
14.	$7ab\times(-2cd)$	$=-14abcd$
15.	$(-6ax)\times(+4ab)$	$=-24a^2bx$
16.	$(-1)\times(xy)$	$=-xy$
17.	$(2cd)\times(-\frac{3}{2})$	$=-3cd$
18.	$(-5y)\times .1$	$=-.5y$
19.	$(3\pi r^2)\times(2h)$	$=6\pi r^2h$
20.	$(-3ab)\times(-\frac{2}{3}cd)$	$=2abcd$
21.	$(a)\times(-a)$	$=-a^2$
22.	$(-b)\times(-b)$	$=b^2$
23.	$(-2a)\times(3b)$	$=-6ab$
24.	$(a^2)\times a^3$	$=a^5$
25.	$-b^3\times b^2$	$=-b^5$
26.	$\pi r^2\times r$	$=\pi r^3$

Example	Answer	Example	Answer
27. $-3h^2 \times -2h^4$	$=6h^6$	34. $(+2a)^3$	$=8a^3$
28. $\frac{2}{5}a^2b \times 4ab$	$=\frac{8}{5}a^3b^2$	35. $(-3x^2)^2$	$=9x^4$
29. $-4a^2b^2 \times -2a^2$	$=8a^4b^2$	36. $(+3x^2)^2$	$=9x^4$
30. $(+2x)^2$	$=4x^2$	37. $(-3x^2)^3$	$=-27x^6$
31. $(-2a)^2$	$=4a^2$	38. $(+3x^2)^3$	$=27x^6$
32. $(-\frac{5}{2}xy)^2$	$=\frac{25}{4}x^2y^2$	39. $(2a)(-3b)(4c)$	$=-24abc$
33. $(-2a)^3$	$=-8a^3$	40. $(-3x)(-4x^2)(+x)$	$=12x^4$

Division

We learned that $6 \div 3 = 2$, since $2 \times 3 = 6$.

In like manner $\frac{-6}{+3}$ may be found by asking what $+3$ should be multiplied by in order to give -6. We know from our work in multiplication that the required multiplier is -2.

$\frac{+6}{-3} = -2$, since $-3 \times -2 = +6$

$\frac{-6}{-3} = +2$, because $(-3) \times (+2) = -6$.

1. From the above examples, we observe that the rule of signs in division is the same as that in multiplication.

Like signs give plus (+), *unlike signs give minus* (—).

2. The *arithmetical* part of the quotient is found by division as in arithmetic.

$\frac{-18}{+2}$ has an *absolute* value of 9.

$\frac{-12}{-8}$ has an *absolute* value of $1\frac{1}{2}$ or $\frac{3}{2}$

3. The *literal part* may be explained in this way:

$\frac{6 \text{ dozen}}{3} = 2$ dozen; $\frac{6 \text{ doz.}}{3} = 2$ doz.; $\frac{6d}{3} = 2d$.

$\frac{28 \text{ threes}}{7} = 4$ threes; $\frac{28\,(3)}{7} = 4\,(3)$;

In like manner we have:

$$\frac{6\times a}{6}=a; \qquad \frac{12a}{a}=12;$$

$$\frac{ab}{a}=b; \qquad \frac{abc}{ab}=c;$$

$$\frac{4ax}{2x}=2a; \qquad \frac{3xyz}{6xz}=\tfrac{1}{2}y.$$

4. The *law of exponents* is shown thus:

$$\frac{a^3}{a^2}=\frac{a\times a\times a}{a\times a}=a;$$

$$\frac{a^5}{a^3}=\frac{a.a.a.a.a}{a.a.a}=a.a=a^2.$$

We use the following rule for exponents in division: *When powers of the same letter (or base) are divided, we subtract the exponents.*

This rule is expressed by the formula:

$$\frac{a^m}{a^n}=a^{m-n}.$$

Thus we have:

$$\frac{c^4}{c}=c^3; \qquad \frac{ab^5}{b^4}=ab;$$

$$\frac{a^3d^3}{ad}=a^2d^2; \qquad \frac{dc^4e^3}{c^3e}=dce^2.$$

Division of Monomials

Examples:

$$\frac{+12a^3b^2}{-6ab}=-2a^2b; \qquad \frac{-15x^4y^3z}{-20xy^2}=+\tfrac{3}{4}x^3yz \text{ or } \frac{3x^3yz}{4}.$$

Exercises (Oral):

Find the quotients:

1. $\frac{6a}{6}$
2. $\frac{8a}{a}$
3. $\frac{-12c}{-c}$
4. $\frac{-4d}{d}$
5. $\frac{-15x}{-5}$
6. $\frac{20ab}{-4}$
7. $\frac{0}{-2c}$
8. $\frac{3xy}{-3xy}$
9. $\frac{2a^2}{-1}$
10. $\frac{-3cd}{-1}$
11. $\frac{cd}{d}$
12. $\frac{-ac}{-a}$
13. $\frac{-30abc}{-2}$
14 $\frac{16bcd}{-bc}$
15. $\frac{4a^2}{-a^2}$
16. $\frac{a^2b}{a^2}$
17. $\frac{c^2d}{cd}$
18. $\frac{ab^2}{-b^2}$
19. $\frac{2cx}{-2cx}$
20. $\frac{-4ab}{12}$
21. $\frac{10^5}{10^2}$
22. $\frac{3a^2}{4a}$
23. $\frac{16cd^2}{-8d^2}$
24. $\frac{0}{-6a^3}$
25. $\frac{a^2b^3}{ab}$
26. $\frac{-4c^3d}{2c}$
27. $\frac{-x^4y^2}{xy}$
28. $\frac{.6a^3b}{.2ab}$
29. $\frac{3.5a}{35}$
30. $\frac{.5a^3}{-.1a^2}$

Exercises:

Lay a strip of paper over the quotients and write your answers on the strip. Compare results. In case of errors, find the cause and try again later.

1. $\frac{+a^3}{+a^2} = a$
2. $\frac{-b^2}{+b} = -b$
3. $\frac{-ab}{-a} = b$
4. $\frac{-4ab}{+2b} = -2a$
5. $\frac{6bcd}{-3bc} = -2d$
6. $\frac{-8a^2}{-8} = a^2$
7. $\frac{9a^3}{-a^3} = -9$
8. $\frac{-15a^2}{20} = \frac{-3a^2}{4}$
9. $\frac{14x^2y}{-16x} = \frac{-7xy}{8}$
10. $\frac{0}{3a} = 0$
11. $2a^2 \div (-a) = -2a$
12. $(-15a^3b) \div (-5b) = 3a^3$
13. $-16xy \div 8xy = -2$
14. $9c^3 \div (-9c^3) = -1$
15. $(-10a) \div (15a) = -\frac{2}{3}$
16. $\frac{2^4}{2^3} = 2$
17. $10^5 \div 10^3 = 100$
18. $\frac{6xy}{-1} = -6xy$
19. $\frac{-3cv}{1} = -3cv$
20. $\frac{0}{-8a^2} = 0$
21. $\frac{-18x^2y}{-18xy} = x$
22. $4\pi r^2h \div 8 = \frac{1}{2}\pi r^2h$
23. $\frac{10\pi r^2}{5} = 2\pi r^2$
24. $\frac{-12a^3}{10a} = \frac{-6a^2}{5}$
25. $\frac{32ax^3}{-4x^2} = -8ax$
26. $1.6c \div .8c = 2$
27. $1.4ab^2 \div .2ab = 7b$
28. $\frac{.06bc^2}{.3c} = .2bc$
29. $\frac{-2.4d^3}{1.6d} = \frac{-3d^2}{2}$
30. $\frac{-.5x^2y}{-.1xy} = 5x$

Multiplication of Polynomial by Monomial

Consider the following:

$$\begin{aligned}(a+b)+(a+b)+(a+b) &= a+b+a+b+a+b.\\ &= 3a+3b\end{aligned}$$

$$\begin{aligned}\text{Thus, } 3\times(a+b) &= 3a+3b\\ \text{or, } 3(a+b) &= 3a+3b.\end{aligned}$$

$$\begin{aligned}\text{So also } -6(a+b) &= -6a-6b\\ y(a+b+c) &= ya+yb+yc\\ -2y(3a-2b-c) &= -6ya+4yb+2yc.\end{aligned}$$

From these examples you will observe that each term of the polynomial is multiplied by the monomial.

Example: Find the product of $3x-2y+5z$ and $-2ax$.

$$\begin{array}{r} 3x-2y+5z\\ -2ax\\ \hline -6ax^2+4axy-10axz\\ \hline \end{array}$$

You will note that the multiplier $-2ax$ is placed at the *left*-hand side for convenience.

$$\begin{aligned}&-2ax\,(3x-2y+5z)\\ =&-6ax^2+4axy-10axz.\end{aligned}$$

The monomial is generally placed in front, for, if placed at the end, brackets would be required. Compare

$$(3x-2y+5z)-2ax$$

and

$$(3x-2y+5z)\,(-2ax).$$

The first form calls for subtraction, the second, multiplication.

Exercises: Cover answers while working.

Example	*Answer*
1. $a(b+c)$	$=+ab+ac$
2. $-x(x+2)$	$=-x^2-2x$
3. $2c(a-b)$	$=2ac-2bc$
4. $4x(-x+2)$	$=-4x^2+8x$
5. $-3t(-t-t^2)$	$=3t^2+3t^3$
6. $-8a(-1+a)$	$=8a-8a^2$
7. $7c^2(-c+2)$	$=-7c^3+14c^2$
8. $-9ax(ax-4)$	$=-9a^2x^2+36ax$
9. $\pi r(2h+r)$	$=2\pi rh+\pi r^2$
10. $\pi(\mathrm{R}^2+r^2)$	$=\pi\mathrm{R}^2+\pi r^2$
11. $\pi(\mathrm{R}^2-r^2)$	$=\pi\mathrm{R}^2-\pi r^2$
12. $2\pi(4\mathrm{R}-3r)$	$=8\pi\mathrm{R}-6\pi r$
13. $-3c(-2c+1)$	$=6c^2-3c$
14. $-1(5x-2a)$	$=-5x+2a$
15. $-4d(-\mathrm{D}+d)$	$=4d\mathrm{D}-4d^2$
16. $(a+b)c$	$=ac+bc$
17. $(-2x+y)y$	$=-2xy+y^2$
18. $-a(-a+b+c)$	$=a^2-ab-ac$
19. $-6(2x-y+z)$	$=-12x+6y-6z$
20. $4a(-4a+5)$	$=-16a^2+20a$
21. $3c(-2+5c^2)$	$=-6c+15c^3$
22. $(6-2y)(5y^2)$	$=30y^2-10y^3$
23. $(a-2b)(-3a)$	$=-3a^2+6ab$
24. $(2c-3d)(-5)$	$=-10c+15d$
25. $6a(\frac{1}{2}a-3)$	$=3a^2-18a$
26. $8c(\frac{1}{3}a-1)$	$=\dfrac{8ac}{3}-8c$
27. $\frac{1}{2}(2a-3)$	$=\mathrm{a}-\frac{3}{2}$
28. $\frac{1}{2}a^2(6a-3b)$	$=3a^3-\frac{3}{2}a^2b$
29. $.5x(4x-.5y)$	$=2x^2-.25xy$
30. $.1(4a-.1b)$	$=.4a-.01b$

Exercises: Multiply:

1. $\begin{array}{l} x+y \\ \underline{\;x\;\;\;\;} \end{array}$

2. $\begin{array}{l} -a+b \\ \underline{-a\;\;\;\;} \end{array}$

3. $\begin{array}{l} -x-y \\ \underline{-y\;\;\;\;} \end{array}$

4. $\begin{array}{l} 2a-3b \\ \underline{2a\;\;\;\;\;\;} \end{array}$

5. $\begin{array}{l} -3x+2y \\ \underline{\;\;2x\;\;\;\;\;\;} \end{array}$

6. $\begin{array}{l} -4x-5y \\ \underline{-3y\;\;\;\;\;\;} \end{array}$

7. $\begin{array}{l} 2ab-3 \\ \underline{-4ab\;\;\;\;} \end{array}$

8. $\begin{array}{l} 6a^2-2 \\ \underline{-5a^2\;\;\;} \end{array}$

9. $\begin{array}{l} 7x^2-2x \\ \underline{4x^3\;\;\;\;\;\;} \end{array}$

10. $\begin{array}{l} -9a^2-2a \\ \underline{-6a^3\;\;\;\;\;\;} \end{array}$

11. $\begin{array}{l} a+b-c \\ \underline{b\;\;\;\;\;\;\;\;\;\;} \end{array}$

12. $\begin{array}{l} 2a-3b+5c \\ \underline{-3c\;\;\;\;\;\;\;\;\;\;} \end{array}$

13. $\begin{array}{l} 5x^2-3xy+2y^2 \\ \underline{-4\;\;\;\;\;\;\;\;\;\;\;\;\;\;} \end{array}$

14. $\begin{array}{l} -6a^2+2ab+b^2 \\ \underline{-4a\;\;\;\;\;\;\;\;\;\;\;\;} \end{array}$

15. $\begin{array}{l} -2.5a+b \\ \underline{-4a\;\;\;\;\;\;\;} \end{array}$

16. $\begin{array}{l} 1.6u-2t \\ \underline{.4u\;\;\;\;\;\;\;} \end{array}$

17. $\begin{array}{l} \frac{1}{2}a-\frac{1}{4}b+c \\ \underline{-6b\;\;\;\;\;\;\;\;\;\;} \end{array}$

18. $\begin{array}{l} \frac{1}{3}x^2-\frac{2}{3}y-y^2 \\ \underline{4x\;\;\;\;\;\;\;\;\;\;\;\;} \end{array}$

19. $\begin{array}{l} .1a-.2b \\ \underline{.5x\;\;\;\;\;\;} \end{array}$

20. $\begin{array}{l} 5v-6.8t \\ \underline{2v\;\;\;\;\;\;\;} \end{array}$

21. $\begin{array}{l} 4v^2-3u^2 \\ \underline{-6a\;\;\;\;\;\;} \end{array}$

22. $\begin{array}{l} .1c-.2d \\ \underline{.5d\;\;\;\;\;\;} \end{array}$

23. $\begin{array}{l} 5R^2-2r^2 \\ \underline{3\;\;\;\;\;\;\;\;\;\;} \end{array}$

24. $\begin{array}{l} 1.2R^2+3.5r^2 \\ \underline{\;.6\;\;\;\;\;\;\;\;\;\;\;\;} \end{array}$

Find the products of:

25. $6a-2a^2+4a^3$ and $2a$

26. $3-5c-2c^2$ and $-4c$

27. $1+2c-3c^2$ and $-c^2$

28. yd and $2-3d+5d^2$

29. $-3ab$ and $-2+5ab$

30. h and $-2r^2+3R^2$

Dividing Polynomials by Monomials

Consider the following:

$$\tfrac{1}{2} \text{ of } 6+\tfrac{1}{2} \text{ of } 4=3+2=5$$

$$\tfrac{1}{2}(6+4)=\tfrac{1}{2}(10)=5.$$

The result in each case is 5.

$$\tfrac{1}{3} \text{ of } 9+\tfrac{1}{3} \text{ of } 15-\tfrac{1}{3} \text{ of } 6=3+5-2=6.$$

But $\tfrac{1}{3}(9+15-6)=\tfrac{1}{3}(18)=6;$

also $\dfrac{9+15-6}{3}=\dfrac{18}{3}=6;$

So, also, $\dfrac{9a+15b-6c}{3}=3a+5b-2c.$

The same result may be obtained by this longer way:

$$\frac{9a+15b-6c}{3}=\frac{9a}{3}+\frac{15b}{3}+\frac{-6c}{3}=3a+5b+(-2c)$$
$$=3a+5b-2c.$$

Examples:

1. $\dfrac{-4x+8y-20z}{-4}=+x-2y+5z.$

2. $\dfrac{30a^3-12a^2+6a}{6a}=5a^2-2a+1.$

3. $\dfrac{-28a^2b+7ab}{-7ab}=4a-1.$

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Checking results.

A simple check is obtained by multiplying the quotient by the divisor, the result being the dividend.

Thus in 1. $-4\,(+x-2y+5z)=-4x+8y-20z$;
and in 2. $6a\,(5a^2-2a+1)=30a^3-12a^2+6a$.

Exercises: Find the quotients:

1. $\dfrac{6a+12b}{6}$
2. $\dfrac{10a-20b}{5}$
3. $\dfrac{-14x+21y}{-7}$
4. $\dfrac{6-18t^2}{6}$
5. $\dfrac{25c^2d-5}{-5}$
6. $\dfrac{ab+ac}{a}$
7. $\dfrac{xy-3y}{+y}$
8. $\dfrac{a^2-a^3}{a}$
9. $\dfrac{-4ax-8ay}{4a}$
10. $\dfrac{x^2y-3xy}{xy}$
11. $\dfrac{-6ax+12ayx}{6ax}$
12. $\dfrac{-4ac-8cda}{-4ac}$
13. $\dfrac{a^3+a^4}{a}$
14. $\dfrac{c^2d-cd^2}{-cd}$
15. $\dfrac{-6ab-6a}{-6a}$
16. $\dfrac{3a-6+9a^2}{-3}$
17. $\dfrac{1-a-a^2}{-1}$
18. $\dfrac{5x-10y+15}{-5}$

Exercises: Cover answers when doing the following exercises. Study any errors made.

Example: *Answer*

1. $\frac{2cd-c}{-c} = -2d+1$
2. $\frac{4a-8b}{-2} = -2a+4b$
3. $\frac{4r^2+4rh}{4r} = r+h$
4. $\frac{4v+3w}{-1} = -4v-3w$
5. $\frac{-3a^2-2a}{-1} = 3a^2+2a$
6. $\frac{5t^2-t^3}{-1} = -5t^2+t^3$

Example: *Answer*:

7. $\frac{2a-3a^2}{-a} = -2+3a$
8. $\frac{6R-4r}{-2} = -3R+2r$
9. $\frac{6\pi R^2-2\pi r^2}{2} = 3\pi R^2-\pi r^2$
10. $\frac{-4ab+2ab^2}{2ab} = -2+b$
11. $\frac{a-2a^2+5a^3}{a} = 1-2a+5a^2$
12. $\frac{-c^2+c-c^3}{+c} = -c+1-c^2$

Example:. *Answer*:

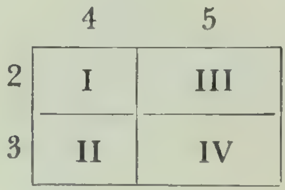
13. $\frac{5t^2-6t^3-4t}{-t} = -5t+6t^2+4$
14. $\frac{ab+ac+ad}{a} = b+c+d$
15. $\frac{-ab+ac-a}{-a} = b-c+1$
16. $\frac{-ab-2ab^2}{-a} = b+2b^2$
17. $\frac{-4ab+6a^2b}{2ab} = -2+3a$
18. $\frac{2ab-3a^2b-ab^2}{-a} = -2b+3ab+b^2$
19. $\frac{6x^3-12x^2+18x^4}{-3x} = -2x^2+4x-6x^3$
20. $\frac{7x-14x^2+21x^3}{7x} = 1-2x+3x^2$

Multiplication (Binomial × Binomial)

Let us take the case $(a+b)(c+d)$, which is the way we write $(a+b)\times(c+d)$. It means $(a+b)$ times $(c+d)$. Suppose we wish to find an equivalent expression in simpler form.

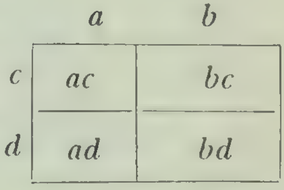
Interpreted geometrically. $A=lw$ gives the area of a rectangle with dimensions l and w. Now let us apply this formula to the figure on the right.

$$
\begin{aligned}
A=lw&=(4+5)(2+3)\\
&=(4+5)\times(2+3)
\end{aligned}
$$



But the area is composed of four rectangles.

$$
\begin{aligned}
A&=\mathrm{I}+\mathrm{II}+\mathrm{III}+\mathrm{IV}\\
&=4\times2+4\times3+5\times2+5\times3
\end{aligned}
$$



If we replace 4, 5, 2, 3 by the letters a, b, c, d, we have a figure like the one on the right, and this rectangle has an area represented by $(a+b)(c+d)$. This rectangle has an area equal to the areas of the four rectangles enclosed within the large one.

Interpreted geometrically, then,

$$(a+b)(c+d)=ac+ad+bc+bd$$

NOTE: A binomial has two terms. *Example*: $3x-2y$.

Interpreted algebraically.

$$
\begin{aligned}
(a+b)(c+d)&=(a+b)\text{ times }(c+d)\\
&=a\text{ times }(c+d)+b\text{ times }(c+d)\\
&=a(c+d)+b(c+d)\\
&=ac+ad+bc+bd.
\end{aligned}
$$

This is the method to use in finding the product of two binomials.

Example 1. $(3a-2b)(4c-5d)=3a(4c-5d)-2b(4c-5d)$
$=12ac-15ad-8bc+10bd.$

The first part of the product, $3a(4c-5d)=12ac-15ad.$

The second part of the product, $-2b(4c-5d)=-8bc+10bd.$

Example 2.

$$(-4x+3y)(7a-6b)=-4x(7a-6b)+3y(7a-6b)$$
$$=-28xa+24xb+21ya-18yb$$

Exercises:

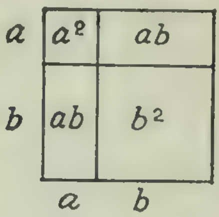
Find the products.

1. $(a+c)(b+d)$
2. $(a+c)(b-d)$
3. $(a-c)(b+d)$
4. $(a-c)(b-d)$
5. $(-a+c)(-b-d)$
6. $(-a-c)(-b-d)$
7. $(2a+b)(3c+d)$
8. $(2a+3b)(5c+6d)$
9. $(4x-2y)(5t+6s)$
10. $(6x-4y)(8t-s)$
11. $(5x-y)(7t-2s)$
12. $(3c+2a)(4d-y)$
13. $(-3c+2a)(4d-y)$
14. $(-3c+2a)(-4d-y)$
15. $(-a+2b)(3c+4d)$
16. $(A+B)(a+b)$
17. $(A-2B)(a-2b)$
18. $(a+2)(b+3)$
19. $(a-4)(b+3)$
20. $(-c+3)(d-7)$
21. $(-x-5)(-y+2)$
22. $(2a-6)(3b+5)$
23. $(5-a)(5-b)$
24. $(6+2x)(8-5y)$
25. $(7x-1)(-2y+8)$
26. $(3a-4)(2-6b)$
27. $(9-6x)(3y-5)$
28. $(a^2+b^2)(a+b)$
29. $(-3ab+2)(2cd-6)$
30. $(-5xy+12)(-6cd-4)$

Square of a Binomial

In the expression $(a+b)(c+d)$, if we replace c with a, and d with b, the expression becomes $(a+b)(a+b)$ and this is written $(a+b)^2$ and is read: '$(a+b)$, quantity squared'.

It represents the *square of a binomial.* As before, we shall interpret this geometrically, and then consider the algebraic method of finding the product.



The figure on the right shows this square divided into four parts, two rectangles and two squares.

Thus $(a+b)^2=a^2+2ab+b^2$.

This result may be obtained algebraically in this manner:

$$\begin{aligned}(a+b)^2=(a+b)(a+b)&=a(a+b)+b(a+b)\\&=a^2+ab+ab+b^2\\&=a^2+2ab+b^2.\end{aligned}$$

In place of the usual four terms we have three, a trinomial.

Exercises:

Find the products:

1. $(c+d)(c+d)$
2. $(2x+y)(2x+y)$
3. $(a+2b)(a+2b)$
4. $(a+x)^2$
5. $(8a+y)^2$
6. $(4a+3x)^2$
7. $(2a+5b)^2$
8. $(2x+7)(2x+7)$
9. $(8a+5)(8a+5)$
10. $(6x+1)(6x+1)$
11. $(9y+1)^2$
12. $(1+2x)(1+2x)$
13. $(1+5y)^2$
14. $(6+ab)(6+ab)$
15. $(cd+2)^2$
16. $(3ax+5)^2$
17. $(4a^2+2)^2$
18. $(6x^2+7)^2$
19. $(9a^2+6b)^2$
20. $(8c^2+7a^2)^2$
21. $(\frac{1}{2}a+b)^2$

Rule for Squaring a Binomial

(1) The product consists always of three terms.

(2) Two of these terms are squares, the square of the first term of the binomial and the square of the second term.

(3) The remaining term is twice the product of the two terms of the binomial.

Thus $(5x+2y)^2$ produces the sum of:

(1) The square of $5x$, that is, $25x^2$.
(2) The square of $2y$, that is, $4y^2$.
(3) Twice $(5x \times 2y)$, that is, $20xy$.

So $$(5x+2y)^2=25x^2+20xy+4y^2.$$

The product is written with the highest power of x followed by the next power.

RULE: *The square of a binomial equals the sum of the squares of its terms together with twice the product of those terms.*

Square of $(a-b)$

$a-b$ is the same as $a+(-b)$; so that by replacing b with $-b$ in the last result, we should obtain the value of $(a-b)^2$.

$$(a+b)^2=a^2+2ab+b^2$$
$$(a-b)^2=a^2+2a(-b)+(-b)^2=a^2-2ab+b^2.$$

The product obtained from $(a-b)^2$ differs from that of $(a+b)^2$ only in the sign of one term.

The same general rule applies to this case:

The square of a binomial equals the sum of the squares of its terms together with twice their product.

Exercises:

Find the products (note examples 28-30):

1. $(a-x)(a-x)$
2. $(x-y)^2$
3. $(2a-x)(2a-x)$
4. $(3x-a)^2$
5. $(4a-b)^2$
6. $(y-5b)^2$
7. $(y-8a)^2$
8. $(2c-3d)^2$
9. $(5c-6y)^2$
10. $(8a-9b)^2$
11. $(a-1)^2$
12. $(1-x)^2$
13. $(3b-1)^2$
14. $(10a-1)^2$
15. $(1-7a)^2$
16. $(ab-5)^2$
17. $(3ax-8)^2$
18. $(2x^2-9)^2$
19. $(6-8a^2)^2$
20. $(2a^2-3b^2)^2$
21. $(1-5t)^2$
22. $(2A-3B)^2$
23. $(4R-2r)^2$
24. $(-a+b)^2$
25. $(-2x+3y)^2$
26. $(-7a+5c)^2$
27. $(-3x+2y)^2$
28. $(-a-b)^2$
29. $(-5a-6b)^2$
30. $(-2x-7)^2$

Application to Arithmetic

Example **1.** To find the square of 41.

$$\begin{aligned} 41^2 &= (40+1)^2 = 40^2 + 2\,(40)\,(1) + 1^2 \\ &= 1600 + 80 + 1 \\ &= 1681 \end{aligned}$$

Example **2.** To find the square of 28.

$$\begin{aligned} 28^2 &= (30-2)^2 = 900 - 120 + 4 \\ &= 784 \end{aligned}$$

Exercises: Find

1. 21^2 2. 81^2 3. 32^2 4. 91^2
5. 19^2 6. 38^2 7. 59^2 8. 99^2

The Product $(a+b)\ (a-b)$

This is the *product of the sum and the difference of two numbers.*

$$\begin{aligned} (a+b)\,(a-b) &= a\,(a-b) + b\,(a-b) = a^2 - ab + ab - b^2 \\ &= a^2 - b^2. \end{aligned}$$

Exercises:

Work out fully, as in the above example:

1. $(2a+5b)\,(2a-5b)$ 4. $(8f+3h)\,(8f-3h)$
2. $(6x+3y)\,(6x-3y)$ 5. $(10ab+6)\,(10ab-6)$
3. $(7c-2d)\,(7c+2d)$ 6. $(1-4cd)\,(1+4cd)$

The result in each case is a binomial consisting of the squares of the two terms appearing in the brackets, and in each case a minus sign separates these squares.

Thus, '*the product of the sum and the difference of two quantities equals the difference of their squares*'.

Type: $(a+b)\,(a-b) = a^2 - b^2.$

Exercises (*oral*):

Give products directly as the difference of squares:

1. $(a+x)(a-x)$
2. $(a-y)(a+y)$
3. $(y+x)(y-x)$
4. $(2a+b)(2a-b)$
5. $(3a-4b)(3a+4b)$
6. $(6x-2y)(6x+2y)$
7. $(7a-4)(7a+4)$
8. $(3a+1)(3a-1)$
9. $(6xy+5)(6xy-5)$
10. $(3A+B)(3A-B)$
11. $(1+a)(1-a)$
12. $(1-5c)(1+5c)$
13. $(7+8x)(7-8x)$
14. $(2c^2+d)(2c^2-d)$
15. $(9-2R^2)(9+2R^2)$
16. $(5a^2+2b^2)(5a^2-2b^2)$
17. $(4+a)(-4+a)$
18. $(b+3a)(-b+3a)$
19. $(-2x+3y)(2x+3y)$
20. $(-2+3y)(2+3y)$

Application to Arithmetic

Example **1.** To find the product 51×49.

$51\times49=(50+1)(50-1)=50^2-1=2500-1=2499$

$38\times42=(40-2)(40+2)=40^2-2^2=1600-4=1596.$

Exercises: Find the products:

1. 31×29
2. 41×39
3. 89×91
4. 99×101
5. 53×47
6. 121×119
7. 32×28
8. 33×27
9. 64×56

$(a+b)^2$; $(a-b)^2$; $(a+b)(a-b)$.

Exercises: Expand:

1. $(x+2)(x+2)$
2. $(c+5)^2$
3. $(x-6)^2$
4. $(3a+5)(3a-5)$
5. $(2a-3)^2$
6. $(1-9d)(1+9d)$
7. $(5a+2b)^2$
8. $(3x+2)^2$
9. $(3c-2)(3c+2)$
10. $(1+x)^2$
11. $(4-t)^2$
12. $(1+5l)^2$
13. $(ab+2)^2$
14. $(xy-4)^2$
15. $(ax+3)(ax-3)$
16. $(a+\frac{1}{2})^2$
17. $(b-\frac{1}{3})^2$
18. $(2b+\frac{1}{3})^2$
19. $(a+\frac{1}{2}b)^2$
20. $(x+\frac{1}{2})(x-\frac{1}{2})$
21. $(c-\frac{1}{4}d)^2$
22. $(\frac{1}{2}a+\frac{1}{2})^2$
23. $(a+\frac{3}{2})(a-\frac{3}{2})$
24. $(2x-\frac{1}{3})(2x+\frac{1}{3})$
25. $(1+\frac{1}{5}a^2)(1-\frac{1}{5}a^2)$
26. $(a+.5)^2$
27. $(b-.2)^2$
28. $(.3+a)^2$
29. $(x-.6)(x+.6)$
30. $(.1a+.2b)^2$

Other Products

Example: $(2x-8y)(3x+5y)$.

First method:
$$\begin{aligned}(2x-8y)(3x+5y) &= 2x(3x+5y)-8y(3x+5y)\\ &= 6x^2+10xy-24xy-40y^2\\ &= 6x^2-14xy-40y^2.\end{aligned}$$

Second method: The work may be arranged in vertical or column form:
$$\begin{array}{r} 3x+5y \\ \underline{2x-8y} \\ 6x^2+10xy \qquad\quad \\ \underline{-24xy-40y^2} \\ \underline{6x^2-14xy-40y^2} \end{array} \qquad \text{or} \qquad \begin{array}{r} 2x-8y \\ \underline{3x+5y} \\ 6x^2-24xy \qquad\quad \\ \underline{+10xy-40y^2} \\ \underline{6x^2-14xy-40y^2} \end{array}$$

Exercises (multiply):

1. $(2a+3b)(4a+5b)$
2. $(4b+3c)(2b-6c)$
3. $(5x+8a)(3x-2a)$
4. $(4y-b)(2y+b)$
5. $(7a-6c)(a-c)$
6. $(3c+x)(3c+x)$
7. $(5y-7z)(8y-z)$
8. $(4t+1)(3t-5)$
9. $(6-8V)(5+7V)$
10. $(8+6R)(2-3R)$

11. $\begin{array}{r} 6a-5b \\ \underline{2a+8b} \end{array}$

12. $\begin{array}{r} 3x+2y \\ \underline{2x-y} \end{array}$

13. $\begin{array}{r} 6y-2c \\ \underline{6y-5c} \end{array}$

14. $\begin{array}{r} -5y+2c \\ \underline{3y-8c} \end{array}$

15. $\begin{array}{r} 9a+b \\ \underline{3a-2b} \end{array}$

16. $\begin{array}{r} 11a-3b \\ \underline{a+\ b} \end{array}$

17. $\begin{array}{r} 9x-5 \\ \underline{3x+6} \end{array}$

18. $\begin{array}{r} 4t-1 \\ \underline{3t+1} \end{array}$

19. $\begin{array}{r} 1-8D \\ \underline{1+6D} \end{array}$

You should now be able to write the product of two binomials, such as are given in the preceding exercise, without writing the middle step.

Example: $(6d+3r)(5d+4r)$.

The first term of the product is the product of the first terms of the two binomials: $6d \times 5d$ or $30d^2$.

The third term of the product is the product of the second terms of the two binomials: $3r \times 4r$ or $12r^2$.

The second term of the product is the sum of two products, namely, the product of the 'outside' terms, $6d \times 4r$ or $24dr$, and the product of the 'inside' terms, $3r \times 5d$ or $15dr$, the sum being $39dr$.

$$(6d+3r)(5d+4r)=30d^2+39dr+12r^2.$$

Exercises:

Using the method above, write products for the following:

1. $(a+2c)(a+3c)$
2. $(x-2y)(x+5y)$
3. $(x+3y)(x-7y)$
4. $(3x+2y)(5x+6y)$
5. $(8x+a)(8x-2a)$
6. $(3a-5b)(3a+8b)$
7. $(2t-7)(5t-7)$
8. $(-a+b)(a-2b)$
9. $(-3x-2w)(-5x+6w)$
10. $(8lw-3)(2lw+5)$

Exercises:

In addition to examples like the above, you will find in these exercises examples of squares of binomials, like $(a+b)^2$ and $(a-b)^2$, as well as the product of a sum and a difference, like $(a+b)(a-b)$.

1. $(3a+2b)(a-b)$
2. $(5a+b)(5a+b)$
3. $(6c-d)(3c+2d)$
4. $(A-B)(2A+3B)$
5. $(y-\frac{1}{2})^2$
6. $(7x-8y)(7x+y)$
7. $(2a+3b)(5a+7b)$
8. $(5x-6y)(5x+6y)$
9. $(7a+2b)^2$
10. $(8y+2)(9y-3)$
11. $(v-3w)^2$
12. $(A-a)(A+a)$
13. $(5a-A)(2a+6A)$
14. $(-a+b)(-a-b)$
15. $(10t+v)(10t+v)$
16. $(5y-3)(5y-3)$
17. $(1-8ab)(1+8ab)$
18. $(c+1.5)^2$
19. $(2R+3r)(5R-6r)$
20. $(4y^2-3)(2y^2+1)$

Additional Practice:

Individual students or groups of students may prepare additional exercises for the class, at the same time preparing answers.

Removal of Brackets

Example 1. Simplify: $2(a+b)-5(2a-b)$.

This expression is a *difference between two products,* the product $2(a+b)$ and the product $5(2a-b)$. One method is to write these products as $2a+2b$ and $10a-5b$ and then subtract one from the other, thus: $(2a+2b)-(10a-5b)$. The result is $2a+2b-10a+5b$ or $-8a+7b$. It will be found easier to consider the second product as $-5(2a-b)$ which gives $-10a+5b$. This is then taken with $2a+2b$.

Here is the work:

$$2(a+b)-5(2a-b)=2a+2b-10a+5b$$
$$=-8a+7b.$$

Example 2. If $x=A+B$ and $y=2A-4B$, find the value of $4x-6y$ in terms of A and B.

$$4x-6y=4(A+B)-6(2A-4B)$$
$$=4A+4B-12A+24B$$
$$=-8A+28B.$$

Exercises:

Give answers to the following, in the simplest form:

1. $a+(a-b)$
2. $2-(a-2)$
3. $a+2(a-1)$
4. $c-4(c+1)$
5. $2x+5(x+1)$
6. $7x-3(x-y)$
7. $4-2(1-a)$
8. $5+6(-a+1)$
9. $(c-3)-(3+x)$
10. $2(b+3)+(b-1)$
11. $(a-7)-2(a+1)$
12. $3(1-a)-4(2-3a)$
13. $(a+2)-(2a-1)$
14. $(5-a)-(a-5)$
15. $(t-3)+2(1-t)$
16. $4(d+5)-3(d-6)$
17. $5(-c+2)+2(-d+1)$
18. $8(v+w)-6(-v-2w)$
19. $(a+b-5)-2(a-b)$
20. $4(-2c+d)-6(-5c+d)$
21. $3(-1+x)-(2-a)$
22. $-(a-b)+(a+b)$
23. $5(t-2)-3t+4d$
24. $1-2(-a+1)$

When $A=x-2$ and $B=2x-3$ find values for:

25. $A+B$
26. $A-B$
27. AB
28. $2A+B$
29. $A-4B$
30. $3A-B$
31. $2A-2B$
32. $5A+6B$
33. $-4A-3B$

Evaluating Expressions

Example 1. Find the value of $3x-\frac{1}{2}(x+1)$ when $x=-2$.

$$\begin{aligned}\text{The expression} &= 3(-2)-\tfrac{1}{2}(-2+1)\\ &= -6-\tfrac{1}{2}(-1)\\ &= -6+\tfrac{1}{2}\\ &= -5\tfrac{1}{2}.\end{aligned}$$

Example 2. Evaluate $5a^2-3b^2$ when $a=-3$ and $b=4$.

$$\begin{aligned}\text{The expression} &= 5(-3)^2-3(4)^2\\ &= 5\times 9-3\times 16\\ &=45-48\\ &= -3.\end{aligned}$$

Exercises:

Evaluate the following expressions for the given values:

1. $2a-3$ when $a=4$.
2. $5-6a$ when $a=2\frac{1}{2}$.
3. $2x-8$ when $x=-3$.
4. $\frac{1}{2}x+6$ if $x=-2$.
5. $3(x-2)$ if $x=1$.
6. $8(4+3x)$ if $x=-1$.
7. $5x-(x+2)+2x$ when $x=4$.
8. $4y+9(y-5)$ when $y=2$.
9. $7-3(b-2)$ when $b=0$.
10. x^2+3x+4 if $x=4$.
11. $2a^2+5a+3$ if $a=3$.
12. $1-2t-5t^2$ when $t=5$.
13. $4a+5b$ when $a=2$, $b=-1$.
14. $6a-2x$ if $a=0$, and $x=-3$.
15. $3c-2d$ if $c=\frac{1}{2}$ and $d=\frac{1}{4}$.
16. x^2-4y^2 if $x=y=-2$.
17. $\dfrac{2x-3y}{x+y}$ when $x=4$, $y=2$.

*18. $\dfrac{c^2-cd}{2c-d}$ when $c=-1$, $d=0$.

*19. $\dfrac{ab}{a-b}$ when $a=b=-1$.

*20. $2(x-3)^2$ when $x=\frac{1}{2}$.

Review Exercises

1. If $a=5$ and $b=6$, find the value of $\frac{a^2+2b.}{2a-b}$

2. If $x=2$, $y=1$, $a=0$, evaluate $3x^2-ay+a^2$.

3. One book costs x cents. Give in dollars the cost of y books.

4. Subtract $3a-10$ from $10-3a$.

5. Remove brackets and collect terms: $(2a-3)-(4-a)+(-2a+1)$.

6. Simplify $-\frac{3}{4}c(-c+4)$.

7. Divide $(16a^2-8a-4)$ by -4.

8. Find the product of $2c-3d$ and $-5c+6d$.

9. Multiply as indicated $-3ab(5a^2-6b^2)$.

10. Subtract $3x-2y$ from 0.

11. The product of two expressions is $-2c^3+14c^2$. One is $-2c^2$. What is the other?

12. If $a=-2$ and $b=3$, find the value of $\frac{a+b}{a-b}$

13. If $x=-1$, $y=-5$, $a=2$, evaluate $2x^2-4ay-y^2$.

14. Add $2a+4b$, $3a-b$, $-7a+3b$, $5a-7b$.

15. From $6ab-3bc+4ac$ take $-7ca+4cb-3ba$.

16. Remove brackets and collect terms:

$$x^2+(-4x^2+3x)-10+2(3x-2x^2).$$

17. Give the product of $4a^2b$ and $3a-2b+1$.

18. If $C=2x-1$ and $D=5-x$, find the sum of C and D in terms of x.

19. Given $A=7-3x$, $B=2+x$, $C=-5-4x$, give the sum of A, B, and C in terms of x.

20. Find the value of $X-Y$ when $X=-6a+2$ and $Y=a-5$.

21. Multiply the sum of $2c-3d$ and $5c+2d$ by $-cd$.

22. Add $3x-2y+5$ to $4-3y-2x$ and subtract $x-y-1$ from the sum.

23. What must be added to $3a-5$ to give $2a+6$?

24. What must be subtracted from $c-3d$ to give $5c+d$?

25. Find the expression whose factors are $3-7a$ and $2+8a$ and arrange in descending order.

Remedial Exercises

1. If $x=2$, $y=3$, find the value of $6x^2-8y$.

2. Find the value of $\dfrac{2ab-3b^2}{5a-b}$ when $a=-3$ and $b=-1$.

3. From $3a^2-2a+5$ subtract $4a^2-6$.

4. Simplify $3x-(2-x)+(1-x)-(-2+3x)$.

5. Multiply as indicated $-2a^2(a^2-2a+1)$.

6. Perform the division as indicated: $\dfrac{15c^3-9c^2-3c}{-3c}$.

7. If $-2x$ is one factor of A and $3x-2y$ is the other, find A.

8. Write the square of $5c-3d$.

9. Divide $8a^2b-2ab^2-4ab$ by $2ab$.

10. Evaluate $3c^2-2d^2$ when $c=5$ and $d=-3$.

11. Find the sum of $2x-y$, $y+5x$, $-x+3y$.

12. Subtract $3a-2x+5$ from zero.

13. Remove brackets and collect terms in the following expression: $5(2a^2-3a)+2a-3(a^2-1)$.

14. Give the product of $9x^2-3x+5$ and $-x$.

15. If $A=3x-2y$ and $B=2x+y$, find the value of 2AB in terms of x and y.

16. The factors of C are $7x-5$ and $2x+3$. Write C as an expression in x, arranging the terms in ascending order.

17. What must be added to -6 to give -9?

18. What must be added to $5x-2$ to give $8x+7$?

19. What must be subtracted from a to give b?

20. What must be subtracted from $2c-d$ to give $5c+6d$?

21. Add the following pairs of expressions:

$$\begin{array}{r} 5x-2y+1 \\ \underline{-3x+6y-1} \end{array} \qquad \begin{array}{l} 2a^2-3ab+6b^2 \\ \underline{8a^2 \qquad\quad -b^2} \end{array} \qquad \begin{array}{r} 2-x \\ \underline{-5+x-2y} \end{array}$$

22. Subtract the lower expression from the upper:

$$\begin{array}{l} 3a-7 \\ \underline{8a+6} \end{array} \qquad \begin{array}{r} 5c^2-3cd+\ d^2 \\ \underline{c^2-3cd+2d^2} \end{array} \qquad \begin{array}{l} 8x \\ \underline{2x-3y} \end{array}$$

23. Which is the correct value of $6-2(a+5)$: $-2a-4$ or $4a+20$ or $4a-20$?

Vocabulary Test

Explain the following terms, giving examples:

Monomial
Binomial
Trinomial
Polynomial
Descending order
Ascending order
Absolute value
Coefficient
Exponent

Achievement Test

1. If West is taken as the positive direction, how would you indicate 16 miles East?

2. Find the sum of $3x-2y$, $\frac{1}{2}y-x$, $\frac{3}{4}x-y$.

3. Subtract $2x-x^2+5$ from $3x^2-8+x$.

4. Take -1 from the sum of $3a^2-2a$ and $6a-3$.

5. Remove the brackets and simplify:
$$(2c-d)-(4d+6c)+(-c+d)-(-2c+3d).$$

6. Select the correct product for $(4y-1)^2$: (a) $16y^2+1$; (b) $16y^2-8y+1$; (c) $4y^2-4y+1$.

7. Multiply as indicated: $-7x\,(3x^2-5x+2)$.

8. Divide $(4a^2-6a^3-2a)$ by $-2a$.

9. Give the product of $3a^2-5$ and $6a^2+4$.

10. Remove brackets and simplify:
$$2\,(3x-2y)-3\,(-2x+y)-(x-y).$$

11. Select the correct answer: $(-3x^2)^3 = -3x^5$, $3x^6$, $-9x^5$, $9x^6$, $9x^5$.

12. Give the value of $\dfrac{a^2b^2-a^2b+ab^2}{-ab}$.

13. Expand $(4a-3b)^2$.

14. Find the sum of $a+.5$, $.25a-1.5$, $1-1.75a$.

15. From $3c^2+.5c-1.2$ take $4c+.6-.1c^2$.

16. What must be added to $4x-y$ to give $3x+2y$?

17. What must be subtracted from $2x$ to give $-x+y$?

18. Evaluate $4ab-a^2+b^2$ when $a=b=-1$.

19. If $A=2c-d$ and $B=c+4d$, find the value of $2AB$.

20. Evaluate $(1-3t)^2-(2+t)^2$.

CHAPTER VIII

THE EQUATION

1. The type of equation with which you are most familiar is the *formula.* This particular form of equation expresses a rule or statement in algebraic language. For example, $d=rt$, is what we call the *distance formula.* In this formula we cannot give to d, r, and t any values whatever. The values of d, r, and t must be related in such a way that the value of d equals the product of the values of r and t. It is important to remember that any sets of values will not do, yet that there are a great many sets possible, such as: 6, 2, 3; 12, 4, 3; 10, 20, $\frac{1}{2}$, etc.

2. If, in such a formula, we know that $d=12$, and $r=4$, we then have the result: $12=4t$, or $4t=12$. This equation is satisfied for only one value of t, namely, 3, and no other values will do. In other words, the equation is true on condition that $t=3$, and so we call it a *conditional equation* or an *equation of condition.* It is generally known simply as an *equation,* and we say that 3 *satisfies* the equation.

3. A third kind of equation has been met, one such as:

$$(a+b)^2=a^2+2ab+b^2.$$

You will find by trial that no matter what values a and b may have, the statement is always true. This kind of equation is called an *identity* or an *identical equation.* $3a(h-5)=3ah-15a$ is another example. The sign $\equiv$ is sometimes used to indicate an identity.

Summary:

An *identity* is true for any set of values of the literal numbers which appear in the equation; a *formula* is true only for certain sets of values of the literal numbers; whereas an *equation* which contains only one literal number is true for only one value of this number.

Review Exercises:

Try solving the following equations by inspection. Think what value of the unknown is needed to make each equation 'balance':

1. $x+4=10$
2. $a-2=3$
3. $2c=27$
4. $x+1=18$
5. $y-3=0$
6. $18=4d$
7. $\frac{x}{2}=8$
8. $\frac{n}{3}=5$
9. $.5a=1.2$
10. $0=30-\frac{1}{2}d$

Language of the Equation

The *unknown* is the name given to a literal number in an equation, such as x or a or c as in the preceding exercise.

To *solve* an equation means to find the number which must be put in place of the unknown in order to make the equation true. For example, in the first example of the last exercise, we find $x=6$. Hence $x+4=10$ becomes $6+4=10$, and this we find is true.

The *left side* or *left member* of the equation is the expression to the left of the sign of equality. The *right side* or *right member* is the expression on the right of the equality sign.

The value of the unknown which makes the equation true is called the *root* of the equation. This root is said to satisfy the equation, since both sides reduce to the same numerical quantity when the root is substituted for the unknown.

NOTE: The word *solution* is used in two ways, sometimes being the name given to the process of solving the equation, at other times being another name for the *root*.

To *check* your answer means to see if the equation is *satisfied* when your answer is substituted for the unknown in the equation. In the checking shown in this text, L.S. will mean left side, and R.S. right side.

Exercises (oral):

1. When $C=10$ and $\pi=3\frac{1}{7}$, what equation is derived from the formula $C=\pi d$? Name the unknown in the resulting equation.

2. State what is meant by solving the equation $5s=8$. What is the root of this equation? In checking your answer, what value would you substitute for s?

3. Find by substitution which, if any, of the numbers $\frac{1}{2}$, 6, 18 is a root of $\frac{2}{3}x-5=7$.

4. What is meant by saying that an equation is *satisfied*? What number will satisfy $5x-6=14$? Name a value which will not satisfy it?

5. Which of the following values, 2, 0, $5\frac{1}{4}$, 6, $5\frac{3}{4}$, will satisfy the equation $18=4d-5$?

Making Equations

Equations are used in mathematics for the solving of problems. Before developing a systematic method for solving problems, we shall have some practice in the making of equations from the information given in the problem.

REPRESENTING EQUATIONS ALGEBRAICALLY

Examples:

Four times the number y is written $4y$.

Ten per cent of a sum of money x is written $.1x$.

A number which is 3 less than five times b is written $5b-3$.

Exercises (oral):

Represent the following by using algebra:

1. One-half the cost of a book, if the cost is n.
2. Two-thirds of the selling-price y.
3. Five per cent of the principal p.
4. Two and one-half times the cost price b.
5. One hundred twenty-five per cent of a sum of money x.
6. A number which is 15 more than another number m.
7. A number ten less than y.
8. A number x more than 5.
9. A number which is b less than 16.
10. Four more than twice t.
11. Eight less than 15 per cent of x.
12. An angle which is 40° more than another angle k.
13. If the width of a rectangle is denoted by r and its length is $1\frac{1}{2}$ times its width, represent its length in terms of r.
14. Represent the perimeter of the rectangle of Question 13.
15. The sum of two numbers is 23. One being denoted by c, represent the other in terms of c.
16. The ages of two brothers total 29 years. If one is d years old, how old is the other?
17. Two girls have ages which added make h years. If one is 14 years old, represent the age of the other.
18. If the selling price of a car is x dollars and the agent's commission is 5%, represent the commission in terms of x.

Making Equations

Example **1.** If $\frac{3}{4}$ lb. of tea costs 45¢, find the cost of 1 lb.

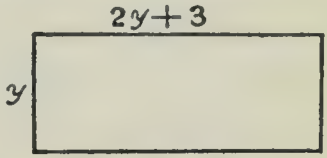
Solution: Let the cost, in cents, of 1 lb. be represented by x. Then the cost of $\frac{3}{4}$ lb. $= \frac{3}{4}x$. But the cost of $\frac{3}{4}$ lb. $=$ 45¢.

Then $\frac{3}{4}x = 45$

is our equation.

Example 2. The length of a rectangle is 3 in. more than double its width, and the perimeter is 36 in. Find the dimensions.

Let the number of inches in the width be denoted by y.



Then the length will be $2y+3$.
Hence the perimeter is
$2(2y+3)+2y$;
but the perimeter is 36 in.
Then $2(2y+3)+2y=36$.

NOTE: We do not write y in. or x¢ in these examples. It is understood that x represents the *number* of cents in the cost of a pound, and that y is the *number* of inches in the width of the rectangle.

Exercises:

Without solving them, make equations for the following problems:

1. Find the cost of a hat if 8 such hats cost $22.

2. If 1½ lb. of butter cost 60¢, find the cost of 1 lb.

3. One number is 5 more than another, and their sum is 19. Find the numbers.

4. One number is 5 times another and their sum is 72. What are the two numbers?

5. Two angles together make 90°, and one is 20° more than the other. What are the angles?

6. One angle is 3½ times another, and together they make up 45°. Find the size of each.

7. The length of a rectangle is 3 times its width, and the perimeter is 180 yd. Find its dimensions.

8. One angle of a triangle is 10° more than another, while the third angle equals the sum of these two. Find the size of each.

Solving Equations

The rules for solving equations are best illustrated by means of a balance, on one side of which is some object and, on the other, corresponding weights.

Suppose, to begin with, that the object balances the weight. If this balance is to continue, any *addition* made to one side must be met by an equal addition to the other. If we take away any amount from one side, an equal weight must be *subtracted* from the other. Again, if we increase the amount on one side by 2, 3, 4, or any number of times, we must likewise, *multiply* the weight on the other. Lastly, if we decrease the amount on one side (*division*) we must do the same to the other.

Applying these simple principles to equations, we have the following four rules or axioms:

(1) If the same quantity be added to each side of an equation, the two sides will still be equal.

(2) If the same quantity be subtracted from each side of an equation, the two sides will still be equal.

(3) If each side of an equation be multiplied by the same quantity, the two sides will still be equal.

(4) If each side of an equation is divided by the same quantity the two sides will still be equal.

One or more of these four axioms will be used in the following examples. (The examples will demonstrate the occurrence of negative quantities as roots).

Example 1. Solve $2x-3=-9$
Add 3 to each side $2x-3+3=-9+3$
$2x=-6$
Divide each side by 2 $x=-3$
Check: L.S.$=2(-3)-3=-6-3=-9.$
R.S.$=-9.$
Then L.S.$=$R.S.

Example 2. Solve $5-4a=8$

Subtract 5 from each side $5-4a-5=8-5$

$$-4a=3$$

Divide each side by -4 $\frac{-4a}{-4}=\frac{3}{-4}$

$$a=-\frac{3}{4}$$

Check: $\text{L.S.}=5-4\left(-\frac{3}{4}\right)=5+3=8.$
$\text{R.S.}=8.$

Example 3. Solve $\frac{1}{2}x-3=5+\frac{3}{4}x$.

Add 3 to each side $\frac{1}{2}x-3+3=5+\frac{3}{4}x+3$

$$\frac{1}{2}x=8+\frac{3}{4}x$$

Subtract $\frac{3}{4}x$ from each side $\frac{1}{2}x-\frac{3}{4}x=8+\frac{3}{4}x-\frac{3}{4}x$

$$-\frac{1}{4}x=8$$

Multiply each side by 4 $-x=32$

Multiply each side by -1 $x=-32$

Check: $\text{L.S.}=\frac{1}{2}(-32)-3=-16-3=-19.$
$\text{R.S.}=5+\frac{3}{4}(-32)=5-24=-19.$

Solving Equations

Before working the equations in the exercise which follows, try to think of the equations in the following way:

(1) $x+5=8.$

x is a number which will give 8 when 5 is added to it. Or again, x is a number 5 less than 8.

(2) $y-6=1.$

The quantity y is such that 6 must be subtracted from it in order to give 1. Or again, y is 1 more than 6.

(3) $\frac{1}{2}c=9.$

One-half of the number c equals 9.

(4) $2a+3=15.$

The quantity a is such that twice a requires 3 more to make it equal 15. Or, twice a is 3 less than 15.

Exercises:

Solve the following equations, noting which of the four axioms are being used. Check each answer:

1. $x+5=8$
2. $y-6=1$
3. $\frac{1}{2}c=9$
4. $2a+3=15$
5. $\frac{a}{2}+3=8$
6. $\frac{1}{4}a-6=0$
7. $1\frac{1}{4}c+2=7$
8. $-12y=6$
9. $18-6c=0$
10. $0=12+4r$
11. $8a-2=6a+6$
12. $7c+3=10+8c$
13. $21+5x-18=3+4x$
14. $1-y=4y+1$
15. $7a-2a+4=2a-5$
16. $2y-3+4y=6+4y$
17. $-5t+15=0$
18. $a-\frac{2}{3}a-1=0$
19. $3c-.5=2.5$
20. $1.5-a=1.5$

By substituting the values given, find which answer is correct:

	Suggested roots
21. $5x+15=0$	$0, 3, -3, 5$
22. $6c-2=-8$	$1\frac{2}{3}, 1, -1, \frac{2}{3}$
23. $3x+5-x=17$	$3, 4, 5, 6$
24. $3-8t+4t=19$	$5\frac{1}{2}, -5\frac{1}{2}, 4, -4$
25. $2(x-3)=-10$	$3, -2, -8, 2$

Equations with Brackets

Review Exercises:

Remove brackets and collect terms:

1. $3a-(a+2)$
2. $(4-2a)-(5a-1)$
3. $6b-(-b+4)$
4. $6(2+3x)$
5. $6-4(-1+y)$
6. $4(d-3)-2(6-d)$

Note these points particularly:

(1) When removing brackets preceded by a minus sign, change the signs of *all* quantities within the brackets.

(2) When a number or letter comes immediately before a bracket, all terms within the brackets are multiplied by that number or letter. Example 4, above: $6(2+3x)$ $=12+18x$.

(3) In Example 5, above, we do not subtract 4 from 6. We first multiply $(-1+y)$ by -4 and then collect terms, $6+4-4y$, which equals $10-4y$.

Exercises:

Solve the following equations:

1. $2a+(3a-4)=16$
2. $3b-(b+5)=-2$
3. $12a-(-a+3)=11$
4. $6y-2-(5-y)=0$
5. $2(x+1)=3$
6. $6(a-3)=5(a+2)$
7. $(a-3)+4=4-(a-1)$
8. $1=2-(a+2)-(3-a)$
9. $3+(y-1)=3-(y-1)$
10. $17a+5(2-3a)=18$
11. $3(a-6)-2(4-a)=0$
12. $-2-(-y+1)=3$
13. $6-2(a-3)=14$
14. $5+4(y-1)=1$
15. $-2+7(-1+x)=2$
16. $5-5(c-2)=0$

*17. $(a+2)^2=a^2+5$

*18. $(y-2)^2=(y-3)^2+1$

*19. $(2a+1)^2-(2a-3)^2=0$

*20. $3(2x-5)^2-6x(2x-3)=0$

Solve the following problems:

1. If 4 is added to a certain number and the sum is multiplied by 5, the result is 40. What is the number?

2. If 6 is added to a number and the sum then doubled, the result is 28. What is the number?

3. When 5 is subtracted from a number and the difference doubled, the result is 28. What is the number?

4. When 12 is subtracted from a certain number and the difference so obtained multiplied by 7, the final result equals 7. What is the number?

Equations with Fractions

Fractions may be written in several ways:

One-half of c may be written $\frac{1}{2}c$ or $\frac{c}{2}$

$\frac{5}{3}x$ is also written as $\frac{5x}{3}$ and seldom as $1\frac{2}{3}x$.

$\frac{1}{2}(a+3)$ may be written as $\frac{a+3}{2}$. Check by letting $a=5$.

$\frac{3}{5}(x-3)$ sometimes appears like this: $\frac{3(x-3)}{5}$

The examples which follow will illustrate the methods of solving equations containing fractions.

Example 1. Solve $\frac{2x}{3}=5$.

Multiplying both sides by 3: $3\times\frac{2x}{3}=3\times5$.

Cancelling, $2x=15$ Check:

Dividing both sides, $x=7\frac{1}{2}$ L.S. $=\frac{2\times7\frac{1}{2}}{3}=\frac{15}{3}=5$

$=$ R.S.

Example 2. Solve $\frac{7a}{15}=\frac{2}{3}$

Check:

Multiplying, $15\times\frac{7a}{15}=15\times\frac{2}{3}$ L.S. $=\frac{7}{15}\times\frac{10}{7}$

Cancelling, $7a=10$ $=\frac{2}{3}$

Dividing, $a=1\frac{3}{7}$ $=$ R.S.

Example 3. Solve $\frac{3y}{4}=\frac{5}{6}$

Two different methods are shown.

$$\frac{3y}{4}=\frac{5}{6}$$

$$4\times\frac{3y}{4}=4\times\frac{5}{6}$$

$$3y=\frac{10}{3}$$

$$y=\frac{10}{3}\div 3$$

$$y=\frac{10}{9}$$

$$\frac{3y}{4}=\frac{5}{6}$$

We choose the smallest number which contains 4 and 6 as factors, namely, 12, their L.C.M.

$$12\times\frac{3y}{4}=12\times\frac{5}{6}$$

$$9y=10$$

$$y=\frac{10}{9}$$

Check:

$$\text{L.S.}=\frac{3y}{4}=\frac{3}{4}\times\frac{10}{9}=\frac{5}{6}=\text{R.S.}$$

Example 4. Solve $\frac{3}{5x}=\frac{2}{7}$

Multiplying both sides by $35x$, the L.C.M. of $5x$ and 7,

we have $35x\left(\frac{3}{5x}\right)=35x\left(\frac{2}{7}\right)$

Cancelling $7\,(3)=5x\,(2)$

$$21=10x$$

$$x=2\tfrac{1}{10}.$$

Check:

$$\text{L.S.}=\frac{3}{5\times 2\frac{1}{10}}=\frac{3}{10\frac{1}{2}}$$

$$=\frac{3}{\frac{21}{2}}\quad=\frac{2}{7}$$

Exercises:

Solve the following equations, checking each result:

1. $\frac{a}{2}=5$
2. $\frac{a}{6}=7$
3. $\frac{x}{12}=4$
4. $\frac{x}{7}=1$
5. $\frac{x}{8}=-2$
6. $\frac{2a}{5}=8$
7. $.5a=1.3$
8. $\frac{4b}{3}=1$
9. $\frac{4b}{7}=0$
10. $1.05x=420$
11. $2=\frac{3c}{8}$
12. $\frac{1}{3}=\frac{4a}{3}$
13. $0=\frac{2d}{5}$
14. $\frac{5a}{4}=a+2$
15. $\frac{3t}{2}-5=6$
16. $\frac{4t}{7}+2=3t$
17. $\frac{6-2x}{3}=5$
18. $8+\frac{3y}{4}=2y$
19. $\frac{5a}{12}=\frac{3}{8}$
20. $\frac{6y}{15}=\frac{4}{10}$
21. $.2a-1.2=.1a$
22. $\frac{2c}{3}+1=\frac{5c}{4}$
23. $5-\frac{c}{2}=6+\frac{c}{3}$
24. $\frac{2}{a}=\frac{3}{5}$
25. $\frac{2}{x}=\frac{1}{2}$
26. $\frac{3}{2c}=7$
27. $\frac{5}{2}=\frac{3}{4y}$
28. $.8=\frac{1}{c}$
29. $\frac{2}{3x}=\frac{5}{6}$
30. $\frac{9}{8y}=\frac{7}{12}$

*31. $\frac{1}{x}+\frac{1}{2}=3$

*32. $\frac{2}{x}-5=\frac{3}{4}$

*33. $\frac{3}{2x}-\frac{1}{4}=\frac{5}{8x}$

Equations with Fractions and Brackets

If fractions and brackets appear in an equation it is advisable to remove the brackets first, but as you gain experience with such equations you will develop other methods of solution .

Example 1. Solve $x-\frac{1}{2}(x+1)=3$.

Removing brackets, $x-\frac{x}{2}-\frac{1}{2}=3$.

Multiplying by 2,
$$\begin{aligned}2x-x-1&=6\\ x-1&=6\\ x-1+1&=6+1\\ x&=7.\end{aligned}$$

Check: L.S.$=7-\frac{1}{2}(7+1)=7-4=3=$R.S.

Example 2. Solve $\frac{a+4}{3}-\frac{a+12}{6}=\frac{a-2}{12}$

First method: $\frac{1}{3}(a+4)-\frac{1}{6}(a+12)=\frac{1}{12}(a-2)$.

Removing brackets, $\frac{a}{3}+\frac{4}{3}-\frac{a}{6}-\frac{12}{6}=\frac{a}{12}-\frac{2}{12}$

Multiplying by 12,
$$\begin{aligned}4a+16-2a-24&=a-2\\ 2a-8&=a-2\\ 2a-8+8-a&=a-2+8-a\\ a&=6.\end{aligned}$$

Second method: $\frac{a+4}{3}-\frac{a+12}{6}=\frac{a-2}{12}$

Multiply by 12: $\frac{12\,(a+4)}{3}-\frac{12\,(a+12)}{6}=\frac{12\,(a-2)}{12}.$

Cancel:
$$\begin{aligned}4\,(a+4)-2\,(a+12)&=a-2\\4a+16-2a-24&=a-2\\2a-8&=a-2\\a&=6.\end{aligned}$$

Note: $\frac{a+4}{3}$ has the same meaning as $\frac{1}{3}\,(a+4)$.

Exercises:

Practise removing brackets from the following expressions, as shown in the first one:

1. $\frac{a}{3}+\frac{a+5}{4}=\frac{a}{3}+\frac{1}{4}(a+5)=\frac{a}{3}+\frac{a}{4}+\frac{5}{4}$

2. $\frac{y+2}{4}-\frac{y-3}{6}$

3. $\frac{c-2d}{5}-\frac{c-d}{7}$

4. $\frac{a+5}{12}+\frac{2-3a}{15}-\frac{1-a}{4}$

*5. $12+\frac{2x+9}{5}-\frac{x+3}{7}$

Equations with Fractions and Brackets

Exercises:

Solve:

1. $\frac{x+3}{2}=5$

2. $\frac{x-1}{4}=8$

3. $y=\frac{2y-2}{4}$

4. $\frac{1}{2}(y+3)=\frac{3y}{5}$

5. $\frac{1}{2}\,(a-2)+\frac{1}{3}\,(a-3)=3$

6. $\frac{c-4}{6}+\frac{c+3}{4}=28$

7. $\frac{x-4}{2}+10=0$

8. $\frac{a-12}{2}=-4$

*9. $\frac{5}{2}(a-3)-\frac{8}{3}(a+1)=10$

*10. $\frac{3a+2}{11}=4-\frac{8a+3}{9}$

*11. $2-\frac{2a-1}{3}=7$

*12. $\frac{1}{3}(2y-13)-\frac{1}{4}(6y-3)=1$

*13. $\frac{a}{6}-\frac{2}{3}=\frac{1}{12}(a-13)$

*14. $\frac{a+6}{3}-7=\frac{1-a}{4}$

*15. $\frac{x}{2}-5=\frac{x+3}{4}$

*16. $\frac{x}{3}+\frac{1}{2}-\frac{1}{4}(x+2)=\frac{1}{3}$

*17. $\frac{d}{4}-d=5-\frac{d+1}{2}$

*18. $\frac{c}{6}-\frac{c+5}{3}+\frac{13}{6}=0$

*19. $\frac{a}{3}-\frac{a+2}{4}=5$

*20. $\frac{2a-1}{5}-\frac{a-8}{15}-6=0$

Exercises:

By substituting the given values for the unknown, without solving the equations, find the correct root. Do you find any equations in which more than one value is correct?

		Suggested roots
1.	$\frac{3a+1}{2}=5$	$\frac{11}{3}$, 3, -3, $\frac{-11}{3}$
2.	$\frac{1}{4}(3+c)=\frac{3c}{10}$	5, 10, 15, 0
3.	$\frac{y-4}{6}-y=1$	10, 1, 2, -2
4.	$3(y-1)-\frac{y+1}{2}=0$	$\frac{7}{5}$, 1, -1, 0
5.	$\frac{3}{x}-\frac{2}{x}=4$	1, 4, $\frac{1}{4}$, $-\frac{1}{4}$
*6.	$\frac{x(x+2)}{2}=1\frac{1}{2}$	-2, -3, 1, -1

Solving Problems

We now develop a plan for solving problems by the equation method, and shall consider various types of problems which may be worked by means of such a plan. Two examples will be worked in detail, and a plan then made.

Example 1. A man uses 65 ft. of fence to enclose a rectangular garden which is 20 ft. longer than it is wide. What is the width of the garden?

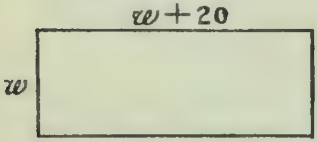
(1) The *known* facts are: Perimeter 65 ft. and length 20 ft. more than the width. The *unknown* fact is the width.

(2) We shall let w=the width.

(3) Then length $=w+20$, and perimeter $=2w+2(w+20)$.

(4) Since the perimeter is given as 65 ft.,
we can write an equation $2w+2(w+20)=65$

(5) Solving, $2w+2w+40=65$



$4w+40-40=65-40$

$4w=25$. Width $=6\frac{1}{4}$ ft.

$w=6\frac{1}{4}$. Length$=26\frac{1}{4}$ ft.

(6) We check the result from the problem:

$$\text{Perimeter}=2\left(26\tfrac{1}{4}+6\tfrac{1}{4}\right)=2\left(32\tfrac{1}{2}\right)=65.$$

Example 2. A man's daily wage is three times that of a boy's. The man works 5 days, the boy 4 days, and together they earn $38. Find the daily wage of each .

(1) *Known facts.* Man's wage 3 times boy's. Man worked 5 days, boy 4. Total earnings $38. *Unknown facts.* Daily wage of each.

(2) We shall let x=boy's daily wage .

(3) Then man's daily wage $=3x$. The boy earns $4x$, the man $15x$.

(4) The equation then is $4x+15x=38$

(5) Solving, $19x=38$

and $x=2$

Hence the boy's daily wage is $2 and the man's $6.

(6) Check the problem: $5\times 6+4\times 2=30+8=38$.

In each of the two examples worked, six steps were made. The plan we shall now give will have six similar steps.

Plan for Solving Problems

(1) By reading the problem carefully, separate the known facts (the data) from the unknown facts.

(2) Let x or any convenient letter represent the unknown quantity, or, if there are more than one, select the one on which the others seem to depend.

(3) Express the other quantities in the problem in terms of x or the selected letter.

(4) Look for two ways of expressing the same thing, and thus form an equation.

(5) Solve the equation.

(6) Check the result, not from the equation, which may by chance be wrong, but from the original problem.

This is a general plan and sometimes may have to be changed to suit certain kinds of problems. If you follow the plan as much as you can you will learn how to analyse a problem and construct the required equation.

The problems given are arranged in groups, such as *number* problems, *percentage* problems, problems from *geometry,* etc.

Number Problems

The first exercise gives you a short practice in changing words into symbols.

Exercises (*oral or written*):

1. If the sum of two numbers is 10 and one is y, what is the other?

2. If the sum of two numbers is x and one is 4, what is the other?

3. If the difference of two numbers is 15 and the greater is C, what is the other?

4. If d is the difference between two numbers and 6 is one, what is the other?

5. If a number is 8 more than x, what is it?

6. If a number is 9 less than y, how would you express it?

7. If n is a number, how would you express twice n? one-third of n?

8. One number is 5 more than another number x. Write the sum.

Number Problems

Exercises:

Applying the method given for solving problems, work the following:

1. Five times a number added to three times the same number gives 72. Find the number.

2. If 5 is added to three times a certain number, the result is 155. What is the number?

3. If 23 be subtracted from four times a number, the remainder is 73. What is the number?

4. One-half of a number with 18 added gives 25. Find the number.

5. If 17 is taken away from one-third of a number, the remainder is 1. Find the number.

6. One number is three times another number, and their sum is 100. What are the two numbers?

7. A class of 39 pupils contains twice as many girls as boys. How many boys are in the class?

8. A house and lot together cost $4,000. The house cost $3,000 more than the lot. Find the cost of each?

9. If 32 be subtracted from seven-eighths of a number the remainder equals 17. What is the number?

10. The difference between three-quarters of a number and one-third of it equals $22\frac{1}{2}$. What is the number?

Without actually solving the following problems, select the correct answer from among those suggested:

***11.** Find the number such that twice the square of the number with 4 equals 76. (Number is 3, 6, -6.)

*12. If 6 be added to the square of a number, the result equals five times the number. (1, 2, 3.)

*13. If from the square of a number, three times the number be subtracted, the result equals 4. (1, —1, 4, —4.)

Percentage Problems

We write 3% of 50 as $.03\times 50$.

Then 3% of y is $.03y$,

125% of b is $1.25b$,

80% of $(x+50)$ is $.8\,(x+50)$.

Example 1. A man sold a house for $2300, making a profit of 15% of the cost. Find the cost.

Solution: Let the cost in dollars be x.

Then 15% of x is $.15x$.

We form an equation,

$$x+.15x=2300.$$

We may also write directly

$$115\%x=2300$$

that is

$$1.15x=2300$$

Then

$$x=\frac{2300}{1.15}=2000.$$

Thus the cost is $2000.

Example 2. A consignment of goods cost a wholesale dealer $1800. He sold them, making a profit of 10% of the selling price. What was the selling price?

Solution:

Let the selling price be x.

The profit $=10\%$ of $x=.1x$;

The cost is then $x-.1x$

Then

$$x-.1x=1800$$
$$.9x=1800$$
$$x=2000$$

The cost price is $2000.

Percentage Problems

1. 24% of a sum of money is $84. Find the sum of money.

2. After allowing a discount of 15% a merchant sold goods at $191.25. Find the list price.

3. A furniture dealer gained 6% of the cost when he sold a dining-room suite for $132.50. What did it cost him?

4. What sum of money must be invested at 2½% to give an annual income of $1,700?

5. A sum of money is invested at 6% interest. At the end of the year, principal and interest total $270.30. Find the sum invested.

6. Mr. Smith invested a certain sum at 4% and the same amount at 5%. The total interest from the two investments for one year was $78.30. What sum was invested at each rate?

7. Another year he invests a certain sum at 3% and twice that sum at 4%, and receives as total interest for the year $132. What sum was invested at each rate?

8. A man invests $800 at 2½% and a second sum of money at 3%. From the two investments his annual income is $56. What sum was invested at 3%?

***9.** Mr. White borrows a certain sum of money at 6% interest per annum, and another sum which is $1,200 greater than the first at 7% interest. On the two amounts he pays a yearly interest of $188. How much was borrowed altogether?

***10.** A man invests $12,000, part at 2% and the rest at 3%. The total interest for the year is $290. How much is invested at each rate?

***11.** I borrow $2,500, part at 5% and the rest at 6%, paying $131 interest each year on the whole sum. How much was borrowed at each rate?

***12.** The income at 4% on a certain sum of money is equal to the income on a sum which is $200 greater at 3%. What are the two sums of money?

Mensuration Problems

Problems dealing with areas, perimeters, volumes, etc., may be solved by the use of equations. Make use of diagrams whenever possible and employ formulas.

Example: The area of a triangle is 54 sq. in., and the height is three-quarters of the base. Find the length of the base and the height.

Known facts: Area $=54$; height$=\frac{3}{4}$ base.

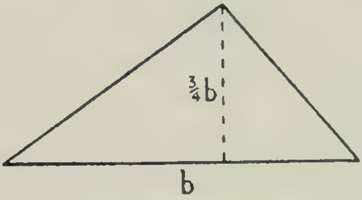
Unknown: Base, height.

Let the base be represented by b, then the height$=\frac{3}{4}b$.

Formula for area $\quad A=\frac{1}{2}ba$

Substituting $\quad 54=\frac{1}{2}b(\frac{3}{4}b)$

This equation gives $\quad 54=\frac{3}{8}b^2$



Removing fractions $8\times54=8\times\frac{3}{8}b^2$

$$432=3b^2$$
$$b^2=144$$
$$b=12.$$

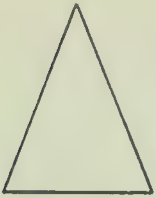
The length of the base is 12 in., the height 9 in.

Check: Area$=\frac{1}{2}ba=\frac{1}{2}\times12\times9=54$.

Exercises:

1. The length of a rectangle is three times its width, and its area is 147 sq. in. What are its dimensions?

2. Find the dimensions of a rectangle whose areas is 375 sq. in. and whose length is $1\frac{2}{3}$ times its width.



3. A certain triangle has two equal sides (isosceles) and the third side, or base, is one-half of either of the equal sides. If the perimeter is $172\frac{1}{2}$ cm., what is the length of each side.

4. In an isosceles triangle whose perimeter is 46 ft., each of the equal sides is 5 ft. more than the third side. Find the length of each side.

5. Find the dimensions of a rectangle whose perimeter is 18 in. and whose length is $2\frac{1}{2}$ in. more than its width.

6. The three angles of any triangle together make up 180°. Two of the angles are equal, and the third angle is 15° less than either of the other two. Find the number of degrees in each angle.

*7. The length of a rectangle is 3 in. more than its width. If its width in increased by 1 in. and its length diminished by 2in., its area remains the same. Find the dimensions of the original rectangle.

Miscellaneous Problems

1. Two boys together earned $2.80. One worked three-quarters as long as the other. How should they share the money?

*2. In a boy's bank there are 37 coins, some 5¢ pieces, the rest 10¢ pieces. If the value of all the coins is $2.80, how many of each are there?

*3. Ruth and Mary had equal sums of money. After Ruth had given Mary 10¢, Mary had then three times as much as Ruth. How much had each to start with?

*4. Four hundred persons attended a school concert, children paying 10¢ each and adults 25¢ each. If the total receipts were $79, how many children paid admission?

Proportion

Since $\frac{2}{3}=\frac{4}{6}$, we say that the numbers 2, 3, 4, 6 are in *proportion* or are *proportional.* The four numbers are called *proportionals.* Using letters, we say that a, b, c, d are in proportion if

$$\frac{a}{b}=\frac{c}{d}.$$

If we have three numbers, 5, 6, 15, and wish to find a fourth number to make up a proportion, we represent this fourth number by x. Then,

$$\frac{5}{6}=\frac{15}{x}.$$

Multiplying both sides of this equation by $6x$, we obtain:

$$5x=90$$

so that

$$x=18.$$

Thus, 5, 6, 15, and 18 form a proportion:

$$\frac{5}{6}=\frac{15}{18}$$

Exercises:

Solve the equations:

1. $\frac{2}{5}=\frac{4}{x}$

2. $\frac{3}{12}=\frac{1}{x}$

3. $\frac{20}{5}=\frac{64}{x}$

4. $\frac{1.5}{3}=\frac{2}{x}$

5. $\frac{100}{15}=\frac{50}{x}$

6. $\frac{x}{8}=\frac{5}{20}$

7. $\frac{x}{9}=\frac{2}{11}$

8. $\frac{6}{7}=\frac{x}{28}$

Find which of the following sets of numbers form proportions:

9. 8, 3, 56, 21

10. 9, 6, 12, 18

11. 1, 2, 3, 4

12. 5, 7, 2.5, 3.5

13. 1.2, 6.4, 8, 42

14. $1\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{1}{4}$, $4\frac{1}{2}$

15. If 3 books cost 75¢, find what 11 of the same kind would cost. Let x be the cost in cents. Then

$$\frac{x}{75}=\frac{11}{3}.$$

16. If the rent of a room for 12 days is \$15, how much would the rent be for 7 days at the same rate?

17. If 16 ft. of wire weigh 2½ lb., how much will 20 ft. of the same wire weigh?

18. If, during a rainstorm, 5¼ tons of water fall on 3 acres, how many tons would fall on 7 acres?

19. If an electric current costs $17.60 for 27 hours, how much would the current cost for 16 hours?

20. If a seeder plants 3 bushels of wheat in 1½ hours, in how many hours would it plant 20 bushels?

21. If 6 dozen pencils are used by 40 children in 2 months, how many pencils would they require for 9 months?

22. A man received $6.30 for 9 hours' work. How much does he receive for 15 hours' work at the same rate?

23. Mr. Ford raises 650 bushels of wheat on a 25-acre field. How many bushels could he raise, at that rate, on a 35-acre field?

24. At a certain time a post 8 ft. high casts a shadow 5 ft. long. What will be the height of a tree which, at the above-mentioned time, casts a shadow 35 ft. in length?

Deriving New Formulas

In solving equations certain axioms are used. To each side of any equation you may add or subtract the same quantity. You may also multiply or divide both sides of an equation by the same amount. These axioms are used in the working of the examples which follow.

Example 1. The length of a rectangle is 5 ft. and its area is 10 sq. ft. Find its width.

$$\begin{aligned} A &= lw \\ 10 &= 5w \\ \text{or } 5w &= 10 \\ \frac{5w}{5} &= \frac{10}{5} \\ w &= \frac{10}{5} \\ w &= 2 \end{aligned}$$

$$\begin{aligned} A &= lw \\ \text{or } lw &= A \\ \frac{lw}{l} &= \frac{A}{l} \\ w &= \frac{A}{l} \\ w &= \frac{10}{5} \\ w &= 2 \end{aligned}$$

In the solution on the preceding page on the left we substitute our given values directly in the formula $A=lw$, obtaining $w=2$.

In the solution on the right, we first derive the formula $w=\frac{A}{l}$. Knowing that $A=10$ and $l=5$, we find $w=2$.

Example 2. The formula $s=c+p$ gives the selling price in terms of the cost price and the profit. Derive a formula for the profit p.

$$c+p=s.$$

Subtracting c from each side,

$$c+p-c=s-c.$$

Thus $$p=s-c.$$

This is the formula required.

Example 3. In how many years will the simple interest on \$200 at 5% be \$400?

We seek a formula for the time t, in terms of principal, interest, and rate. From the interest formula,

$$i=prt,$$

that is, $$prt=i,$$

dividing both sides by pr, we have

$$\frac{prt}{pr}=\frac{i}{pr}$$

$$t=\frac{i}{pr}.$$

Then, substituting values for i, p, and r,

$$t=\frac{40}{200\times .05}=4.$$

Thus, the required time is 4 years.

Example 4. From the mathematical formula

$$s(1-r)=a$$

derive a formula for r.

Removing brackets: $s-sr=a$.

Subtracting s from each side of the equation,

$$s-sr-s=a-s,$$

we have $$-sr=a-s.$$

Changing all the signs, which is equivalent to multiplying or dividing both sides by -1, we obtain

$$sr=s-a.$$

Then
$$r=\frac{s-a}{s}.$$

Thus, if $s=4$ and $a=2$,

$$r=\frac{4-2}{4}=\frac{1}{2}.$$

(NOTE: The process shown in the four examples just given is sometimes called *'changing the subject of the formula'*.)

Exercises:

Derive a formula for the letter indicated.

1. $d=rt;\ t=$

2. $p=a+b+c;\ b=$

3. $c=\pi d;\ d=$

4. $i=prt;\ r=$

5. $p=4s;\ s=$

6. $V=bh;\ b=$

7. $C=2\pi r;\ r=$

8. $w=4st;\ t=$

9. $s=c+p;\ p=$

10. $l=c-s;\ s=$

11. $A=p+prt;\ r=$

12. $A=p\,(1+rt);\ p=$

13. $T=2\pi rh+2\pi r^2;\ h=$

14. $l=a+6d;\ d=$

15. $S=n\,(w-1);\ w=$

16. $w=s+np;\ n=$

17. $s=w-np;\ p=$

18. $l=a+(n-1)d;\ a=$

19. Using the formula obtained in Example 11, find the rate at which \$300 will amount to \$450 in 5 years at simple interest.

20. In Example 16, find n when $s=3$, $p=2$, $w=83$.

21. Using the formula obtained for a in Example 18, find a when $l=33$, $n=15$, $d=2$.

22. With the same formula as in Example 21, find a when $l=10$, $n=14$, $d=4$.

Changing Formulas Involving Fractions

Example: From the formula for the area of a triangle derive a formula for the height of a triangle.

The formula is $A=\frac{1}{2}ba$.

This may be written $\frac{1}{2}ba=A$.

Remove fractions by multiplying both sides of the equation by 2: $2\times\frac{1}{2}ba=2\times A$

$$ba=2A.$$

Divide by b: $$a=\frac{2A}{b}.$$

Exercises:

Derive formulas for the letters indicated.

1. $V=\frac{1}{3}Bh$; B
2. $A=\frac{1}{2}ba$; b
3. $V=\frac{1}{3}\pi r^2h$; h
4. $S=\frac{1}{2}n\,(a+l)$; a
5. $a=\frac{1}{2}\,(b+1)$; b
6. $V=\frac{1}{3}Bh$; h
7. $C=\frac{5}{9}\,(F-32)$; F
8. $V=\frac{1}{3}e^2h$; e

*9. $x=\dfrac{y-2}{3}$; y

*10. $s=\dfrac{n\,(a+l)}{2}$; l

Making use of the formulas found above, complete the following table.

	Original Formula	*Given*	*Find*
1.	$V=\frac{1}{3}Bh$	$V=16$; $h=2\frac{1}{2}$	B
2.	$A=\frac{1}{2}ba$	$A=2.9$; $a=3.6$	b
3.	$V=\frac{1}{3}\pi r^2h$	$V=200$; $r=7$	h
4.	$S=\frac{1}{2}n\,(a+l)$	$S=300$; $n=20$; $l=24$	a
5.	$a=\frac{1}{2}\,(b+1)$	$a=3.5$	b
6.	$C=\frac{5}{9}\,(F-32)$	$C=25^\circ$	F
7.	$V=\frac{1}{3}e^2h$	$V=82$; $h=6$	e
8.	$x=\dfrac{y-2}{3}$	$x=5$	y
*9.	$S=\dfrac{n\,(a+l)}{2}$	$S=210$; $n=15$; $a=1$	l
*10.	$C=\frac{5}{9}\,(F-32)$	$C=0$	F
*11.	$C=\frac{5}{9}\,(F-32)$	$C=200$	F

Review Exercises

Solve the following ten equations:

1. $7a-4=3a$
2. $3.2x-5=3x+1$
3. $\frac{5x}{4}=4$
4. $0.3y-5=1.3$
5. $4\frac{1}{2}a+1.5=10.5$
6. $5a-1=4a-(3a+5)$
7. $4y-3(y+1)=2(y-3)$
8. $.5(10x+4)=.25(x+8)$
9. $\frac{a+6}{3}=\frac{a+12}{6}+\frac{a+1}{5}$
10. $\frac{1}{7}(4c+3)-\frac{c+5}{3}=4$

By substituting the suggested values for the unknowns, without solving the equation, select the proper roots in the next three questions:

11. $(10x-3)-(8x-7)=-4;$ $x=4, -4, 0, 1.$
12. $6-5(a+1)=0;$ $a=1, -1, \frac{1}{5}, -\frac{1}{5}.$
13. $1-\frac{2c+2}{3}=\frac{-c}{2};$ $c=0, 1, 2, 3.$

Use algebraic symbols in the next four questions:

14. The difference of two numbers is b and the smaller one is c. What is the larger number?

15. John and Ralph have y¢ each. John gives Ralph m¢. How many cents has each then?

16. Mary is t years old now. How many years ago was she w years old?

17. How far would you drive in d hours at the rate of k miles per hour?

Solve the next three problems:

18. The length of a rectangle is $2\frac{1}{2}$ ft. more than its width, and its perimeter is 39 ft. Find its dimensions.

19. If 25 be subtracted from three-quarters of a number, 47 will be left. What is the number?

20. A certain sum of money is invested at 3% and an equal amount at 4%. The total income is \$112. What sum is invested at each rate?

21. From the formula $p=a+b+c$ derive a formula for c. Then find the value of c if $p=24.3$, $a=10.1$, $b=8.4$.

22. From the formula $V=\pi r^2 h$ derive a formula for h. Hence find the height of a cylinder with a radius of 3 inches and a volume of 88 cubic inches.

Remedial Exercises

Solve the following equations:

1. $.3x+.5=1.7$
2. $\frac{3a}{5}=9$
3. $.8x-1.8x=2$
4. $6c-4=4$
5. $3(y+1)=0$
6. $8c=4+3(c+2)$
7. $10a-4=8-2(a-6)$
8. $5(a+6)-4(a-9)=0$
9. $\frac{a-4}{6}=\frac{a-1}{12}$
10. $x-2=\frac{1}{2}(x-1)-\frac{5}{2}$

In the next two questions, you are to find which of the three suggested answers is correct. Do this, not by solving the equation, but by substituting for the unknowns:

11. $\frac{a-4}{5}+1=\frac{a}{4}$; $a=0, 4, 1, -1$
12. $6x=6-2(x-1)$; $x=2, -2, 1, -1$

In the next three questions, use letters to represent the required quantities:

13. The sum of two numbers is represented by C. One of them is t. What is the other?

14. The radius of a circle is y units of length. What is its area expressed in terms of y?

15. Travelling at p miles per hour, how far would you go in q hours?

Solve the next five problems:

16. The sum of two numbers is 80 and their difference is 4. What are the numbers?

17. Jack has 38¢ and Tom has 16¢. How much must Jack give Tom, so that each will have the same amount?

18. I bought a radio for $120, which was 80% of the regular price. What was the regular price?

19. One sum of money is invested at 3%. A sum which is twice the first sum is invested at 4%. The total yield is $33. What are the two sums of money?

20. The height of a triangle is half its base, and its area is 16 sq. in. What is the length of the base?

21. From the formula $A=2\pi rh$, derive one for h. If $A=132$ and $r=3$ find h.

22. From $A=\frac{1}{2}ba$, find a formula for b. Hence find the base of a triangle whose area is 16 sq. ft. and altitude 12 ft.

Vocabulary Test

Explain the following terms, giving an example of each:

Conditional equation
Identity
To satisfy an equation
Unknown
To solve
Root
Solution

Achievement Test

Solve the following equations:

1. $2a+5=3a$

2. $\frac{1}{4y}=8$

3. $0=2(x-5.3)$

4. $(3c-2)-(c-1)=8$

5. $\frac{7a}{3}=\frac{14}{9}$

6. $3+\frac{2x}{5}=8$

7. $8-2(x-3)=16$

8. $4-\frac{a}{3}=\frac{a}{2}-1$

9. $\frac{x}{2}+1.5=\frac{3x-1}{2}$

10. $\frac{1}{3}(b-3)=\frac{2b-5}{4}$

From the three suggested roots given for each equation select the correct one:

11. $\frac{3}{2x}=1\frac{1}{2}$; $x=2, -1, 1$

12. $6-2(c+1)=-2$; $c=1, 2, -2$

13. $2(x-1)-\frac{x+1}{3}=0$; $x=\frac{5}{7}, 0, \frac{7}{5}$

14. $.4a-1.2=0$; $a=3, .3, .03$.

15. If x centimetres equal y in., find in terms of x and y the number of centimetres in 1 ft.

Solve these problems:

16. The sum of two numbers is 57 and one is 3 less than the other. What are the numbers?

17. One side of a room is 12 ft. wider than another, and the perimeter is 96 ft. Find its dimensions.

18. I bought 60 stamps, some 2¢, the others 3¢. If they cost in all $1.44, how many of each did I buy?

19. The profits of a business are shared by the two partners, Mr. Stuart receiving $2 for each $3 received by Mr. Thompson. What is each partner's share of profits amounting to $1,275?

20. From the formula $c=2\pi r$ derive a formula for r. Hence find the radius of a circle with a circumference of 10 inches.

CHAPTER IX

LINES

Review of Symbols and Abbreviations

Give the symbol or abbreviation for each of the following words:

(1) Angle
(2) Angles
(3) Triangle
(4) Triangles
(5) Square
(6) Straight
(7) Right
(8) Perpendicular
(9) Degree

The class should discuss these abbreviations and agree on the use of the same symbols by all members of the class.

Review of Vocabulary

Give the meaning of each of the terms listed below, either in words, or by means of a figure, or both. All should have been previously used by you. They will occur frequently in this year's work in geometry:

(1) Line
(2) Line-segment
(3) Straight line
(4) Horizontal line
(5) Vertical line
(6) Oblique line
(7) Angle
(8) Acute angle
(9) Obtuse angle
(10) Right angle
(11) Straight angle
(12) Reflex angle
(13) Triangle
(14) Acute-angled triangle
(15) Obtuse-angled triangle
(16) Right-angled triangle
(17) Equilateral triangle
(18) Isosceles triangle
(19) Scalene triangle
(20) Square
(21) Rectangle
(22) Included or contained angle
(23) Vertex of an angle
(24) Altitude

Class discussion:

The results of the above review will suggest topics which may briefly be discussed by teacher and pupils. The list of topics which follow are merely suggestions and may be added to as the need becomes apparent:

1. The difference between a line and a line-segment, and methods of indicating the indefinite length of a line.

2. A straight line as one that does not change direction, or a line that is the shortest distance between two points.

3. An angle as the amount of rotation of a line about a point, or the difference in direction of two lines which meet at a point.

4. Relations between a point, a line, and a plane surface.

Measurement of Lines

Each pupil should have a ruler or straight-edge, graduated in: (*a*) inches and tenths of an inch, (*b*) centimetres and millimetres; a pair of compasses; and a protractor.

Exercises:

Without instructions being given as to how to measure, the following lengths should first be estimated by each pupil, then measured by groups, as indicated. The English system should be employed: yards, feet, inches.

1. The length of the school building, and the width of the school yard. (This should be done by one small group of pupils).

2. The width of the classroom. (By another group).

3. The height or width of the blackboard. (By one or two pupils).

4. The width of the pupil's desk. (By each pupil).

5. The length of the text-book used for mathematics.

6. The length of this line ————————————————

7. The length of this line ———

Using the metric system, each pupil should measure these lines:

8. ——————————————————————————

9. —————————————

10. ——

Class Discussion:

1. A comparison of each pupil's estimated lengths with the measured lengths will indicate the extent of his ability to estimate.

2. From the measurements made in the exercises there should develop the idea of measurement as being an approximation. Compare the lengths given for Exercises 2 and 10 in the above list. Discuss reasons why the width of the room should be given perhaps to the nearest inch, while the length of the line in the tenth measurement should be given at least to the nearest tenth of an inch, and preferably to a finer approximation.

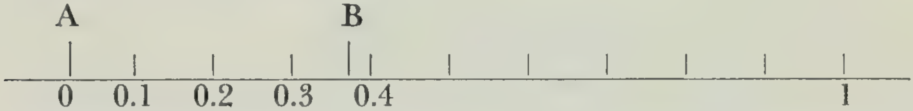
3. Agree that this year, for geometrical figures occurring in exercises, an attempt will be made to measure lines in the English system to the nearest hundredth of an inch, and in the metric system to the nearest half millimetre.

The use of compasses in drawing lines to given lengths and in measuring lines should be taught and stressed. Thus, to draw a line AB, 3 in. long, first draw a line approximately more than 3 in.

Open compasses to a distance of 3 in., as given by the straightedge, and, beginning at any point A, cut off a distance AB equal to 3 in.

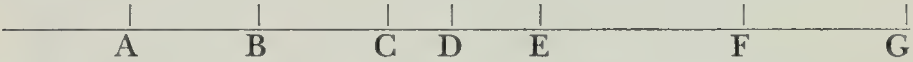
In the same way, to measure a line, first obtain the distance by opening the compasses the required length, then transfer to the straight-edge for reading.

The method of reading hundredths of an inch should be demonstrated on the blackboard by the use of an enlarged figure of a section of the ruler.



The distance from 0 to 1 represents 1 in., while the distance between 0 and 0.1 represents $\frac{1}{10}$ in. Each of these tenths may then be considered as capable of division again into ten equal parts, each being one-hundredth. Thus the distance AB on the above scale represents approximately .37 in.

Exercises:



1. Measure to the nearest hundredth of an inch the distances AB, BC, CD, DE, EF, FG. Add your results. Measure the whole line AG and compare the two answers. Would you expect any variation? If so, why?

2. Draw three or four lines of varying lengths in your exercise-book. Measure each and note results on another page. Change with your neighbour and measure his lines. Compare measurements.

3. A group of six to eight pupils should each measure the same line independently. One of them should write all the measurements on the blackboard and take the average. Each one of the group should then notice how near to the average his measurement is. This method of averaging results should be frequently used.

Measuring Lines (Metric System)

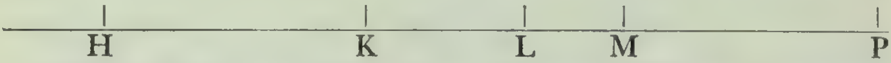
The metric system of length should be reviewed.

Exercises (*oral*):

1. Change 1 metre to centimetres and to millimetres.
2. Explain the abbreviations m., cm., mm.
3. Change 3.5 m. to centimetres.
4. How many millimetres in 14.6 cm.?
5. Represent 237 mm. as centimetres.
6. Express 6 cm. 8.5 mm. as centimetres.

Exercises:

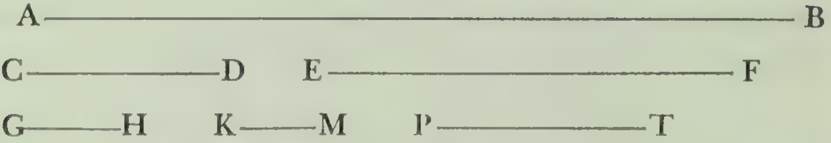
1. Measure HK, KL, LM, MP to the nearest half-millimetre.



Add the four measurements. Measure HP and compare with the sum of the four parts.

(For those pupils having errors in this exercise, a blackboard demonstration is necessary, showing how to read to .5 mm.)

2.



Measure the six lines given above: (1) in inches; (2) in centimetres and millimetres; in each case to the approximation agreed on.

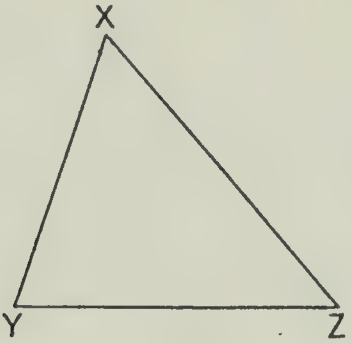
From your results, calculate: (1) the number of centimetres in 1 in.; (2) what fraction of an inch 1 cm. is. Ten answers for each case should be placed on the blackboard by a pupil and an average taken. The class should memorize:

1 cm.=.39 in. and 1 in.=2.54 cm.

3. (*a*) By measurement, using compasses and ruler, find the perimeter of △XYZ in inches.

(*b*) In the same manner, find its perimeter in centimetres.

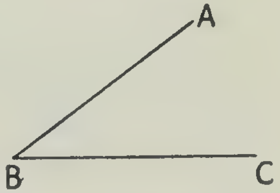
(*c*) Divide answer to (*a*) by answer to (*b*). What should the result be?



Review of Angles

An angle may be considered as (*a*) the difference in direction of two straight lines, or (*b*) the amount of rotation of a straight line with one end fixed in position.

Exercises (*oral*):



1. Name the angle at the right. (∠ABC or ∠CBA, but not ∠CAB or ∠ACB).

2. Name the vertex, and the arms of the angle.

3. What is understood by 1°?

4. How many degrees are formed when a line makes one complete rotation or revolution?

5. How many degrees for one-half a revolution?

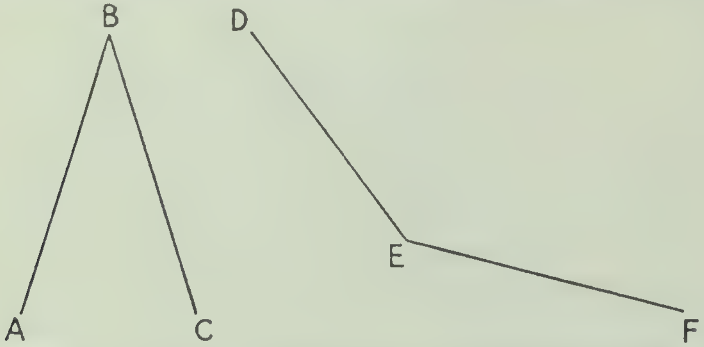
6. What part of a complete rotation is made when the angle is 90°?

7. When the arms of an angle are perpendicular to each other, what is the size of the angle?

Measuring Angles

Exercises:

1. Estimate the number of degrees in these angles: Measure the angles to the nearest half-degree. Take an average of ten answers given by the class and compare your answer with the average.



2. Practise drawing angles such as 23°, $75\frac{1}{2}$°, 121°. Have your neighbour measure them in order to check.

3. Pass around the class drawings of angles and have them measured and results averaged.

Review of Constructions

You should be familiar with the three constructions which follow.

(1) To Bisect a Line

Exercises:

1. Draw a line 3.52 in. in length, bisect it and measure accurately the two parts. Check by arithmetic.

2. Repeat the exercise with a line 10.65 cm. in length.

(If your results in Exercises 1 and 2 are not accurate, practise making this construction until you are satisfied with the degree of accuracy you are obtaining).

3. Give a written explanation of your construction, as follows:

Let AB be a given line.

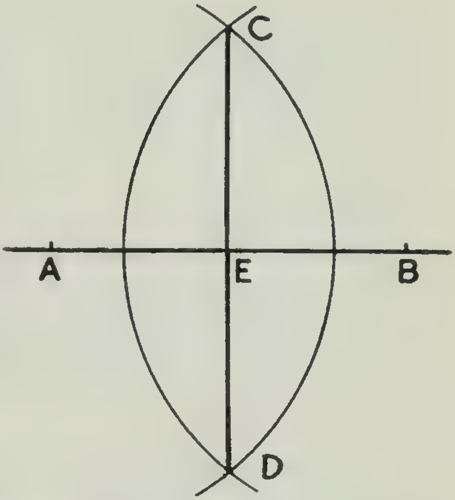
Required to bisect AB.

Construction: With centres A and B and equal radii describe arcs of circles intersecting at C and D.

Join CD, cutting AB at E.

Then AB is bisected at E.

(NOTE: In practice, only small portions of the arcs are drawn near C and D).



4. Measure, with your protractor, ∠AEC and ∠BEC. Your results will show that CD not only bisects AB, but also bisects it at right angles. Thus CD is the perpendicular bisector of AB. (CD $\perp$ AB).

5. Draw a line CD of any length and bisect it. Give a written explanation as shown in the sample.

(2) TO ERECT A PERPENDICULAR

Exercises:

1. Draw a line AB 3 in. in length. Cut off AC=2 in. Using only ruler and compasses, at C erect a line CD perpendicular to AB. Measure ∠ACD.

2. The line XY is 8.65 cm. long. The point Z is 3.7 cm. from X. Erect a line at Z perpendicular to XY. Measure both angles formed at Z.

The two constructions in the last exercises are particular examples of the following general construction: 'At a given point in a given straight line to erect a perpendicular'.

The written explanation should be somewhat as follows. (Supply your own figure to fit this explanation):

Let AB be the given line and C the given point in AB.

Required: To erect at C a line ⊥ AB.

Construction: With centre C and any radius, describe arcs cutting AB at D and E.

With D and E as centres and equal radii, describe arcs intersecting (above or below AB) at F.

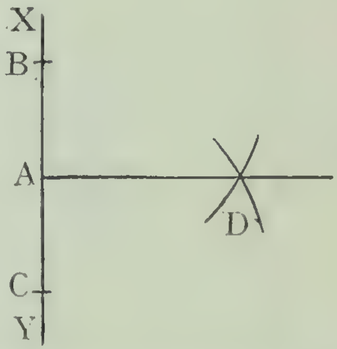
Join CF.

Then CF ⊥ AB.

(NOTE: If the given point C is too near to an end of the line to obtain a satisfactory figure, produce AB, or BA, that is, continue the line AB in either direction).

Exercises:

1. Supply a written explanation to fit the adjoining figure.



2. On a line AB of length 4 cm. erect at A and B lines perpendicular to AB. Make these perpendiculars each 4 cm. long and have them lying on the same side of AB. (See note above). Join their ends. What figure is formed?

3. Using the construction of Exercise 2, make a square with side 2.31 in. Measure all four sides as accurately as you can.

4. Draw a line 3.64 in. long. At each end of the line erect perpendiculars each 1.3 in. long, so that they both lie on the same side of the line. Join their ends. What figure is obtained?

5. Construct a rectangle 6.05 cm. long and 3.75 cm. in width. Find its perimeter by measurement of the four sides. Check by substituting in the formula:

$$p=2\,(l+w).$$

(3) TO DRAW A PERPENDICULAR FROM A GIVEN POINT TO A GIVEN LINE

Summary of the construction: AB is the given line, C is the point outside the line. With C as centre an arc is drawn, cutting AB at D and E. D and E are used as centres, and with equal radii arcs are drawn intersecting at F, preferably on the side of AB away from C. (Why?)

When C is joined to F, a perpendicular is obtained.

Exercises:

1. Supply a figure for the above construction, and write an explanation in proper geometrical form.

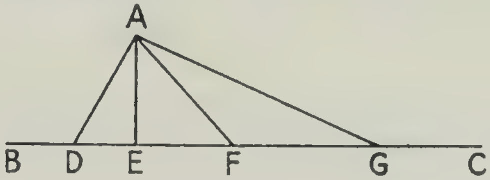
2. Draw any line CD. From a point F, not on CD, draw a line FO perpendicular to CD. From F draw three other lines meeting CD in G, H, K. By measurement, find which of the lines FO, FG, FH, FK is the shortest.

DISTANCE OF A POINT FROM A LINE

When we speak of the distance of a point from a line, we mean, in geometry, its perpendicular distance. The construction given above enables you to find the distance of a point from a line. All that is necessary is to draw a perpendicular and measure the length of this perpendicular lying between the given point and the given line.

Draw a line of any length. Place three points at different places, either above or below the line. By constructing perpendiculars, find how far each point is from the line.

In the adjoining figure, four lines have been drawn from A to BC. By measurement, find which is the shortest line.



Satisfy yourself that the following statement is true:

'Of all lines drawn from a point to a line, the perpendicular is the shortest'.

Exercises:

1. Draw an acute-angled triangle XYZ. From X, Y, and Z draw perpendiculars to the opposite sides. How do these three lines intersect?

2. Using an obtuse-angled triangle, repeat the construction. In this case you will require to produce some of the sides of the triangle. Do you find that the three perpendiculars pass through the same point?

3. Use a right-angled triangle. What difference is there? Do the three perpendiculars pass through the same point? If so, what point is it?

Parallel Lines

We say that an angle is formed when two straight lines meet, and it is assumed that the two lines have different directions. If two lines have the same direction, they will not meet. Such lines are said to be parallel. Later we shall have a more exact definition. Examples of parallel lines are: (1) street-car rails; (2) the ruled lines in your exercise book; (3) the opposite sides of a rectangle.

Exercises (oral):

1. Name two parallel streets in your town.
2. What parts of a sidewalk represent parallel lines?
3. Point out pairs of parallel lines in your classroom.
4. Give examples of parallels found on your desk.
5. Suggest other everyday illustrations.

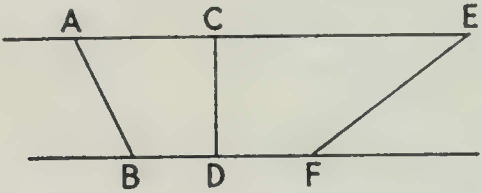
Class discussion:

From the illustrations supplied, the following conditions for parallel lines should be made clear:

(1) Parallel lines are straight.
(2) They lie in the same plane. (Review plane surface.)
(3) They have the same direction.

(4) They never meet when produced in either direction.

(5) They remain the same distance apart. (Explain that the distance between two parallel lines is the perpendicular distance between them. Thus, in the figure, CD is the distance).

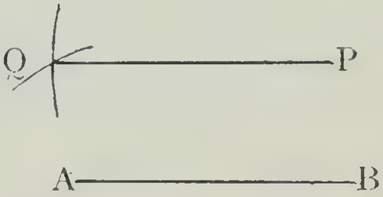


Construction: Through a given point to draw a line parallel to a given straight line.

Let AB be the straight line and P the given point.

Required: To draw through P a line parallel to AB.

Construction: With centre A and radius BP describe an arc.



With centre P and radius BA describe an arc cutting the other arc at Q.

Join PQ.

Then QP is parallel to AB, and it passes through P.

Note that in the last line of the preceding construction, we stated that QP is parallel to AB. We might have said PQ is parallel to BA. Why? Sometimes we indicate the direction of a line by an arrow mark.

Thus A———→———B would be read 'the line AB', whereas A———←———B is referred to as 'the line BA'.

The symbol used for parallel is ||. Do you see that this is just as appropriate for parallel lines as ⊥ is for perpendicular?

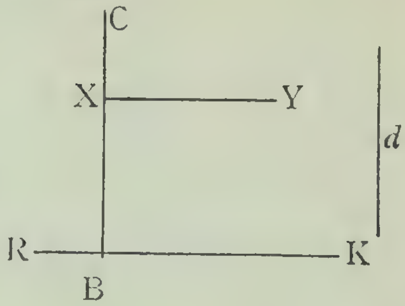
Thus AB || QP is read: 'AB is parallel to QP'.

Construction: To draw a line parallel to a given straight line at a given distance from the line.

Let RK be the given line and d the given distance.

Required: To draw a line parallel to RK at a distance d from the line.

Construction: At any point B in RK erect BC ⊥ RK.



From BC cut off BX equal to d. At X erect XY ⊥ BC. Then XY || RK.

(NOTE: For clearness in demonstrating, the arcs necessary for erecting perpendiculars have been left out of the figure. The pupil should show such construction lines).

Exercises:

1. Draw any line AB. Draw a line parallel to AB and 1.2 in. from it. Give written explanation.

2. Draw a vertical line CD, and draw a line parallel to it and 4.6 cm. from it. Give explanation.

3. Draw any line XY, and construct two lines each parallel to XY and 1.73 in. away from it. By measuring the distance between these two parallels at different points, see if they are parallel to each other .

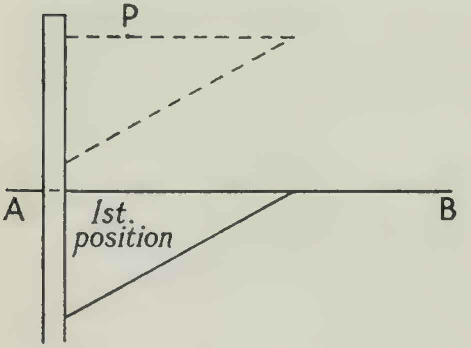
Construction: Develop the following method for drawing through a given point a line parallel to a given line. (This will be an alternative to that given on page 259). From the given point, draw a perpendicular to the given line. Erect a line perpendicular to this new line at the given point.

Make the construction and write your explanation.

Set-square Method of Drawing Parallel Lines

It is generally the object in geometry to make all constructions with only a ruler, compasses, and a pencil. In practice, other methods are used. The following is a quick method of drawing parallels. For it we need a ruler and a set-square.

Required: To draw through P a line parallel to AB.



Method: Lay an edge of the set-square along AB. Then lay the straight-edge in the position shown in the figure. Slide the set-square along the straight-edge until the point P is reached, as shown in the figure by the dotted line. A line is then drawn through P by means of the set-square.

Discussion:

Why is the line drawn through P parallel to AB? (Consider the direction of AB with respect to the straight-edge).

Problem:

To find the distance between two parallel lines. Consider the following two methods:
(1) Draw a perpendicular.
(2) Use a set-square and a straight-edge.

Note again what is meant by the distance between two parallels.

Exercises: (Use a set-square for Nos. 1 to 4).

1. Draw through any point A a line parallel to BC.

2. Draw three parallel lines, each $\frac{1}{2}$ in. from its neighbour.

3. Construct a rectangle 4 in. by 3 in. and divide it by parallels into twelve squares, each one inch square.

4. Draw a triangle ABC, with the base BC 6 cm. long. The other two sides may be of any length. Mark off, along BC, points 1 cm. apart. Through these points draw lines parallel to AC and let them meet AB. Measure accurately the six parts into which you have divided AB.

Construction: To divide a straight line into a given number of equal line-segments.

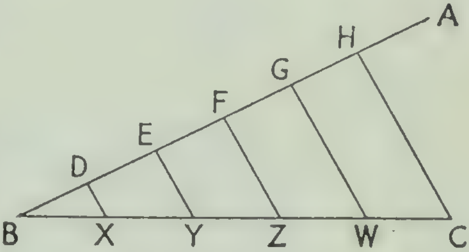
You learned how to bisect a line, that is, to divide it into two equal line-segments. You could repeat the construction to divide the line into 4 equal parts, or into 8, 16, 32, etc., equal parts.

The following method enables you to divide a line of given length into any number of equal parts:

Let BC be the given line.

Required: To divide BC into five equal line-segments.

Construction: Through B draw any other line BA, produced as far as necessary. With any radius, mark off five equal parts, BD, DE, EF, FG, GH . Join HC.



Through D, E, F, and G draw lines parallel to HC. Let those parallels meet BC in X, Y, Z, W.

Then BC is divided into five equal line-segments.

Exercise:

Practise the above construction, and be able to write a clear explanation of your work.

Measure the distances between the parallel lines.

Dividing a Line in a Given Ratio

Review:

A ratio compares quantities of the same kind, as, 35¢ and 70¢, 16 oz. and 3 oz., 4½ in. and 3.2 in. Similarly, lines may be compared as regards length, but their lengths must be expressed in the same unit, as, 3.6 cm. and 2.5 cm.

Example: This last ratio is expressed by means of the fraction $\frac{3.6}{2.5}$ or $\frac{36}{25}$

Exercises:

Express by means of a fraction the ratio of:

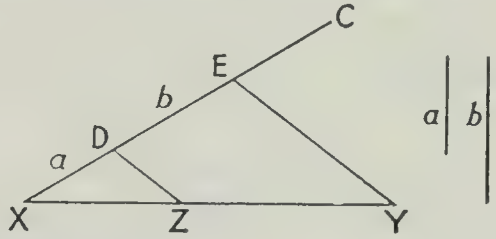
1. $6 to $12
2. $1.26 to 14¢
3. 6¼ lb. to 8 oz.
4. 3½ in. to 7 in.
5. 4.5 cm. to 2.25 cm.
6. 1 cm. to 1 mm.

7. Measure the lines AB and CD in inches, and express the ratio of their lengths as: (1) a common fraction; (2) a decimal fraction.

A________________B C______________________D

8. Repeat the exercise in No. 7 for other pairs of lines.

Construction: To divide a line in the ratio of two given lines.



Let XY be the given line.

Required: To divide XY in the ratio of *a* to *b*.

Construction: Through X draw a line XC of any length. Mark off XD=*a* and DE=*b*.

Join EY. Through D, draw DZ || EY.

Then XY is divided at Z in the ratio of *a* to *b*.

That is, the ratio $\frac{XZ}{ZY}$ = the ratio $\frac{a}{b}$

(Measure XZ and ZY in centimetres. Express the ratio of XZ to ZY as a decimal fraction. Measure the lines *a* and *b* in the same manner, and express the ratio of *a* to *b* also as a decimal fraction. How do these ratios compare? Should they be approximately equal? If they are not equal, draw your own figure and see if you can obtain a more accurate construction).

Exercises:

In each case, check your work by measurement.

1. Draw a line 4.2 in. in length and bisect it.

2. Divide a line 10.35 cm. long into 6 equal parts.

3. Draw a line 3.75 in. in length and divide it in the ratio of 4 to 5.

4. AB is 9.2 cm. long. Divide it in the ratio of 2 to 3.

Problem: To cut off $\frac{2}{3}$ of a given line.

The part cut off contains $\frac{2}{3}$, so the part remaining will be $\frac{1}{3}$. The method, then, is to divide the line into three equal parts, and the part containing two of these thirds is $\frac{2}{3}$ of the line. Make the construction.

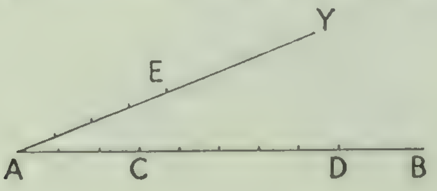
5. Draw a line 2.6 in. long and cut off $\frac{4}{5}$ of it.

6. The line CD is 3 in. long. Draw another line which is $\frac{5}{6}$ of CD.

7. Draw a line $\frac{5}{4}$ of a given line XY.

Problem: By means of geometry to find the value of x in the equation $\frac{3}{5}=\frac{4}{x}$

The fraction $\frac{3}{5}$ represents the ratio of 3 to 5.



Suggestion: Draw any line AB of any length. Cut off 8 equal distances, AC containing 3 of them, and CD the remaining 5. Draw any line AY, and on it mark off 4 equal parts. Can you complete the construction to find x?

8. Solve geometrically and check by calculation:

(*a*) $\frac{2}{3}=\frac{5}{x}$ (*b*) $\frac{4}{3}=\frac{1}{x}$ (*c*) $\frac{3}{x}=\frac{1}{4}$ (*d*) $\frac{4}{x}=\frac{7}{5}$

9. If 4 books cost $5, find by the method of geometry the cost of 6 books.

10. A boy 5 ft. in height casts a shadow 6 ft. 3 in. long. At the same time what length of shadow would be cast by a man 6 ft. tall?

11. A telephone pole 30 ft. high casts a shadow of 20 ft. What length of shadow would you cast at the same time?

12. A metal bar 3 ft. long weighs $5\frac{1}{2}$ lb. What is the weight of 5 feet of the same kind of bar?

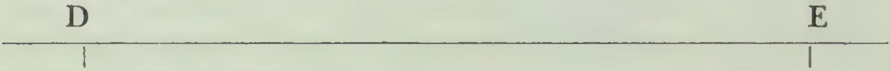
Review of Chapter IX

Topics for Class Discussion:

1. Relations between a point, a line, a plane surface.
2. Estimation of distances.
3. Measurement as being approximate.
4. The metric system as compared with the English system.
5. Consideration of an angle as being the amount of rotation.
6. The distance of a point from a line.
7. The meaning of parallel lines. Practical use of parallels.
8. Summary of the methods of drawing parallel lines.
9. Ratio and proportion as applied to straight lines.

Review Exercises:

1. Measure the line-segment DE in: (*a*) inches; (*b*) centimetres.



From your measurements calculate: (1) the number of centimetres in 1 in., and (2) the decimal fraction of an inch equivalent to 1 cm. State the equivalents you have memorized.

2. From a point O draw five lines OA, OB, OC, OD, OE in any directions. Measure to the nearest half-degree the five angles so formed. Write down their sum.

3. Bisect a line FG. Give a written explanation.

4. From a point C draw a line perpendicular to the straight line XY . Give the usual written explanation.

5. The line WZ is 2.83 in. in length, and the point T is 1.04 in. from W and lies on WZ. At T erect a line perpendicular to WZ. Give a written explanation.

6. Using only compasses and ruler, draw a rectangle 8.65 cm. long and 4.3 cm. wide. Measure its diagonal. Give all construction lines, but give no written explanation.

7. Through a given point draw a line parallel to a given line. Use compasses and ruler, and give written explanation.

8. Show how to trisect a given line TM. Explain your construction.

9. The line BC is 12.5 cm. in length. Divide it in the ratio of 3 to 2.

10. Draw any line XY. On one side of it draw AB parallel to XY and 1.34 in. distant from it. On the other side of XY draw CD parallel to it and 1.62 in. from it. Measure the distance between AB and CD.

CHAPTER X

ANGLES

Review:

1. Give two ways of defining an angle.
2. Define acute, obtuse, and right angles, using figures to illustrate.
3. How would you explain 1° with reference to angles?
4. How many degrees in a straight angle?
5. What is a reflex angle?
6. Give examples of (*a*) complementary angles, (*b*) supplementary angles.
7. Show how to bisect an angle.

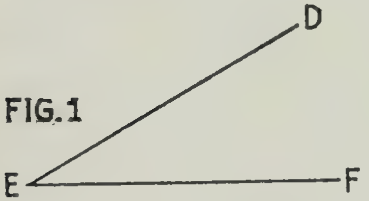


FIG. 1

You should know that the angle in Fig. 1 may be referred to as ∠DEF or ∠FED. It may also be referred to as ∠E, where there is no danger of confusion.

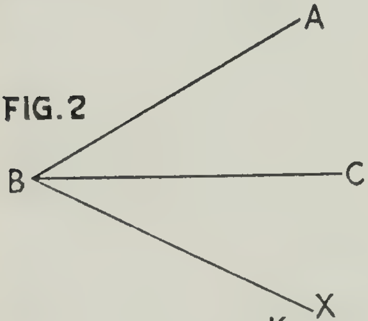


FIG. 2

There would be doubt as to what angle is meant if, in Fig. 2 we were to write ∠B.

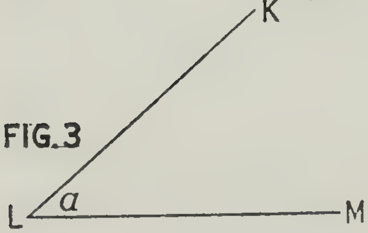


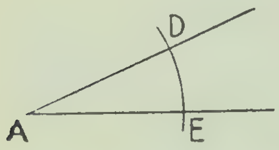
FIG. 3

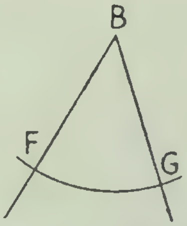
Small letters are sometimes used to indicate angles. Thus in the third figure it is permissible to write ∠*a* in referring to ∠KLM.

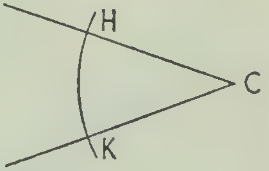
TESTS FOR EQUALITY OF ANGLES

(1) The protractor may be used to test equality of angles.

(2) With a pair of compasses only, we can compare the sizes of angles.







Suppose we are given the three angles A, B, C. It is required to find which, if any, are equal.

Construction: With the vertices of the angles as centres and the same radius in each case, draw arcs to cut the arms of the angles, as shown in the figures. The distances between D and E, between F and G, and between H and K may then be compared by using the compasses.

If the distance $DE = FG$, then $\angle A = \angle B$. If, on the other hand, HK is greater than DE, $\angle C$ is greater than $\angle A$.

NOTE: We have used the word vertices in the above explanation. This is the plural form of vertex. Learn to spell the word and to use it.

We introduce here some new symbols.

The sign $>$ means "is greater than", and the sign $<$ means "is less than".

Thus, in the above explanation we might have written

If $HK > DE$, then $\angle C > \angle A$.

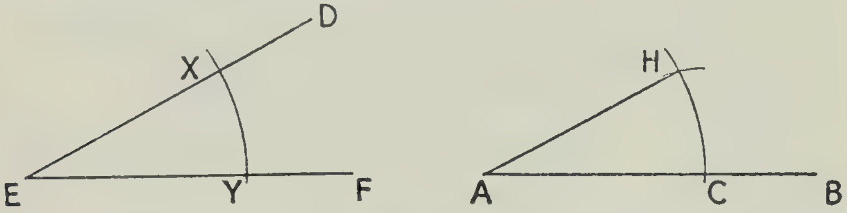
We could express the same meaning in this way:

If $DE < HK$, then $\angle A < \angle C$.

To Make an Angle Equal to a Given Angle

A simple method of making an angle equal to a given angle is to measure the given angle with a protractor, and make an angle of that size wherever it is required.

We wish, however, to make this construction without the use of the protractor. The test given above, for finding if angles are equal, suggests the method we shall use. Can you think of it before reading further?



Let DEF be the given angle.

Required to draw at A on the line AB an angle equal to ∠DEF.

Construction: With centre E and any convenient radius, describe an arc cutting the arms of ∠DEF at X and Y.

With centre A and the same radius, describe an arc cutting AB at C.

With centre C and radius XY describe an arc cutting the first arc at H.

Join AH.

Then ∠HAC=∠DEF.

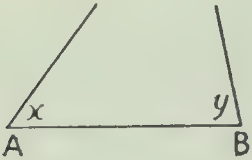
As a test of the accuracy of the construction, measure both angles with a protractor and compare.

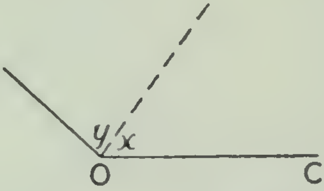
Exercises:

1. At a given point B in a given straight line CD, make an angle equal to a given acute angle KLM. Write out your explanation.

2. With your set-square or protractor draw a right angle. At the point F in a given line FG, using compasses and ruler only, make an angle equal to the right angle.

3. Draw an obtuse angle and transfer this angle, by the above method, to a point C in a line XY. Give a written explanation.





4. At the point O in the line OC, make an angle equal to the sum of the angles at A and B. The dotted line and the small letters *x, y* suggest the method.

5. Using the method of No. 4, transfer the three angles of any triangle to a point. Discuss the result.

6. Repeat the exercise with the four angles of any quadrilateral.

Complements and Supplements

Review:

1. Give the complement of each of these angles:
10°, 15°, 48°, 75°, 89°, 14½°, 1¼°, 14.2°.

2. Give the supplement of each of these:
60°, 90°, 120°, 32°, 178°, 16½°, 1¼°, 100.3°.

3. The following pairs of angles are either complementary or supplementary or there is no such relation. Classify them in this manner:

(*a*) 130°, 50°
(*b*) 28°, 72°
(*c*) 105°, 75°
(*d*) 14½°, 75¼°
(*e*) 1°, 179°
(*f*) 44.3°, 45.7°

4. If *a* and *b* are complementary angles state a formula which expresses this condition.

5. Give a similar formula when *a* and *b* are supplementary angles.

Adjacent Angles

When angles have a common vertex and a common arm and lie on opposite sides of the common arm, they are said to be adjacent. Thus *a* and *b* are adjacent angles in each of these four figures:

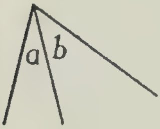


FIG. 1

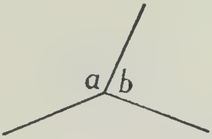


FIG. 2

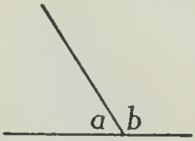


FIG. 3

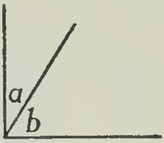


FIG. 4

State to which figure this formula applies,

$$a+b=90;$$

and to which this does,

$$a+b=180.$$

(Note that in these formulas we do not write 90° or 180°).

In Fig. 3 $\angle a$ is the supplement of $\angle b$. In this figure we have one straight line meeting or standing on another straight line. This leads to the general statement:

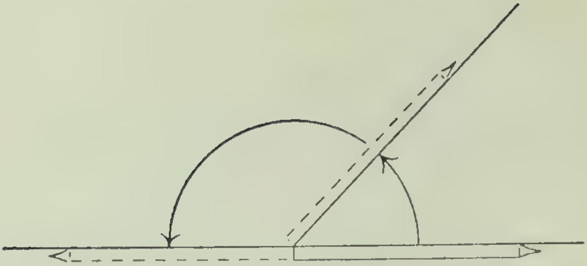
'If one straight line meets another straight line, the adjacent angles so formed together equal two right angles.'

This statement may be demonstrated in various ways:

(1) Draw a number of figures in which one line stands on another. With your protractor, measure the angles and find the sum of each pair.

(2) Considering the straight line as a straight angle, its two parts make up 180°.

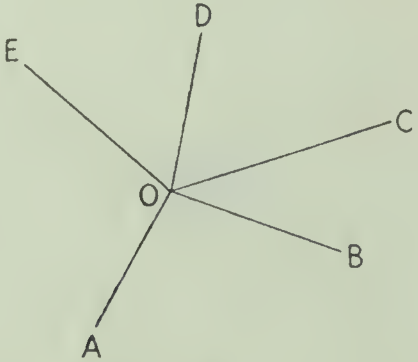
(3) By rotating a pencil through the two angles, it will be found that the pencil makes exactly one half of a complete revolution. (See figure on next page).



Angles at a Point

Draw a figure similar to the one below, on a larger scale.

(1) Measure the five angles at O, and add the results.



(2) Lay a pencil on OA. Keep one end fixed at O, while the pencil is rotated through the five angles AOB, BOC, etc. When the pencil completes the rotation through the fifth angle, EOA, it will be back in its starting position. It has then made one complete revolution, and this is an angle of 360°.

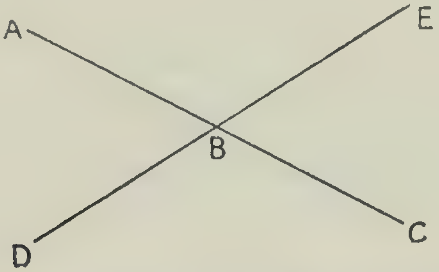
Discussion:

In measuring each of these five angles to the nearest half-degree, do you think your result should be exactly 360°? If you were able to measure to the nearest tenth of a degree, would you still expect to get 360° for your answer?

Vertically Opposite Angles

When two straight lines intersect, the pair of angles opposite to one another are said to be *vertically opposite.*

In the figure, ∠ABD and ∠EBC are vertically opposite angles. Name the other pair.



Vertically opposite angles are equal. This may be shown in two ways:

(1) Measure ∠ABD and ∠EBC. Are they approximately equal?

(2) Lay your ruler along ABC. Hold it fixed at the point B, and rotate until it falls on DBE. Part of the ruler on one side of B represents the line BA, and the remaining part represents BC. It is clear that the BA part of the ruler rotates through the same size of angle as the BC part. Then ∠ABD and ∠EBC represent the same amount of rotation, and so are equal.

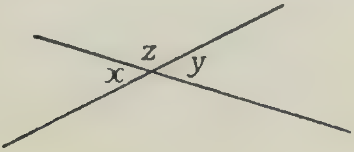
(3) Discuss this algebraic proof in class:

$x+z=180$ (See figure below)

$y+z=180.$

Then $x+z=y+z.$ (Why?)

So $x=y.$

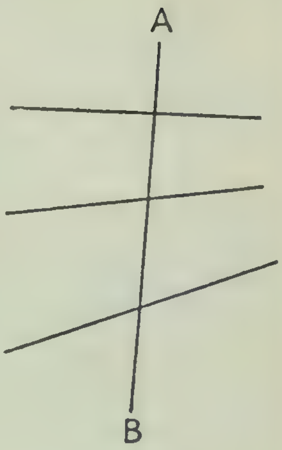


Transversals

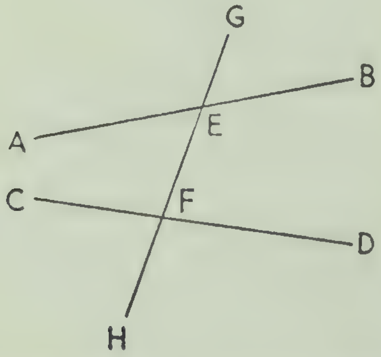
A transversal is the name given to a straight line which crosses a number of other straight lines.

In the figure to the right the line AB is a transversal. The marginal line in one of your ruled exercise books is another example. Can you suggest practical illustrations of transversals?

Can you suggest practical illustrations of transversals?

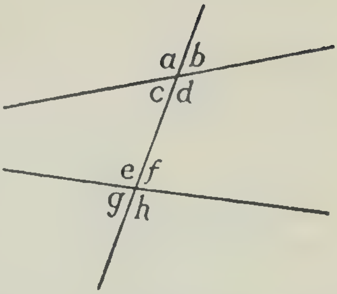


We now consider the case of a transversal GH cutting only two straight lines, AB and CD at E and F.



Without including straight or reflex angles, we have eight angles formed, four at E and four at F. We shall apply names to these angles. To make the matter clearer to you, another figure has been drawn on the next page, with the eight angles designated by the small letters, *a*, *b*, etc.

We classify the angles as follows:



(1) Interior angles: *c, d, e, f.*

(2) Exterior angles: *a, b, g, h.*

(3) Angles on the same side of the transversal: *a, c, e, g* in one group, and *b, d, f, h* in another.

(4) Corresponding angles: *a* and *e, c* and *g, b* and *f, d* and *h.* Note that one of a pair of corresponding angles is exterior, the other interior; both lie on the same side of the transversal; and one is at one intersection and the other at the second intersection.

(5) Alternate angles: One pair is *c* and *f,* while the other pair is *d* and *e.* Note that alternate angles are interior; that they lie on opposite sides of the transversal; and that one is at one intersection and the other at the second intersection.

(6) Interior angles on the same side of the transversal: One pair is *c* and *e,* the other pair is *d* and *f.*

Exercise:

1. With reference to the figure on the preceding page, classify the following:

$\angle$AEF and $\angle$EFD; $\angle$HFD and $\angle$FEB;
$\angle$BEF and $\angle$EFD.

2. In the same figure, are the:

(*a*) corresponding angles equal?
(*b*) alternate angles equal?

3. By drawing other figures test the statement that, in general, pairs of such angles are unequal.

4. Draw freehand figures in which the corresponding angles appear to be equal. In such figures, are the alternate angles also equal?

Transversal on Parallel Lines

In this section we consider the effect of a line cutting two *parallel* lines.

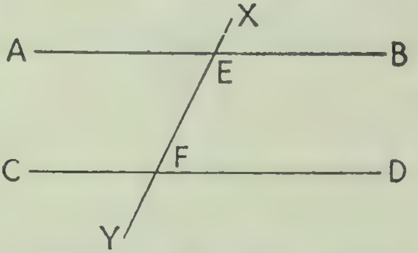
A. Corresponding Angles

Draw AB and CD, parallel lines, with the transversal XY.

1. Name one pair of corresponding angles.

2. Measure these angles. Are they equal?

Fig. 1

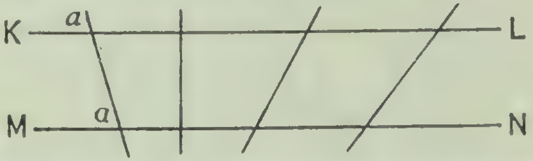


3. Name and measure three other pairs of corresponding angles. What do you conclude?

4. Draw a figure in which AB and CD are not parallel, and measure a pair of corresponding angles. Are they equal?

5. Draw a figure, like the one below with KL and MN parallel. Mark pairs of corresponding angles with letters *aa*, *bb*, etc. Is $a=a$, $b=b$, etc.?

Fig. 2



6. Discuss in class and demonstrate, by rotating your ruler or pencil, that the corresponding angles are equal when a transversal cuts two parallel lines.

Conclusion: Experimentally we have found that *when one straight line cuts two parallel lines, corresponding angles are equal.*

B. Alternate Angles

1. In Fig. 1, on the preceding page, name two pairs of alternate angles.

2. Measure and compare their size.

3. Draw a figure in which AB and CD are not parallel and compare alternate angles as regards size.

4. Draw a diagram like Fig. 2, and mark pairs of alternate angles *a a, b b,* etc. Is $a=a$, $b=b$, etc.?

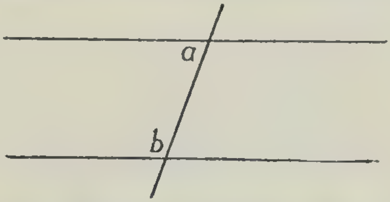
5. By rotation can you demonstrate that, in Fig. 1, alternate angles are equal?

Conclusion: Experimentally we have found that *when one straight line cuts two parallel lines, alternate angles are equal.*

C. Interior Angles on the Same Side of the Transversal

1. Draw two parallel lines, and a third line cutting them. Mark the angles *a* and *b* as in the figure. What kind of angles are these: acute, obtuse, right? Do they appear equal?

2. Measure $\angle a$ and $\angle b$. Are they equal?



3. Mark in your figure another pair of interior angles lying on one side of the cutting line. Measure them with your protractor. Are they equal? If not, what name can you give them when they are considered together?

4. Draw a figure similar to Fig. 2 on the preceding page, and measure pairs of angles like *a* and *b* above. Are they pairs of complementary or supplementary angles?

5. Try the rotation method.

Conclusion: Experimentally we have found that *when one straight line cuts two parallel lines, a pair of interior angles on the same side of the cutting line together equals* 180° *or two right angles.*

NOTE: Each conclusion, we say, has been arrived at experimentally. In more advanced geometry, you will be able to prove these statements theoretically.

The three statements which follow may be used as tests of parallel lines. You will at a later time learn theoretical proofs.

1. If corresponding angles are equal, the lines cut by the transversal are parallel.

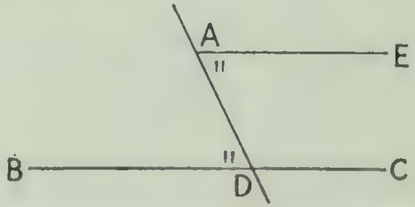
2. If alternate angles are equal the lines are likewise parallel.

3. If a pair of interior angles on the same side of the cutting line are supplementary, the lines are parallel.

These three statements suggest three ways of drawing parallel lines. A sample construction is given. Before looking at it, try to construct a line to pass through a given point A parallel to a given line BC, by using each of the three statements just given.

Drawing Parallels

Example: To draw through a given point a line parallel to a given line.



Let A be the fixed point and BC the given line.

Required: To draw through A a line parallel to BC.

Construction: Through A draw any line cutting BC at D. At A make ∠DAE=alternate ∠ADB. Then AE || BC.

(We have two lines AE and BC cut by the transversal AD, and we know that the alternate angles are equal. It follows that the lines AE and BC are parallel. EA, of course, may be produced if required.

Exercises:

1. Through A draw a line parallel to BC, using corresponding angles.

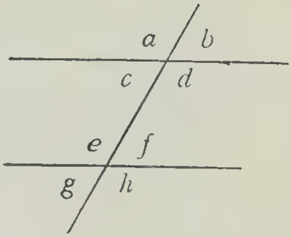
2. Try making the construction, using interior angles. Which of the three methods is the easiest?

3. Draw a line BC and take a point A outside BC. Draw a transversal AD, perpendicular to BC. Is it just as easy now to use interior angles as it is to use alternate or corresponding angles? Why? Do you recognize this as one of your earlier methods of drawing parallels? (See page 276).

4. AB and CD are each perpendicular to XY. Apply the three tests given on the last page to show that AB is parallel to CD.

Class Discussion:

Use a blackboard drawing of the figure at the right to review the various types of angles dealt with in this chapter. It may also be employed in introducing the class to simple theoretical geometry.



Review of Chapter X

Topics for Discussion:

(1) Kinds of angles dealt with in this chapter.
(2) Methods of transferring or copying angles.

(3) Angles formed by a transversal and parallel lines.
(4) Conditions for parallels.
(5) Methods of drawing parallel lines.

Review Exercises:

1. Bisect a given straight line, explaining your construction.

2. Bisect a given angle and give a written explanation.

3. Draw a perpendicular from a given point to a given straight line. Explain the steps in your construction.

4. Divide a given line in the ratio of two given lines, explaining your method.

5. Draw any three angles and explain how you would compare their sizes without the use of your protractor.

6. At a given point in a straight line make an angle equal to a given angle. State your construction.

7. Draw any two lines, not parallel, with a transversal not perpendicular to either, thus forming eight angles. Mark the angles with small letters. From your figure name:

(*a*) 8 pairs of adjacent angles;
(*b*) 4 pairs of vertically opposite angles;
(*c*) 8 pairs of supplementary angles;
(*d*) 4 obtuse angles ;
(*e*) 4 acute angles;
(*f*) 2 sets of alternate angles;
(*g*) 4 sets of corresponding angles.

8. State three ways of testing for parallel lines, derived from the work of this chapter.

9. Through a given point draw a line parallel to a given line, making use of corresponding angles. Give a written explanation.

CHAPTER XI

TRIANGLES

Review:

1. Define a triangle. (Note the symbol $\triangle$).

2. Explain the terms: vertex, vertices, sides, base, perimeter, altitude (or height), area, included angle.

3. Give a formula for finding the perimeter of a triangle.

4. What formula have you used for obtaining the area?

5. Classify triangles with reference to the sides.

6. Classify triangles with respect to the angles.

7. Draw on the blackboard: (*a*) an acute-angled isosceles triangle; (*b*) an obtuse-angled isosceles triangle; and (*c*) a right-angled isosceles triangle.

8. Draw a scalene triangle which is: (*a*) acute-angled; (*b*) obtuse-angled; (*c*) right-angled.

9. Express an opinion as to the following possibilities: (*a*) acute-angled equilateral; (*b*) right-angled equilateral; (*c*) equilateral isosceles; (*d*) right-angled acute.

Congruent Triangles

When two triangles are identical in shape and size, we say they are *congruent*. Draw, free-hand, two triangles which are approximately congruent. Name these triangles ABC and DEF in such a way that:

$AB=DE \qquad \angle A=\angle D$
$BC=EF \qquad \angle B=\angle E$
$CA=FD \qquad \angle C=\angle F$

If you were to cut out two congruent triangles, one could be fitted on the other exactly.

Exercise:

Draw a triangle ABC, with these dimensions:

AB=4 cm.	∠ABC=83°
BC=5 cm.	∠ACB=41°
CA=6 cm.	∠BAC=56°

Did you use all six dimensions in drawing the triangle? In the work which follows you will see that you can draw a triangle without knowing the sizes of all its sides and angles.

Construction of Triangles

Exercise:

For the purposes of this exercise, the class may be divided into four groups, and one problem assigned to each group.

1. Draw △XYZ having XY=3 in. and XZ=2 in. Compare the resulting triangles to see if they are congruent.

2. Construct △DEF with DE=3 in. and ∠DEF=30°. Compare results.

3. Draw △ABC with ∠B=40° and ∠C=50°. Are the triangles drawn by the group equal in all respects?

4. Draw △GHK with ∠G=30°, ∠H=60°, ∠K=90°. Compare the resulting triangles by measuring the sides.

Conclusion: You found that you could draw a triangle the same as that of your neighbour without using all six dimensions, three sides and three angles. In the exercise given above, you will find that in no case were all the triangles of one group congruent. The next three constructions show how triangles may be drawn with the minimum amount of information.

Case 1. *To draw a triangle when given the three sides.*

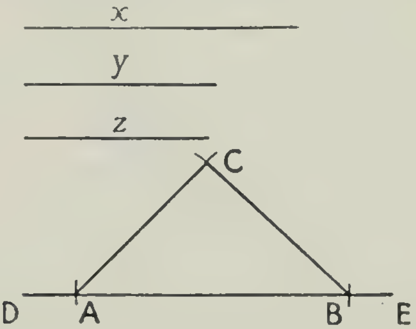
As this is partly review work, the class should be divided into groups, each group doing one of the following exercises, and comparing their resulting triangles for congruency, by measuring angles or by cutting out.

Exercises:

Draw triangles with the following dimensions: (*a*) 3 in., $2\frac{1}{4}$ in., 2 in.; (*b*) 8 cm., 5 cm., 6 cm.; (*c*) $2\frac{3}{4}$ in., 4 in., $1\frac{3}{4}$ in.

General method:

Let x, y, z be the lengths of the sides of the triangle ABC. It is required to construct △ABC.



Construction: Draw line DE any length.

Cut off a line-segment AB=x.

With centre A and radius y describe an arc.

With centre B and radius z describe an arc to cut the other at C.

Join AC and BC.

Then ABC is the required triangle.

Case 2. *To draw a triangle when given two sides and the included angle.*

Exercises:

1. Construct △ABC, having AB=3 in., BC=2 in., ∠ABC=30°. To compare all the triangles drawn by the class members, measure the third side and the two remaining angles.

2. Given DE=6.4 cm., DF=5.9 cm., ∠D=105°, draw △DEF. Compare by measuring, as before, the other parts.

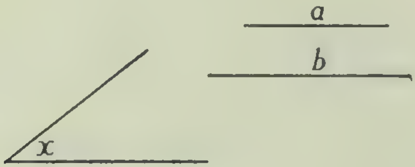
3. If XY=YZ=3.5 cm., and ∠XYZ=$85\frac{1}{2}$°, draw △XYZ. Compare results. What kind of triangle is obtained?

4. Construct a right-angled triangle with the sides containing the right angle 3.05 in. and 2.64 in. Measure the hypotenuse.

5. *General problem.* Construct a triangle, given two sides and the contained angle (included angle).

Method: Let a and b be two sides and x the contained angle.

Required: To construct the triangle.



Construction: Complete the above.

6. Two straight roads diverge at an angle of 30°. One house is situated on one road 500 yd. from the intersection, and another is on the second road 850 yd. from the same intersection. Find geometrically how far apart the houses are.

7. A car, travelling at an average rate of 30 miles per hour, takes one of the roads mentioned in No. 6, and a second car, with an average rate of 25 miles per hour, travels along the other road. They leave the intersection at the same time. Draw a figure, and use it to calculate the distance between the cars $1\frac{1}{2}$ hours after they start.

Discussion: Draw on the blackboard five triangles, each with two of its sides 3 in. and 4 in., but with the included angles (1) 10°, (2) 30°, (3) 80°, (4)140°, (5) 170°. Discuss the size of the third side, and its relation to the contained angle. Consider also the minimum and maximum size of the angle. Use a pair of compasses to illustrate.

Case 3. To draw a triangle when given one side and two angles adjacent to the side.

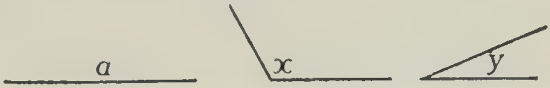
1. Draw $\triangle ABC$ with $BC = 2\frac{1}{2}$ in., $\angle B = 42°$, $\angle C = 61°$. Measure AB, AC, and $\angle A$, and compare results.

2. Construct $\triangle DEF$ having $DF=6.3$ cm., $\angle EDF=15^\circ$, $\angle EFD=125^\circ$. Are all such triangles constructed by the pupils congruent? With three such dimensions would the resulting figure always have the same size and shape?

3. The triangle XYZ has $XZ=4.15$ cm., $\angle X=\angle Z=50^\circ$. Construct it. Measure the remaining sides. Classify the triangle you have made.

4. Construct a triangle on a base of 2.6 in. with angles adjacent to the base 23° and 67°. Measure the remaining angle. State the kind of triangle which is obtained.

5. *General problem.* To construct a triangle, given one side and two adjacent angles.



Let a be the given side and x and y the given angles.

NOTE: An oral as well as a written explanation of these constructions should be given by class members. Demonstrations should be made at the blackboard before groups of the class.

6. On a base of 5 cm. draw a triangle whose base angles each equal 60°. Measure the remaining sides and angle. Classify the triangle.

7. Find, by free-hand sketches, which of the following sets of measurements result in triangles:

(*a*)	(*b*)	(*c*)	(*d*)
$AB=3$ in.	$DE=4$ cm.	$XY=6$ cm.	$KL=2$ in.
$\angle A=40^\circ$	$\angle D=105^\circ$	$\angle X=120^\circ$	$\angle K=90^\circ$
$\angle B=140^\circ$	$\angle E=1^\circ$	$\angle Y=120^\circ$	$\angle L=90^\circ$

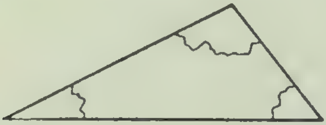
8. Two ports, A and B, are 60 miles apart measured in a direct line, A lying due west of B. One ship steams north from A a distance of 60 miles, the other ship steams northwest from B. Find, by a diagram, how far the second ship travels before it crosses the path taken by the other.

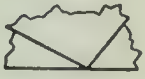
Angles of a Triangle

The sum of the three angles of a triangle may be found as follows:

(1) Experimentally by measuring the angles of various triangles and adding the results.

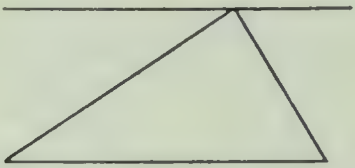
(2) By cutting out a large triangle, tearing off the corners, placing them together as shown in the figure, and observing the result.





(3) By rotating a pencil through the three angles, one at a time, and observing the first direction of the pencil and the final direction.

(4) By drawing a line through the vertex of a triangle parallel to the opposite side. (This is left for a class discussion. Make use of the alternate angles.)



The sum of the three angles of a triangle is 180° *or two right angles.*

Exercises:

(Apply the formula: $a+b+c=180$.)

1. In $\triangle ABC$, $\angle A=21\frac{1}{2}^\circ$, $\angle B=83^\circ$. Find $\angle C$.

2. In $\triangle DEF$, $\angle D=\angle E=\angle F$. Find the size of each.

3. One of the angles in a right-angled triangle is 42°. What is the size of each of the others?

4. Each of the base angles of a triangle is 32°. Find the size of the third angle.

5. State the size of each of the angles in an isosceles right-angled triangle.

6. Each of the base angles of an isosceles triangle is double the third angle. Find all the angles. (Let x=number of degrees in the third angle.)

7. One angle of an isosceles triangle is two-thirds of each of the other two. Find the size of each.

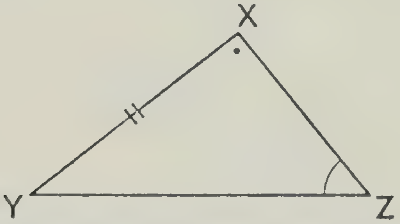
8. The angles of a triangle are represented by $3x$, $4x$, $5x$. Find the number of degrees in each angle.

In view of our statement on the preceding page regarding the angles of a triangle, we now can find the third angle of a triangle when two angles are given. The construction given on page —— can now be extended.

'To construct a triangle, given one side and any two angles.' The actual construction remains the same. Calculation of the third angle, when necessary, may be made by means of the formula:

$$a+b+c=180.$$

Example: Construct △XYZ, with XY=2 in., ∠X=30°, ∠Z=40°.



On drawing a free-hand figure and marking the given parts, we see that we do not have the two angles adjacent to XY, the given side.

So we write:

$$\begin{aligned} X+Y+Z&=180\\ 30+Y+40&=180\\ Y&=110. \end{aligned}$$

Then ∠Y=110°, and the construction may be made by using the method previously learned.

Exercises:

In these exercises you will find examples of all three triangle constructions.

1. Construct $\triangle ADF$, if $\angle A=35°$, $\angle D=72°$, $DF=4.8$ cm.

2. Draw $\triangle BCD$, given $\angle B=\angle C=70°$, $BD=2.54$ in.

3. The line EF is 3 in. The point D is 2 in. from E and $1\frac{1}{2}$ in. from F. Find, by drawing, two positions of D with reference to EF.

4. The lines AF and AG meet at right angles at A. If $AF=4.2$ cm. and $AG=5.1$ cm., find the length of FG. Apply the formula $a^2=b^2+c^2$ to check your answer.

5. The point A is 4.1 cm. from B, while B is 6.3 cm. from C, and C is 5.8 cm. from A. How far is the point A from the line BC?

Congruent Triangles

1. Draw any triangle, naming it ABC. Make another triangle, DEF, in which $DE=AB$, $EF=BC$, $DF=AC$. Measure and compare the angles opposite equal sides.

2. Draw any triangle XYZ. Make another triangle GHK, having $\angle G=\angle X$, $GH=XY$, $GK=XZ$. Measure and compare the sides HK and YZ. Also measure and compare the angles opposite equal sides.

3. Draw any triangle, naming it LMN. Construct $\triangle PQR$ so that $\angle P=\angle L$, $\angle Q=\angle M$, $PQ=LM$. As in 1 and 2, compare the remaining sides and angles.

From the above, and from work previously done, you should understand the statements which follow:

(1) Two triangles are congruent if the three sides of one respectively equal the three sides of the other.

(2) Two triangles are congruent if two sides and the contained angle of one respectively equal two sides and the contained angle of the other.

(3) Two triangles are congruent if a side and two angles of one equal a side and the two corresponding angles of the other.

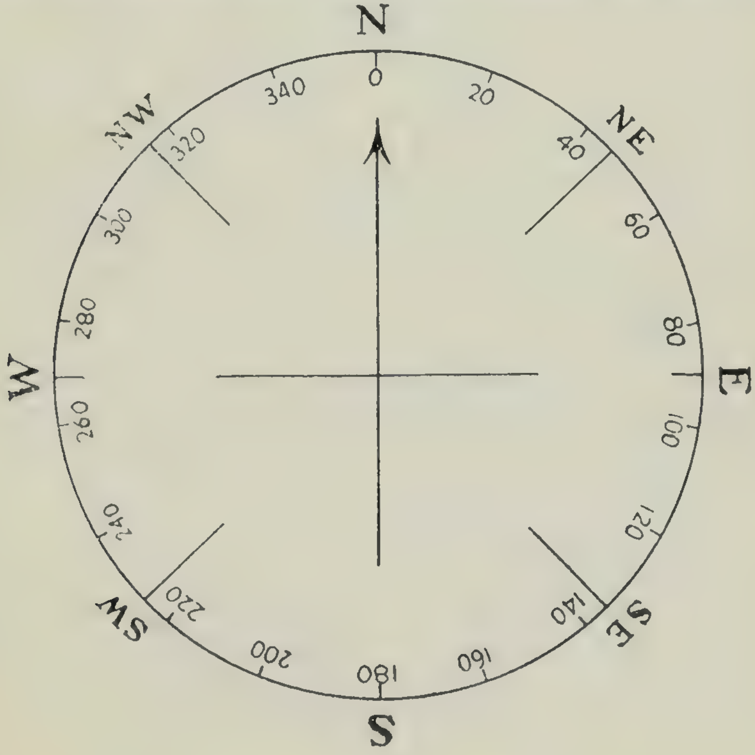
Exercises:

Without making accurate drawings, determine which of the following pairs of triangles are congruent:

1. △s ABC, DEF; AB=EF, BC=DF, AC=ED.
2. △s AXD, BYC; ∠A=∠C, AX=CB, AD=BY.
3. △s BCD, XYZ; ∠B=∠Z, ∠C=∠Y, BC=YZ.
4. △s XYA, KLM; XY=KM, ∠X=∠K, ∠A=∠L.
5. △s AFT, PQR; AF=FT=AT; PQ=QR=PR.
6. △s ABC, XYZ; ∠A=∠X, BA=XY, AC=ZX.
7. △s DEF, RST; DE=RS, EF=ST, ∠D=∠R.
8. △s PQX, ABY; PQ=QX=AB=BY, ∠Q=∠B.

Angles and Triangles in Navigation

In recent years, one section of the science of navigation as it affects the steering of sea-going vessels, has been much



simplified. A sailor no longer finds it necessary to 'box the compass', since he does not now steer by points such as east-north-east, 30° south of east, etc. The modern method of plotting courses is based on the fact that one complete revolution is 360°. The card of the compass is marked off in degrees ranging from 0° at North, through 90° at East, 180° at South, 270° at West, to 360° at North. The direction is then in a clock-wise direction (following the hands of a clock) . Thus if a navigator wishes to sail east, his course is set at 90°, and if he wishes to travel in a south-westerly direction his course is 225°. So also, if the navigator calls for a course of 270°, this means sailing due west.

Note: The navigator has to take into account the fact that the compass needle points to the magnetic north, not the geographical north, as shown on his map or chart. As the necessary correction varies with longitude, it has been left out of consideration in the following problems.

Exercises:

1. A launch sails from Vancouver for a distance of 20 miles on a course of 240°, then changes to a course of 200°, which it keeps for 15 miles. Find by drawing a diagram how far the ship is from Vancouver, and in what direction it lies from Vancouver.

2. A sailing race is held over a triangular course. The first leg of the course is 8 miles in a direction 65°, the second leg is 15 miles in a direction 190°. Find from a figure the direction and the length of the third leg.

3. One vessel leaves port in a direction 10° and steams at an average speed of 15 knots. At the same time a second ship leaves the same port on a course of 265° at a speed of 12 knots. Find, by means of a diagram, the distance in nautical miles between the ships at the end of 5 hours sailing. Give your answer also in miles.

Angle of 60°

On a base of 2 in. construct an equilateral triangle. Each angle is 60°. This gives a method of drawing an angle of 60° without the use of a protractor.

To Construct an Angle of 60°

Construction: Draw any line-segment AB.

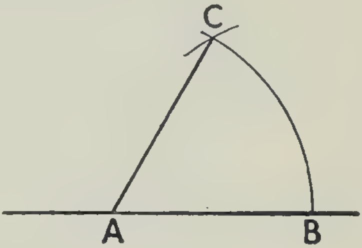
With centre A and radius AB, describe an arc.

With centre B and the same radius, cut this arc at C.

Join AC.

Then ∠CAB=60°.

Why?



Angles of 30° and 90°

To construct an angle of 30°

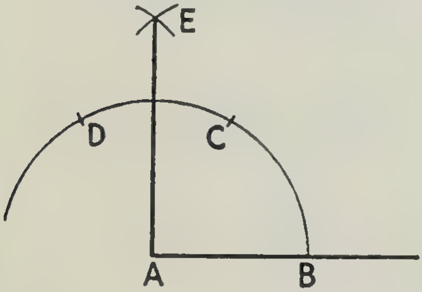
Make an angle of 60° and bisect it.

To construct an angle of 90°

Since 90°=60°+30°, the following construction should be clear:

Draw any line AB.

With centre A and radius AB describe an arc of a circle.



With centre B and the *same* radius cut the arc at C.

With centre C and the same radius, cut it again at D.

(If we joined AC and AD, then ∠DAC would equal 60°. Why?)

With centres C and D and the same radius describe arcs cutting at E. (This is the construction for bisecting an angle, in this case ∠DAC).

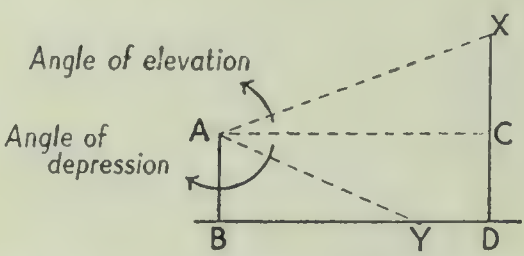
Join AE. Then ∠EAB=60°+30°=90°.

Exercise:

Without using a protractor make angles of 90°, 45°, 120°, 135°, 30°, 15°.

Angles of Elevation and Depression

If you have forgotten the meaning of angles of elevation and depression, the explanation which follows will assist you to recall that meaning.



In the figure you are represented as the observer with your eye as the point A. C is a point on the wall or blackboard, such that the distance CD to the floor is the same as AB. The line AC is then horizontal or level, being parallel to the floor BD. Now, if you look at an object higher up the wall, at X, ∠CAX is *the angle of elevation*. If, on the other hand, you look at an object Y on the floor, ∠CAY is *the angle of depression*.

These angles are always the angles taken at the point of observation, in this case, A, the eye.

In $\triangle ACX$, $\angle ACX=90°$. If we know the sizes of AC and ∠XAC, the triangle may be drawn ,and XC measured.

Exercises:

1. At a point 50 yd. from the foot of a tower, in a horizontal line, the angle of elevation of the top of the tower is 43°. From a drawing, made to scale, find the height of the tower.

2. From the top of a vertical cliff 150 ft. high the angle of depression of a boat on the water is 29°. How far is the boat from the foot of the cliff?

3. From an observation balloon floating 3,000 ft. above a town the angle of depression of an armoured car is seen to be 15°. How far is the car from the town?

*4. A flagpole stands on the top of a building. The observer, standing 50 yd. away from the building, observes the angle of elevation of the foot of the flagpole to be 62°, and that of the top of the flagpole to be 65°. Find, by means of a drawing, the height of the pole.

*5. From the top of a building, the angle of elevation of the top of a church across the street is 40°, and the angle of depression of the foot of the church is 35°. If the street is 100 ft. wide, find the height of the church.

The Exterior Angle of a Triangle

When we speak of the angles of a triangle we usually mean the interior angles.

Thus the interior angles of $\triangle ABC$ are

$$\angle ABC,\ \angle BAC,\ \text{and}\ \angle ACB.$$

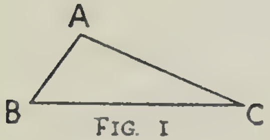


FIG. I

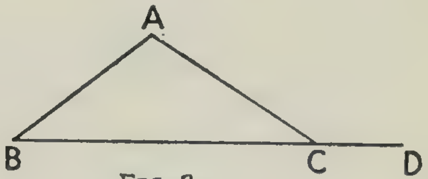


FIG. 2

When we produce a side of a triangle another angle is formed, known as the *exterior angle* of the triangle. It is the angle between one side and the adjacent side produced.

Thus ∠ACD is one of the exterior angles of △ABC. For each triangle there are actually six exterior angles, as shown in Fig. 3. They have been marked *x, x, y, y, z, z.* Can you give a reason why we have marked them in pairs, such as *x, x*? Does this mean that the two angles marked *x* are equal? If you think so, what reason can you give?

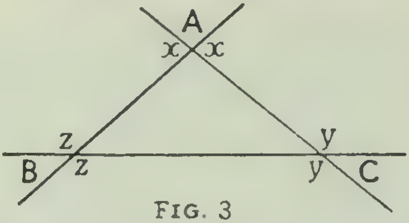


FIG. 3

When we speak of the exterior angles of a triangle we usually mean the angles formed when the three sides are produced in order as shown in Fig. 4. Thus the exterior angles are ∠ACD, ∠BAE, and ∠CBF.

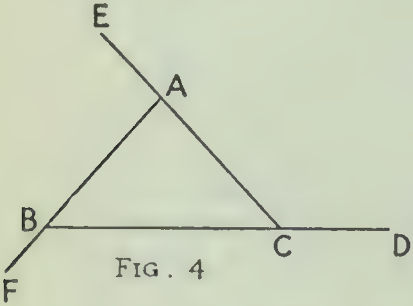


FIG. 4

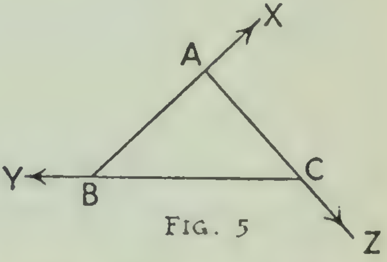


FIG. 5

The sides may also be produced in order as shown in Fig. 5, in which BA, AC, and CB are produced.

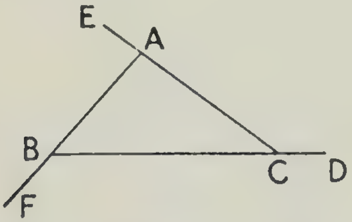
The Sum of the Exterior Angles of a Triangle

1. Experimentally, we may find the sum of the three exterior angles of a triangle by measurement with a protractor. Let each member of the class draw in his work-book any triangle. We should then have a number of different cases. Find by measurement the sum of the exterior angles and compare results. Take an average of ten such results.

2. By rotating your ruler through the three angles in turn and observing the initial and final positions, you should be able to arrive at some conclusion regarding the sum of the three exterior angles. Referring to Fig. 4, the rotation may be made by laying the ruler along CD with one end fixed at C. Develop the other steps yourself.

3. A third method of determining the sum of the exterior angles of a triangle is suggested:

Method: At O in the line OX make an angle $XOY = \angle ACD$. Then adjacent to this angle make $\angle YOZ = \angle EAB$, and similarly with the third angle CBF.



4. The following additional method should form a class discussion.

Method:

$$\angle ACB + \angle ACD = 180°$$
$$\angle CAB + \angle BAE = 180°$$
$$\angle ABC + \angle CBF = 180°$$

Since you know the sum of the interior angles, you should be able to determine the sum of the exterior angles. Complete the proof.

Conclusion: *The sum of the exterior angles of a triangle is* 360°.

Summary: The four methods given above represent different types of proof in geometry.

(1) The protractor was used in the first plan, and approximate results only were possible.

(2) The second method was carried out in a practical way by taking a movable object, such as a pencil or a ruler, to represent a line.

(3) The third method employed compasses instead of a protractor. In this method you employed the construction, previously learned, for copying or transferring angles.

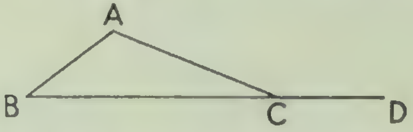
In the three methods summarized above, it was only possible to state the results obtained by the triangles you or your class-mates were able to draw. In other words, your conclusions are true only for as many triangles as you actually drew.

(4) The fourth method, however, is a theoretical one, and is applicable to all possible triangles. This is the type of proof you will use in more advanced work.

The Exterior Angle of a Triangle

There is a definite relation between any one exterior angle of a triangle and certain interior angles. We employ two methods, an experimental one using measurement, and a theoretical one using reasoning.

1. Draw any △ABC and produce BC to D, forming exterior ∠ACD. Measure ∠ACD and also each of the interior angles of △ABC. Study the results. You should see that ∠ACD is the supplement of ∠ACB. What relation has it to either ∠A or ∠B or to the sum of these two angles?



2. Reason as follows:
   (*a*) We know that the three angles of △ABC together equal 180°.
   (*b*) Then any one angle must be the supplement of the sum of the remaining two. Thus ∠ACB is the supplement of ∠A+∠B.
   (*c*) We know also that ∠ACB is the supplement of ∠ACD.
   (*d*) Make your conclusion, and write the whole proof in proper form.

Summary: The exterior angle of any triangle equals the sum of the two interior opposite angles.

Exercises:

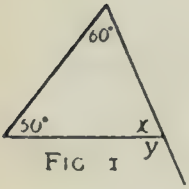


FIG 1

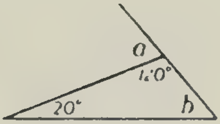


FIG 2

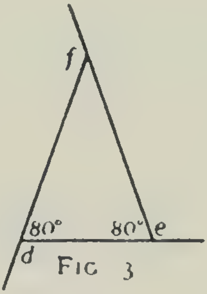


FIG 3

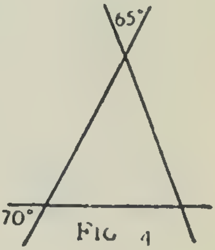


FIG 4

1. In Fig. 1 find the sizes of ∠*x* and ∠*y*.

2. In Fig. 2 find ∠*a* and ∠*b*.

3. In Fig. 3 find ∠*d*, ∠*e*, ∠*f*.

4. In Fig. 4 there are twelve angles, two of which are given. Find the sizes of the remaining ten.

5. Find the size of each of the exterior angles of an equilateral triangle.

6. Two of the exterior angles of a triangle are 60° and 150°. Is the triangle acute-angled or obtuse-angled or right-angled? Is it isosceles, equilateral, or scalene?

The Equilateral Triangle

(1) By definition, an equilateral triangle is one having all its sides equal.

(2) By measurement you find that the three angles of an equilateral triangle are equal to one another, and that each equals 60°. Compare this with the statement that the three angles of *any* triangle together equal 180°.

The Isosceles Triangle

(1) By definition, an isosceles triangle is one having two of its sides equal. Does it follow from this definition that an equilateral triangle is also isosceles?

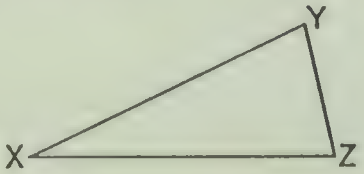
(2) By measurement you find that in an isosceles triangle two angles are equal. It should be noted that the equal angles are opposite to the equal sides. Verify this statement by drawing and measurement.

(3) Check, then learn, the next two statements:

(*a*) *If two sides of a triangle are equal, the angles opposite to them are also equal.*

(*b*) *If two angles of a triangle are equal, the sides opposite to them are also equal.*

(4) By the base of an isosceles triangle we generally mean the side lying between the equal angles. By the sides we generally mean the two sides which are equal. By the vertical angle we usually mean the one not equal to either of the other two. Thus, with reference to the isosceles $\triangle XYZ$, $\angle X$ is the vertical angle, YZ is the base, X is the vertex, $\angle$s Y and Z are the base angles.



Exercises:

1. $\triangle ABC$ is isosceles with $BA=BC=2.3$ in., $\angle A=72°$. Construct the triangle. (Is $\angle A$ the contained angle?)

2. Draw a perpendicular from the vertex of an isosceles triangle to the base. Measure the segments of the base.

3. Bisect the vertical angle of an isosceles triangle and let the bisector meet the base. Measure the segments of the base.

Inequalities

Exercise:

Draw, on the same page, triangles, each with a base of 2 in., with the lengths of the other two sides as follows:

(*a*) $2\frac{1}{2}$ in., $2\frac{1}{2}$ in.; (*b*) 3 in., $2\frac{1}{2}$ in.; (*c*) 1 in., 3 in.; (*d*) $3\frac{1}{2}$ in., 2 in.

Measure the base angles. In each triangle, note which is the greater base angle and note also its position with reference to the greater of the two sides.

Study the figures given below.

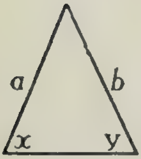


Fig. 1

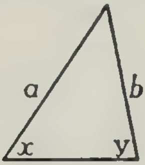


Fig. 2

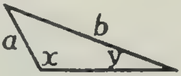


Fig. 3

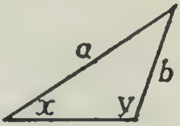


Fig. 4

Using the triangles shown above, check the following statements:

Fig. 1. If $a=b$ then $\angle y=\angle x$.
Fig. 2. If $a>b$ then $\angle y>\angle x$.
Fig. 3. If $a<b$ then $\angle y<\angle x$.

Summary: (1) If two sides of a triangle are unequal, the angles opposite these sides are unequal, the greater side having the greater angle opposite to it.

(2) If two angles of a triangle are unequal, the sides opposite these angles are unequal, the greater angle having the greater side opposite to it.

Exercises:

1. Using the same four figures, make four statements such as: If $\angle x > \angle y$ then $b > a$.

2. $\triangle$ABC has AB=5cm., BC=$2\frac{1}{2}$ cm., AC=3 cm. Arrange the angles in order of size.

3. Without drawing an accurate figure, name the greatest and the least angle of $\triangle$DEF, which has DE=4 in., EF=8 in., DF=6 in.

4. $\triangle$XYZ has $\angle$X=32°, $\angle$Y=80°. Name the three sides in order of magnitude.

The Right Triangle

Oral Discussion:

In a right triangle:

(1) Can you have an obtuse angle?

(2) How many acute angles are there?

(3) What is the greatest angle?

(4) What is the greatest side?

(5) What name is given to the side opposite the right angle?

(6) Which is the longest side?

The Theorem of Pythagoras. A theorem is a statement which is to be proved. The following theorem is called the Theorem of Pythagoras, or the Pythagorean Theorem, because a Greek mathematician named Pythagoras who lived in the sixth century B.C. is said to have first proved it:

The square on the hypotenuse of a right triangle is equal to the sum of the squares on the other two sides.

This statement is expressed by means of the formula

$$a^2=b^2+c^2$$

in which a represents the hypotenuse and b and c the other two sides.

Exercises:

1. Demonstrate the truth of the above theorem by drawing triangles with the following dimensions. In each case measure the hypotenuse.

(*a*) $\angle B=90^\circ$	(*b*) $\angle D=90^\circ$	(*c*) $\angle X=90^\circ$
$AB=2$ in.	$DE=4$ cm.	$XY=6.4$ cm.
$CB=3$ in.	$DF=6$ cm.	$XZ=7.2$ cm.

2. Use the formula to calculate the length of the hypotenuse of each of the triangles in Exercise 1. Compare your calculations with the results found by measurement.

Thus in (*a*) $AC^2=AB^2+BC^2$.

(Note that AB^2 represents the square on the line AB).

Then $AC^2=2^2+3^2=13$,
from which $AC^2=13$,
giving $AC=\sqrt{13}=3.6$.

3. Construct a square on a line $1\frac{3}{4}$ in. in length. Draw in a diagonal and measure its length. Check your measurement by means of the formula $a^2=b^2+c^2$.

Construction of Right Triangles

To construct a right triangle, given the hypotenuse and one side.

Let x be the length of the hypotenuse and y one side.

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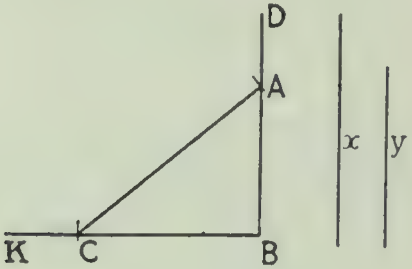
Required: To construct the triangle.

Construction: Draw a right angle, DBK.

From BK, cut off BC$=y$.

With C as centre and x as radius, describe an arc to cut BD at A. Join AC.

Then ACB is the required triangle.



Exercises:

1. Construct △DEF, given ∠DEF$=90°$, EF$=3.5$ cm., DF$=8.9$ cm. Measure the third side and check by the formula.

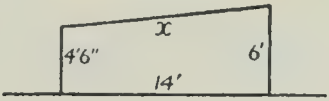
2. Draw △KLM, having KL$=3.45$ in., LM$=1.84$ in., ∠M$=90°$.

3. Construct a rectangle whose sides are 2.72 in. and 1.76 in. Draw a diagonal, measure it, and check by the formula.

4. Two ships sail at the same time from a port, one in a direction 220°, the other 130°. If the first ship travels at 10 miles per hour and the other at 14 miles per hour, find from a diagram how far apart they are in 8 hours time. Verify your answers by means of the formula.

5. The gable of a house is in the form of an isosceles right-angled triangle. If the width of the house is 24 ft., find by measurement the length of the equal sides of the triangle. (In a right isosceles triangle, what sizes are the angles?)

6. The figure below illustrates the end of a shed. Find the width x of the roof: (*a*) by drawing to scale and measuring; (*b*) by calculation.



7. The foot of a ladder 23 ft. long is placed 9 ft. from a vertical wall. How high up the wall will the ladder reach?

8. Find the height of an equilateral triangle of 2 in. side. (The perpendicular from the vertex bisects the base).

*9. The height of an equilateral triangle is 10 cm. Find the length of each side.

Additional Problems with Triangles

When lines pass through the same point they are said to be *concurrent*. The term generally refers to three or more lines.

Exercises:

1. The bisectors of the angles of a triangle are concurrent. Demonstrate this by using a large triangle and making all construction lines as accurately as you can.

2. Find, by construction, if the perpendicular bisectors of the sides of a triangle are concurrent.

3. Draw perpendiculars from the vertices of a large triangle to the opposite sides and see if they are concurrent.

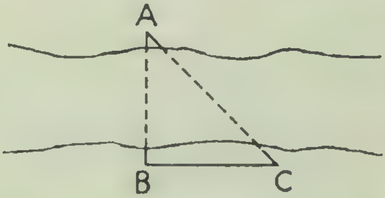
4. Draw a right triangle and find, by construction, where the perpendicular bisectors of the sides meet.

5. Take a large obtuse-angled triangle and draw perpendiculars from the vertices to the opposite sides. You will find it necessary to produce the sides containing the obtuse angle. Are these perpendiculars concurrent? Do they meet inside or outside the triangle?

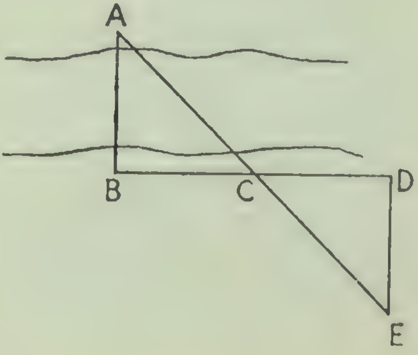
6. Bisect the three sides of an obtuse-angled triangle perpendicularly. Do the bisectors meet inside or outside the triangle?

7. Without making an accurate drawing, state where the perpendiculars drawn from the vertices of a right triangle to the opposite sides meet.

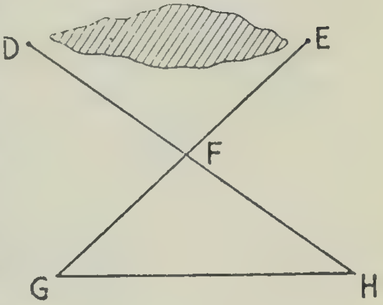
8. A and B are points directly opposite one another on the banks of a river. The distance BC, measured at right angles to AB, is found to be 230 yd. The angle at C measures 45°. What is the width of the river? Can you make this calculation without drawing an accurate figure?



9. The distance between two objects, A and B, when a direct measurement cannot be made, may be determined indirectly as follows: BC is measured off at right angles to AB. CD is a continuation of BC and equals BC. DE is measured at right angles to CD, and E is located in line with A and C. Which line is now the same length as AB? Give reasons for your answer.

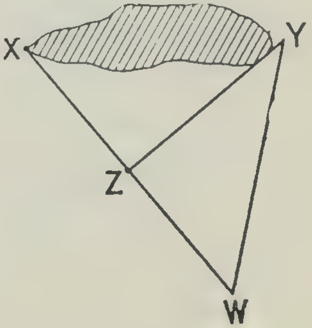


10. The distance across a lake from D to E was found by the following plan: Stakes were driven in the ground at F and G so that EF=FG, with EFG a straight line. Another stake was located at H in such a manner that D, F, and H were sighted in the same line, and FH was made equal to DF. Which line was measured to give the distance DE? Give a clear reason for your answer.

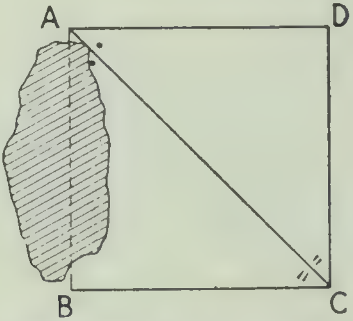


***11.** John wished to find the distance across a circular lake. He proceeded as follows: He located two objects, A and B, lying due north and south from each other on opposite sides of the lake. With the aid of a pocket compass, John walked due east from A a distance of 100 yds., and then walked due south until the object B lay straight west of him. If the distance he walked south was 175 yds., how wide is the lake?

***12.** Explain how the distance from X to Y, shown in the adjoining figure, may be obtained. Give reasons. The point Z is located so that ∠XZY is a right angle. W is then located such that XZW is a straight line, and ZW=XZ.



*13. The distance from A to B, shown in the accompanying figure was obtained as follows: Any point C was taken. The direction of the line CD was determined by making ∠ACD=∠ACB. The direction of AD was found by making ∠CAD=∠CAB. Which line in the figure gives the distance AB? Why?



14. Discuss in class how congruency of triangles assisted in the solutions of Problems 9, 10, 12, and 13.

Review of Chapter XI

Topics for Class Discussion:

(1) The three cases of congruency of triangles.
(2) Methods of constructing triangles.
(3) The sum of the angles of a triangle.
(4) The exterior angles of a triangle.
(5) Triangles with unequal sides and unequal angles.
(6) The Theorem of Pythagoras.

Review Exercises

1. Show how to draw a triangle :
   (*a*) Given three sides;
   (*b*) Given two sides and the contained angle;
   (*c*) Given one side and any two angles.

Give a written explanation for each, in standard form.

2. Construct $\triangle DFK$, given $DF=6.2$ cm., $FK=8.1$ cm., $KD=7.6$ cm.

(*a*) Measure each angle, and give their sum.

(*b*) Bisect each angle.

Show all construction lines, but give no written explanation.

3. Draw $\triangle XYB$, given $\angle XYB=115°$, $XY=2.78$ in., $BY=3.6$ in.

(*a*) Measure the third side.

(*b*) Bisect each side at right angles.

Give no written explanation, but show all construction lines.

4. Given $\angle D=75°$, $\angle E=42°$, $EF=6.8$ cm., draw $\triangle DEF$.

(*a*) Measure DE and DF.

(*b*) Draw perpendiculars from D, E, and F to the opposite sides.

Give construction lines only.

5. Show how to draw an angle of 135°, without using a protractor. Give an explanation of your construction.

6. The side XY of $\triangle AXY$ is produced to B forming the exterior angle AYB. The size of this angle is 142° and $\angle AXY$ is 130°. Find the size of the exterior angles at A and X.

7. Complete the following statements:

(*a*) In $\triangle DXY$, if $DX>DY$ then ——>——.

(*b*) In $\triangle BCD$, if $\angle BCD<\angle BDC$ then ——<——.

(*c*) In $\triangle AFG$, if $AF>FG>GA$, then ——>——>——.

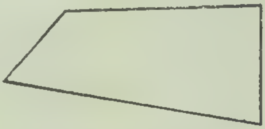
8. (*a*) Show by construction that the bisectors of the three angles of a triangle pass through the same point.

(*b*) Make free-hand drawings showing two other sets of lines in a triangle which are concurrent. Express by means of statements.

CHAPTER XII

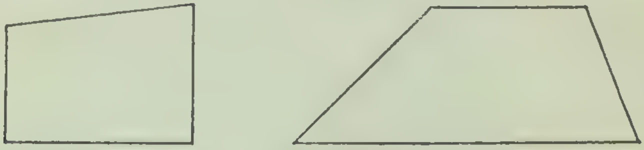
QUADRILATERALS

Quadrilateral is the general name given to any plane figure with four straight lines for sides. In general, the sides are not equal, nor are any sides parallel. Special kinds of quadrilaterals, in which certain sides are equal or parallel, are given special names:



QUADRILATERAL

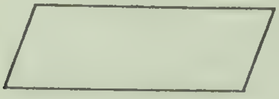
Trapezoid. When a quadrilateral has a pair of sides parallel it is known as a trapezoid (sometimes called a trapezium). An end view of the ordinary shed type of building or 'lean-to' is an example of a trapezoid.



TRAPEZOIDS

Parallelogram. When both pairs of opposite sides of a quadrilateral are parallel, it is called a *parallelogram.* The abbreviation we shall use for parallelogram is $||^m$. The figures given below are samples.

Rectangle. If a parallelogram has its angles right angles it is called a rectangle. Thus a rectangle is a quadrilateral and also a parallelogram.



PARALLELOGRAM

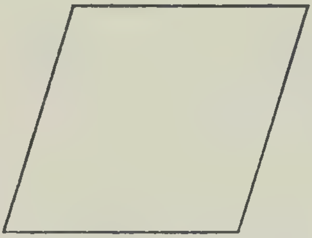
RECTANGLE

Square. If the sides are equal and the angles right the quadrilateral is a square. It is a particular form of rectangle.

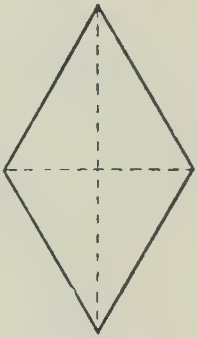
Rhombus. If all the sides of a parallelogram are equal, with no condition as regards the angles, we give it the special name of rhombus.

Diagonal. The line joining a pair of opposite vertices or corners of a quadrilateral is called a diagonal. As we shall see later, the diagonals of rectangles, squares, etc., have special properties.

SQUARE



RHOMBUS



RHOMBUS

Angles of a Quadrilateral

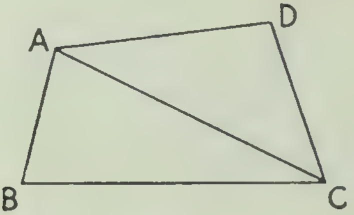
The sum of the angles of a quadrilateral. Four ways are given by which we may find the sum of the angles of any quadrilateral.

(1) Have each member of the class draw any quadrilateral, measure each angle, and find the sum. An average of the results will give an approximate value.

(2) As you did in the case of a triangle, rotate your pencil through the four angles. Note the initial and final positions of the pencil. Did it make one-half a revolution or a whole one?

(3) Transfer the four angles to one point, as you did also in the case of the exterior angles of a triangle.

(4) Divide the quadrilateral into two triangles. The six angles of the two triangles are together equal to the four angles of the quadrilateral. But the three angles of each triangle together equal 180°. Then the six angles taken together make up 360°. Hence, the sum of the angles of any quadrilateral equals 360°, or four right angles, or one complete revolution. Does this agree with the results found by the three other methods?



Given three angles of a quadrilateral, to find the fourth.

Representing the result found above by a formula, we have:

$$a+b+c+d=360.$$

From this formula, if any three of the letters are known, the fourth may readily be found. Thus, if three of the angles of a quadrilateral are 50°, 100°, 175°, we have:

$$\begin{aligned} 50+100+175+d&=360\\ 325+d&=360\\ d&=35. \end{aligned}$$

Thus the fourth angle is 35° .

Exercises:

1. In quadrilateral ABCD, $\angle A=82°$, $\angle B=123°$, $\angle C=101°$. Find $\angle D$.

2. In figure XYZW, $\angle X=\angle Y=90°$, $\angle W=15°$. Find $\angle Z$.

3. In the quadrilateral EFGH, $\angle E=\angle F=\angle G=108°$. Find $\angle H$.

4. If, in figure ABCD, $\angle A=\angle B=\angle C=\angle D$, what kind of quadrilateral is it?

Construction of Quadrilaterals

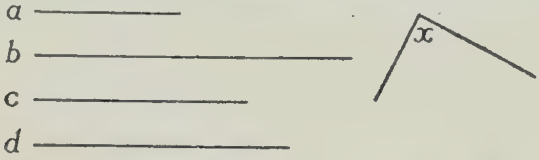
The construction of quadrilaterals depends on the work you have done in constructing triangles.

In the case of a triangle, we are able to complete the figure when we know the lengths of the three sides. If four bars were hinged together to form a quadrilateral, would the figure be fixed or rigid? Would the angles remain the same? Try it by cutting out four strips of cardboard and joining them together. A cardboard box with top and bottom removed will serve also as an illustration. Is the box collapsible? In other words, do the angles vary?

(*a*) *Given four sides and one angle, to construct a quadrilateral.*

Let a, b, c, d represent the sides EF, FG, GH, HE, taken in that order, and let $\angle x$ represent $\angle$EFG.

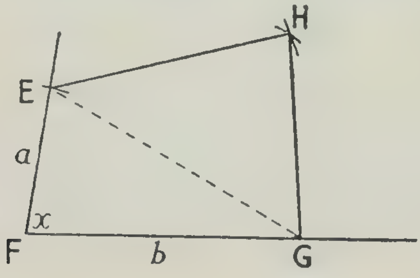
Required: To construct quadrilateral EFGH.



Construction: Draw an angle $=\angle x$, naming the vertex F. From the arms of this angle, cut off parts FE$=a$, FG$=b$.

(Since two sides and the contained angle determine a triangle, we have thus determined $\triangle$EFG, and so we know EG. Knowing EG, GH, and HE, we have the three sides of $\triangle$EGH, and so can determine its position).

With centre E and radius HE or d, describe an arc.



With centre G and radius GH or c, describe an arc.

Join HE and HG.

Then EFGH is the required quadrilateral.

Note: The same four lines, a, b, c, d, if taken in a different order would give a different quadrilateral. So also, if $\angle x$ represented $\angle FEH$ we should have a different figure.

Exercises:

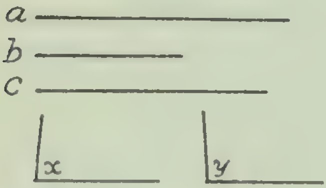
Construct quadrilaterals to the following measurements:

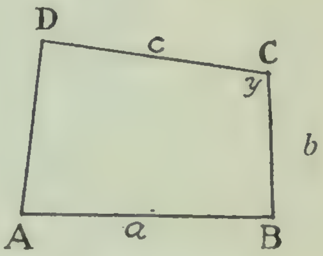
1. $AB = 5.8$ cm., $BC = 4$ cm., $CD = 3.4$ cm., $DA = 3.1$ cm., $\angle ABC = 71^\circ$.

2. $EF = FG = 3$ in., $GH = 1.5$ in., $HE = 3.4$ in., $\angle GHE = 90^\circ$.

3. $XY = YZ = 6$ cm., $ZW = WX = 3.4$ cm., $\angle YZW = 105^\circ$.

(*b*) *Given three sides, and two contained angles.*





Let $AB = a$, $BC = b$, $CD = c$, $\angle ABC = x$, $\angle BCD = y$.

Required: To construct quadrilateral ABCD.

Construction: (A free-hand figure will assist in the construction.)

Draw $AB = a$. At B make an angle $= x$. Cut off $BC = b$.

At C make an angle $= y$.

Cut off $CD = c$. Join DA.

Then ABCD is the required quadrilateral.

Exercises:

Construct quadrilaterals, given the following:

1. ABCD: AB=4.2 cm., BC=2 cm., CD=6.4 cm., ∠ABC=115°, ∠BCD=109°.

2. EFGH: EF=FG=2.6 in., GH=2.1 in., ∠F=73°, ∠G=112°.

3. ABCD: AB=4.35 in., BC=0.82 in., AD=2.36 in., ∠A=92°, ∠B=76°.

(*c*) *Given two adjacent sides and any three angles.*

Analysis: Since three angles are given, the fourth may be found.

We thus know the angle between two adjacent sides whose lengths are given. This angle is drawn first and the given sides marked off along its arms. Can you complete the construction?

Do the three exercises given below, then write the general case.

Exercises:

Construct the quadrilaterals as indicated:

1. DYXE: DY=5.7 cm., YX=7 cm., ∠Y=135°, ∠D=50°, ∠X=62°.

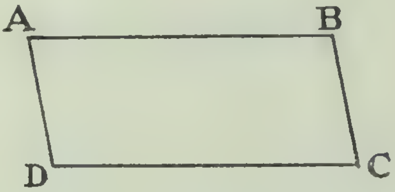
2. ABFG: GF=FB=4 in., ∠G=∠B=120°, ∠F=48°.

3. ABCD: ∠A=∠B=∠C=90°, AB=2.75 in., BC=1.2 in.

PARALLELOGRAM

We define a parallelogram as a quadrilateral with its opposite sides parallel. Its other properties may be found experimentally by examining and measuring a number of different parallelograms.

Using ruler and set-square, let each pupil draw a pair of parallel lines any distance apart. Let him draw another pair of parallel lines crossing these. Name the figure ABCD. Then AB || DC and AD || BC.



1. Measure the sides. Are any equal?

2. Measure the angles. Are any equal?

3. Draw a diagonal dividing the parallelogram into two triangles. Do these triangles appear to be congruent? Cut out one or two figures and separate into triangles. Try fitting together each pair of triangles. Are they equal in all respects?

4. Draw in both diagonals. Measure the parts of these diagonals made when they intersect. What is your conclusion?

Properties of a Parallelogram. Verify the following list of properties:

(1) The opposite sides are equal.
(2) The opposite angles are equal.
(3) The diagonal divides the figure into congruent triangles.
(4) The diagonals bisect each other.

Later, a proof will be found for each of these statements.

Exercises:

Any of the above properties may be used in making the following constructions:

1. Construct $\|^{m}$ ABCD, given AB$=$2 in., BC$=$3 in., $\angle B=75^\circ$.

2. Construct $\|^{m}$ EFGH, given EF$=$7.3 cm., FG$=$4.8 cm., $\angle F=108^\circ$.

3. ABXY is a $\|^m$ having $AB=9.2$ cm., $BX=3.4$ cm., $\angle A=25°$. Construct it and measure the diagonal BY.

4. Construct $\|^m$ XYZW with $XY=YZ=2\frac{1}{2}$ in., $\angle XYZ=125°$. What kind of parallelogram is obtained? Measure the angle formed where its diagonals intersect.

*5. The diagonals AC and BD of a parallelogram intersect at an angle of 40°. If $AC=3$ in. and $BD=4$ in., draw the parallelogram.

6. Draw $\|^m$ EFGH with $EF=8$ cm., $FG=4$ cm., $\angle G=90°$. What kind of parallelogram is obtained?

Rectangle

We define a rectangle as a parallelogram with one right angle. Why do we not say four right angles? The rectangle possesses all the properties found in a parallelogram, and others besides.

Draw a number of rectangles.

(1) Are all the angles right angles?
(2) Are the diagonals equal?
(3) Are the diagonals at right angles to each other?

Conclusions: All its angles are right angles, its diagonals are equal, but not necessarily perpendicular.

Square

We define a square as a rectangle with two adjacent sides equal. Why do we not say in the definition that all its sides are equal? The square possesses all the properties of a rectangle and, therefore, also of a parallelogram. In addition, all its sides are equal, and its diagonals are at right angles to each other. Verify from a figure.

Rhombus

We define a rhombus as a parallelogram with a pair of adjacent sides equal. Why do we not say in the definition that all its sides are equal? By drawing a number of figures and making measurements, verify the statement that the diagonals of a rhombus are perpendicular to each other.

TRAPEZOID

We define a trapezoid as a quadrilateral with one pair of sides parallel. Examine a number of trapezoids. Can you make any statement about their sides, or angles, or diagonals?

Exercises:

In each case ABCD is a quadrilateral. By means of a free-hand figure, find what particular kind of quadrilateral each represents. Call it a quadrilateral if it has no special name:

1. $AB=BC=CD=DA$, $\angle B=30°$.
2. $AD=BC$ and $AB \parallel DC$.
3. $AB=CD$, $AD=BC$, and $\angle B=75°$.
4. $AC=BD$ and $AC \perp BD$.

Make the following constructions:

5. Draw a rectangle whose dimensions are 8.2 cm. and 3.4 cm.

6. Construct a rhombus whose sides each equal 2.3 in., with an angle of 120°.

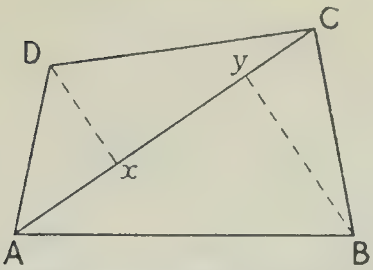
7. The diagonals of a rectangle are each 10 cm. in length and make with each other an angle of 40°. Draw the rectangle.

Areas of Quadrilaterals

Review:

State the formulas used in finding the area of: (*a*) a square; (*b*) a rectangle; (*c*) a parallelogram.

The area of a quadrilateral is obtained by drawing a diagonal, dividing the figure into two triangles. Perpendiculars are then drawn from two of the vertices to this diagonal. These perpendiculars are the heights of the triangles.



Area of quad. ABCD = area of △ABC + area of △ACD

$$= \tfrac{1}{2}BY.AC + \tfrac{1}{2}DX.AC$$

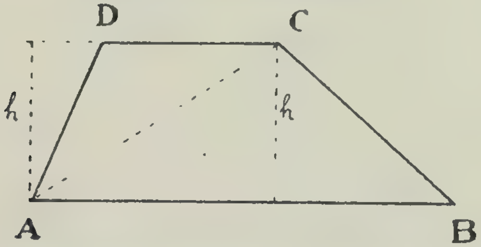
$$= \tfrac{1}{2}AC(BY + DX).$$

No formula need be remembered, each case being worked out like the above.

Exercise:

Draw a number of quadrilaterals and find their areas.

Area of a Trapezoid



To find the area of the trapezoid ABCD, draw the diagonal AC and find the area of each triangle.

Area of △ABC = $\tfrac{1}{2}AB.h$

Area of △ACD = $\tfrac{1}{2}DC.h$

Area of ABCD = $\tfrac{1}{2}h\,(AB + DC)$.

The formula for the area of a trapezoid is

$$A = \tfrac{1}{2}h\,(a + b)$$

in which a and b are the parallel sides, and h the distance between them.

Exercises:

1. Find the area of a trapezoid whose parallel sides are 10 in. and 12 in. and altitude 5 in.

2. The parallel sides of a trapezoid are 18 cm. and 29 cm. and the distance between them is 15 cm. Find its area.

3. If $h=4$ ft., $a=6$ ft., $b=18$ ft., use the formula to find A.

4. The end of a shed is in the form of a trapezoid. It is 8 ft. wide, 6½ ft. high at the back, and 8½ ft. high in front. Find the area of the end.

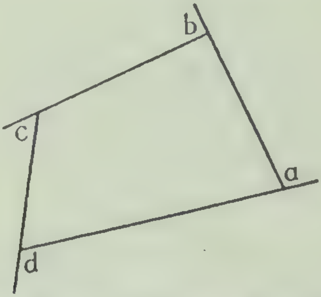
The Exterior Angles of a Quadrilateral

The exterior angles of a quadrilateral may be found:

(1) By measurement with a protractor. (The figure indicates the angles to be measured).

(2) By rotating a ruler through the four angles.

(3) By collecting the angles at one point, as was done with the interior angles.



(4) By adding the interior angles and the exterior angle at each vertex, and subtracting from the total sum the sum of the interior angles.

Conclusion: The sum of the exterior angles of a quadrilateral=360°.

Review of Chapter XII

Topics for Class Discussion:

(1) Kinds of quadrilaterals.

(2) The interior and exterior angles of a quadrilateral.

(3) Construction of quadrilaterals as dependent on triangles.

(4) Properties of parallelograms.

(5) Areas of various types of quadrilaterals.

Review Exercises

1. The information supplied (data) refers in each case to a quadrilateral. Classify them, that is, state whether you have a rectangle, square, etc. Draw free-hand figures if necessary:

(*a*) Both pairs of opposite sides equal, with two angles right.

(*b*) All the sides equal, one diagonal longer than the other.

(*c*) Diagonals bisect each other at right angles, one angle equal to 90°.

(*d*) Only one pair of sides parallel, diagonals unequal.

(*e*) One pair of sides equal and parallel.

2. Construct a rectangle with one side 3.45 in. and and another side half as long. Find its area by using the formula.

3. Construct a square whose perimeter is 21.6 cm.

4. Find the area of the quadrilateral EFGH, given EF=$3\frac{1}{4}$ in., FG=$1\frac{3}{4}$ in., GH=$1\frac{1}{2}$ in., HE=2 in., ∠GHE=120°.

5. Find the perimeter of the quadrilateral AXKL, having LA=35 yd., AX=105 yd., XK=30 yd., ∠A=52°, ∠X=90°.

6. Find, by construction, if the bisectors of the interior angles of a quadrilateral meet at a point.

7. Find, by construction, if the perpendicular bisectors of the sides of a quadrilateral are concurrent.

8. Find, by measurement and also by reasoning, the relation between∠A and ∠B in $\|^{m}$ ABCD.

9. The two diagonals of a quadrilateral divide it into four triangles. State in which types of quadrilateral these four triangles are congruent: (*a*) parallelogram; (*b*) rectangle; (*c*) square; (*d*) trapezoid; (*e*) rhombus; (*f*) any quadrilateral.

10. Draw a trapezoid ABCD, having AB=3.4 in., BC=2.1 in., ∠A=65°, and ∠B=82°.

CHAPTER XIII

THE CIRCLE

Review

1. Discuss this definition: A circle is a closed figure, lying in a plane and bounded by a line such that all points on it are equidistant from a fixed point called the centre.

2. Discuss also this meaning: When a point moves in a plane so as to be at a constant distance from a fixed point, the path traced out is a circle.

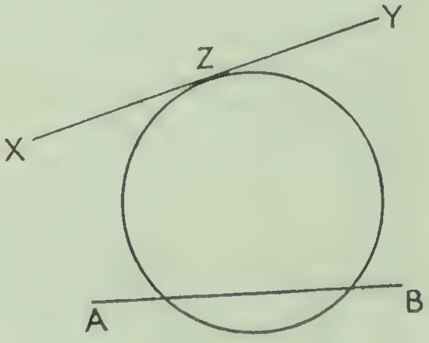
3. Review, using illustrations, the meaning of radius, diameter, circumference, arc, chord, segment, sector, semicircle, quadrant.

4. Review the formulas for diameter, circumference, area:

$$d=2r;\ c=\pi d \text{ or } 2\pi r;\ A=\pi r^2.$$

The Tangent to a Circle

We may define a tangent as a line which touches a circle. Thus XY is a tangent to a circle drawn at the right. It touches the circle at the point Z, and this point is known as the point of contact, or the point of tangency.



Numerous illustrations of tangents may be found in and out of the classroom. For example, a football lying on the ground has the ground as a tangent.

A *secant* may be defined as a line which is of unlimited length, passing through two points in the circumference of a circle. Thus, in the figure, AB is a secant.

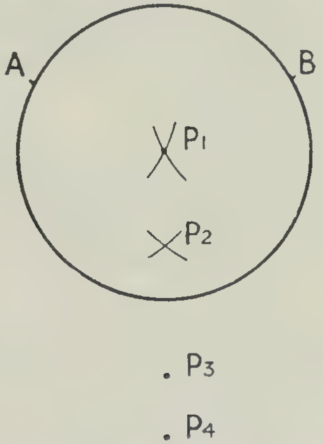
The Circle

(*a*) *One point.*

Through one point any number of circles may be drawn. They may be of any size, and their centres do not occupy any particular position.

(*b*) *Two points. To draw circles through two fixed points.*

Since each circle passes through A and B, these two points are equidistant from the centre. With any radius and with A as centre, describe an arc. With B as centre and the same radius, describe an arc, cutting the first at P_1.



Then P_1 is the centre of the circle with radius P_1A which will pass through A and B. Repeat the process with a longer radius, and a second point P_2 will be obtained which will be the centre of a second circle passing through A and B.

How many centres such as P_1, P_2, etc., may be obtained?

How many circles may be drawn through A and B.

Obtain, in your own figure, centres which lie above AB.

What position do the centres appear to occupy?

Join them in order P_1, P_2, P_3, etc. Is the result a straight line or a curve. Produce it both ways. Join AB.

Where is AB cut by this line? Is the line through P_1, P_2 . . . at right angles to AB? Is the point where it cuts AB the centre of a circle which can be made to pass through A and B?

In more advanced geometry we call such a line as P_1, P_2 . . . a *locus*. It is the location of the centres of all circles passing through A and B.

Conclusion: *The centres of all circles passing through two fixed points lie on the perpendicular bisector of the line joining the points.*

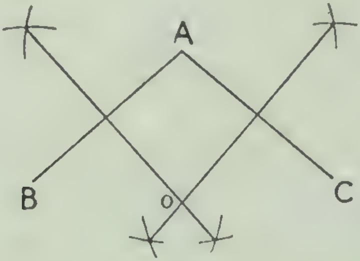
Exercise:

Take two points X and Y 6 cm. apart. Join XY and bisect it at right angles. On the bisector take any point A and with AX as radius describe a circle. Does it pass through Y?

(*c*) *Three points.*

To draw a circle passing through three fixed points.

Let A, B, C be the three fixed points.



Required: To find how many circles may be drawn through A, B, C.

Construction: Experimentally, try to place the point of your compasses in such a position that you can draw a circle through A, B, and C. How many such centres can you obtain?

The centres of all circles passing through two points A and B lie on the perpendicular bisector of the line joining A and B. Join AB and bisect it at right angles. In the same way the centres of circles passing through A and C lie on the perpendicular bisector of AC. Let these two bisectors meet at O. Then, since OB=OA and OA=OC, the point O is equidistant from A, B, C. It is the centre of a circle passing through A, B, and C.

Do you think this is the only circle possible? If B and C are moved farther apart, and if ∠BAC increases in size, where does O move? If the fixed positions of A, B, and C were such that ∠BAC=175°, would O be near or far away? If B, A, and C were in a straight line, where would O be situated?

Conclusion: If three fixed points are not in a straight line, one and only one circle may be drawn, passing through them.

From this conclusion we may say that any three points on the circumference of a circle determine or fix the circle.

To complete a circle when an arc is given. Try this construction yourself, making use of the conclusion found above.

To find the centre of a given circle. Let one pupil draw a circle on the blackboard, removing any sign of the centre. Let another pupil find the centre. The method depends, again, on the principle stated above.

Triangles and Circles

In Fig. 1, on the next page, △ABC is said to be *inscribed* in the circle, and the circle is said to be *circumscribed* about the triangle. The centre of the circumscribed circle is called the *circumcentre.*

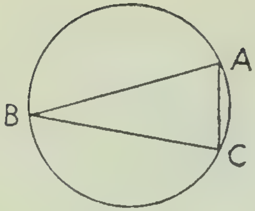


Fig. 1

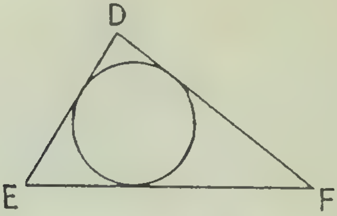
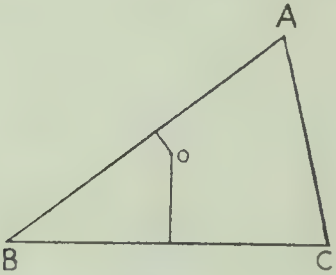


Fig. 2

In Fig. 2, △DEF is said to be circumscribed about the circle, and the circle is said to be inscribed in the triangle. The centre of the inscribed circle is known as the *incentre.*

To circumscribe a circle about a given triangle.



Let ABC be the given triangle.
Required: To circumscribe a circle about △ABC.
Construction: Bisect AB and BC at right angles.
Let the bisectors meet at O.
Then O is the required centre.
With O as centre and OA as radius, describe a circle.
This circle is the circumscribed circle.

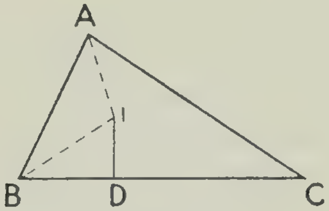
Exercises:

1. Show, by construction, and by reasoning, that the perpendicular bisectors of the three sides of a triangle are concurrent.

2. Draw the circumscribed circle of an obtuse-angled triangle and also of a right-angled triangle. In the case of the right triangle, where is the centre?

Triangles and Circles

To inscribe a circle in a given triangle.



Let ABC be the given triangle.

Required: To inscribe a circle.

Construction: Bisect two angles, A and B, letting the bisectors meet at I.

From I draw ID$\perp$BC.

With centre I and radius ID describe a circle. It should touch the three sides of the triangle. In other words, the sides of ABC should be tangents to the circle.

Exercises:

1. Show by construction that the bisectors of the three angles of a triangle are concurrent.

2. Draw an obtuse-angled triangle and construct the inscribed circle.

3. Construct a right-angled triangle and inscribe a circle.

4. Construct $\triangle$ABC, having AB$=$3 in., BC$=2\frac{1}{2}$ in., CA$=2\frac{3}{4}$ in.

(*a*) Inscribe a circle.

(*b*) Circumscribe a circle.

Are the centres of these circles at the same point? Can you think of a triangle in which the two centres would come together?

Review of Year's Work in Geometry

Topics for Discussion:

(1) Parallel lines, and angles formed by a transversal.
(2) Angles of elevation and depression—practical uses.
(3) Importance, practically, of congruency of triangles.
(4) The Theorem of Pythagoras.
(5) Types of quadrilaterals.
(6) Properties of parallelograms.
(7) Circles passing through two and three points.
(8) Inscribed and circumscribed circles.

Review Exercise:

1. Divide a line 10.2 cm. long in the ratio of 5 to 2.

2. Draw a line parallel to a given line and $2\frac{1}{4}$ in. distant.

3. State three conditions for lines to be parallel, using the angles made by a transversal.

4. Make a statement regarding:

(*a*) The sum of the interior angles of a triangle.
(*b*) The sum of the three exterior angles of a triangle.
(*c*) One exterior angle of a triangle.
(*d*) The sum of the interior angles of a quadrilateral.
(*e*) The sum of the exterior angles of a quadrilateral.

5. Make a statement regarding the diagonals of a (*a*) parallelogram; (*b*) rectangle; (*c*) square; (*d*) rhombus.

6. Show, by a figure, that the perpendiculars drawn from the vertices of a triangle to the opposite sides are concurrent.

7. Construct $\triangle DEF$, given $\angle D = 102°$, $\angle E = 37°$, $DF = 4.6$ cm. Circumscribe a circle about this triangle. Measure its radius.

8. Inscribe a circle in the triangle whose sides are 3 in., 3.4 in., 3.9 in. Measure its radius.

9. The diagonal of a square is 2 in. in length. By construction or otherwise, find the length of a side of the square.

10. State whether the following statements are true or false:

(a) Adjacent angles are always supplementary.

(*b*) A right-angled triangle always contains two acute angles.

(*c*) The diagonals of a rectangle are equal.

(*d*) Since ∠A=∠D, ∠B=∠E, ∠C=∠F, △s ABC and DEF are congruent.

(*e*) Equilateral triangles are also isosceles.

(*f*) Through two fixed points one circle and only one may be drawn.

(*g*) The complement of an acute angle is always obtuse.

(*h*) The sum of the angles of a quadrilateral is 360°.

(*i*) The exterior angle of a triangle is greater than either of the interior opposite angles.

(*j*) The diagonals of every rhombus are equal.

11. Complete the following:

(*a*) Of all lines drawn from a point to a line the ——— is the shortest.

(*b*) The bisectors of the angles of a triangle are ———.

(*c*) The complement of 82° is ———.

(*d*) The supplement of $x°$ is ———.

(*e*) The sum of the three exterior angles of a triangle equals ——— degrees.

(*f*) If each of the base angles of an isosceles triangle is 25°, the third angle is ——— degrees.

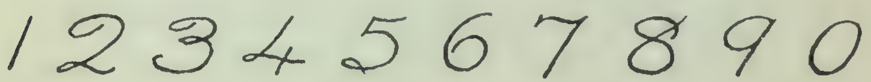
(*g*) In every right triangle there are two ——— angles.

CHAPTER XIV

BUSINESS ARITHMETIC

In the business practice of arithmetic, the first requirement is the ability to make neat and legible figures which any one can read easily. Neatness is the first step to accuracy.

Practise copying the row of figures immediately below until you can make them accurately. To gain skill by practice frequent repetition is necessary.



Drill on rapid addition:

2	7	4	8	3	5	4	9	7	3	7	8	2	9	6	5	8	6
5	4	6	2	9	7	6	2	5	8	6	9	3	4	7	8	2	9
7	2	9	3	7	6	8	9	4	7	1	3	2	8	5	7	6	3
8	1	7	6	5	9	3	2	9	8	4	2	5	7	6	9	5	8
3	8	7	5	6	7	8	7	6	4	6	5	8	6	9	3	8	7
1	7	5	7	6	8	4	6	2	7	7	4	9	3	2	5	1	9
4	6	9	2	4	5	2	5	8	1	3	9	2	7	9	6	8	4
9	9	8	9	7	4	9	7	5	6	1	7	9	3	2	7	5	9
6	3	6	9	8	3	1	2	8	4	5	9	2	5	8	2	9	4
8	5	2	6	4	7	6	7	5	6	6	2	7	3	5	8	7	6
3	7	7	5	9	1	6	3	4	1	9	3	1	8	6	7	2	9
5	6	1	4	7	8	3	8	2	9	7	4	4	1	5	6	7	4
7	9	4	8	5	2	8	4	8	8	6	5	7	6	9	3	8	2
9	3	9	7	8	9	7	9	1	2	8	7	6	5	3	8	1	9
4	5	8	6	5	7	2	5	7	4	3	5	9	4	2	4	3	7
7	8	2	5	6	3	9	6	8	5	2	9	7	8	5	1	8	6
6	9	4	9	6	4	2	4	5	1	4	2	1	9	2	5	6	8
2	3	6	8	3	6	5	7	7	4	5	8	4	6	3	8	2	7
9	5	7	4	9	8	5	8	9	7	8	5	9	4	5	1	6	2
3	4	5	1	3	5	4	6	8	9	3	7	5	6	4	8	1	9

1. Add each column separately, starting at the right. Do not write any results. Practise for concentration and accurate and fast reading, as 11, 18, 26, 32, etc.

2. Drill on horizontal addition in the same way.

3. Use the whole block of figures as an addition question. Write the results as you add. Drill until you can add the 20 numbers in 4 minutes.

Gaining Number Power

1. Add by 8's from 17 to 161.
2. Subtract from 182 by 7's.
3. Learn the 15 times table.
4. Call the right change from a dollar by taking away 18¢, 27¢, 36¢, 29¢, 53¢, 32¢, 46¢, 74¢.

Drill on rapid subtraction:

$$\begin{array}{r} 9\;1\;3\;4\;5\;6\;7\;8\;0\;0\;1 \\ \underline{2\;3\;4\;4\;6\;8\;9\;2\;7\;3\;6} \end{array} \qquad \begin{array}{r} 6\;2\;3\;1\;0\;4\;2\;7\;0\;0\;3 \\ \underline{2\;9\;6\;8\;4\;7\;5\;8\;2\;9\;4} \end{array}$$

Practise until you can do the above in 10 seconds each. As you subtract, move your pencil from right to left without touching the paper. As you practise you will increase the speed of your pencil.

Drill on multiplication:

$$\begin{array}{cccccccccccc} 6 & 8 & 9 & 7 & 4 & 9 & 8 & 6 & 5 & 4 & 2 & 7 \\ \underline{5} & \underline{9} & \underline{7} & \underline{8} & \underline{6} & \underline{4} & \underline{3} & \underline{7} & \underline{9} & \underline{6} & \underline{5} & \underline{8} \end{array}$$

1. Begin at the left side and name the product of each figure in the first line when multiplied by 8, 6, 7, and 9. Next read from the right side to the left. In the same way drill on the second line.

2. Consider the top line one number. Without copying the number, set down the product when it is multiplied by 9.

Drill on division (oral work):

Give quotient and remainder when 624, 387, 569, 471, 286, 397, 504, and 729 are each divided by 4, 7, 8, 5, 6, and 9.

To find 10% of an amount, move the decimal point one place to the left. Read to the nearest cent 10% of $25.37, $56.21, $168.59, $90.03, $64.55, $73.23, $81.76, $5.27, $.86.

To multiply a number by 25, add 2 ciphers and divide by 4. Write the answers when each of the following is multiplied by 25: 64, 78, 83, 151, 89, 263, 57, 613, 493, and 508.

To divide a number by 25, multiply by 4 and point off 2 figures in the quotient. Divide by inspection the following numbers by 25 by this method: 725, 680, 850, 630, 529, 728, 392, and 564.

Group addition:

5 6 4 8 5 6 3 8 4 7
4 2 2 8 2 5 9 6 5 8

2 7 5 4 7 4 6 5 3 2
6 9 6 9 7 7 7 9 7 8

7 3 9 6 5 3 8 3 6 5
8 8 8 7 4 8 8 7 8 9

3 5 7 9 9 4 2 4 7 1
9 7 6 8 9 9 7 2 5 8

6 2 6 5 3 6 5 9 3 5
5 9 9 9 7 6 9 9 8 2

9 6 8 7 8 5 4 6 9 3
7 3 7 2 6 8 8 8 7 9

4 5 4 6 2 9 3 5 2 7
8 9 3 8 9 7 4 7 6 9

Experts in addition learn to save time by grouping figures in series of twos. Begin at the right and add in groups of twos, as 16, 28, 35, 44, etc. Add back from the left and practise continually to gain speed.

Adding two fractions by the short method. $\frac{5}{8}+\frac{3}{5}$. 5×8 gives 40 as the denominator. Then $5\times 5=25$ and $3\times 8=24$. $24+25=49$. The answer is $\frac{49}{40}$ or $1\frac{9}{40}$.

This method is simply a shortened form of the way you were taught to add. Can you explain it?

Using this short method, add by inspection:

$$\tfrac{5}{7}+\tfrac{2}{3},\ \tfrac{4}{9}+\tfrac{3}{5},\ \tfrac{2}{3}+\tfrac{5}{8},\ \tfrac{7}{12}+\tfrac{2}{5},\ \tfrac{8}{9}+\tfrac{1}{4},\ \tfrac{7}{8}+\tfrac{4}{9}.$$

Using the same method, subtract by inspection:

$$\tfrac{4}{9}-\tfrac{1}{2},\ \tfrac{5}{7}-\tfrac{2}{5},\ \tfrac{3}{4}-\tfrac{5}{9},\ \tfrac{7}{8}-\tfrac{2}{3},\ \tfrac{11}{12}-\tfrac{7}{9},\ \tfrac{6}{7}-\tfrac{2}{3}.$$

Subtracting mixed numbers by inspection:

$$\begin{array}{r} 5\frac{1}{8} \\ -2\frac{2}{3} \\ \hline 2\frac{11}{24} \end{array} \qquad \begin{aligned} &5\tfrac{1}{8}-2\tfrac{2}{3}=4+1+\tfrac{1}{8}-2\tfrac{2}{3}=4+1-\tfrac{2}{3}+\tfrac{1}{8}-2 \\ &=4+\tfrac{1}{3}+\tfrac{1}{8}-2=4+\tfrac{11}{24}-2=2\tfrac{11}{24}. \end{aligned}$$

Notice that you can first subtract $\frac{2}{3}$ from 1 by inspection and then add the $\frac{1}{3}$ obtained to $\frac{1}{8}$ by the new method which gives the fractional part of the difference. Then, since $\frac{2}{3}$ is greater than $\frac{1}{8}$, we have had to borrow 1 from 5, so the whole number in the difference will be 2.

Subtract by inspection:

$9\frac{1}{3}-4\frac{3}{4}$, $6\frac{1}{2}-4\frac{7}{10}$, $4\frac{3}{8}-1\frac{5}{6}$, $7\frac{2}{9}-5\frac{5}{8}$, $6\frac{1}{5}-2\frac{7}{8}$.

Add by grouping in twos:

5	6	7	3	8	5	9	7	6	8	2	1	7	6	4
7	8	3	8	5	6	4	2	3	3	4	5	8	9	7
6	2	9	9	7	3	7	6	9	5	3	6	5	2	9
9	5	7	6	4	8	8	3	2	9	1	4	4	3	8
4	9	8	7	8	2	9	6	5	4	8	5	3	9	7
8	4	9	3	9	6	5	8	6	9	2	7	9	5	8
7	5	3	8	8	7	6	2	1	3	9	8	6	4	7
3	1	8	4	2	9	3	7	8	6	2	1	9	3	5
9	7	6	2	7	5	4	5	4	7	9	8	2	1	8
2	8	9	9	5	7	8	6	9	4	2	6	9	9	7
6	5	8	6	9	3	2	8	7	6	5	1	8	4	8
5	9	3	6	2	8	7	5	8	9	9	7	5	9	6

Shortened Forms of Multiplication

To multiply any number by:

$1\frac{1}{4}$	add a cipher and divide by	8
$1\frac{2}{3}$	" " " "	6
$2\frac{1}{2}$	" " " "	4
$3\frac{1}{3}$	" " " "	3
$6\frac{1}{4}$	add two ciphers and divide by	16
$8\frac{1}{3}$	" " " "	12
$12\frac{1}{2}$	" " " "	8
$14\frac{2}{7}$	" " " "	7
$16\frac{2}{3}$	" " " "	6
25	" " " "	4
$33\frac{1}{3}$	" " " "	3
50	" " " "	2

1. Multiply $1\frac{1}{4}$ by 360, 420, 524, 617, 493, and 865.
2. Multiply $1\frac{2}{3}$ by 35, 76, 97, 324, 527, 469, and 783.
3. Multiply 96, 420, 312, 56, 150, and 724 by $6\frac{1}{4}$.
4. Multiply 210, 63, 91, 147, 215, and 186 by $14\frac{2}{7}$.
5. Multiply 102, 144, 186, 216, 544, and 1296 by $16\frac{2}{3}$.
6. Multiply 234, 93, 288, 336, 639, and 529 by $33\frac{1}{3}$.

The difference between a number and 100 is said to be the complement of the number. 37 is the complement of 63 and 16 is the complement of 84.

To multiply two numbers of two digits each under and near 100.

Example: Multiply 93 by 97.

$$\begin{array}{rl} 93 & \text{——complement } 7 \\ \underline{97} & \text{——complement } 3 \\ 9021 & \end{array}$$

Write the product of the complements (7×3) and set to the left the difference between one complement and the other number, as 97—7 or 93—3=90.

Suggestion for algebraic solution: $(100-7)(100-3)$.

Extend the method to multiplying numbers near 1000.

Practice: 95×99, 94×98, 92×96, 97×94, 96×99.

Addition by tens:

42	56	72	94	81	65
38	29	33	43	26	71
27	62	17	62	58	16
53	38	56	19	37	52
65	43	48	37	29	39
34	27	23	42	46	26

Begin on the right.

Add the above figures two columns at a time, as 26, 56, 65, 115, 117, 127, 133, etc.

To multiply by a number of which one part is a certain number of times the other part:

Example:

$$\begin{array}{r} 768 \\ 486 \\ \hline 4608 \\ 36864 \\ \hline 373248 \end{array}$$

Solution: 768×6=4608.
Since 48 is 8×6, 48×768 is
8×6×768 or 8×4608.

Which of the following can be used as multipliers with this method? 237, 567, 819, 456, 847, 369, 708, 568.

Practice on the above method:

$$\begin{array}{r} 987 \\ 427 \\ \hline \end{array} \qquad \begin{array}{r} 768 \\ 648 \\ \hline \end{array} \qquad \begin{array}{r} 1297 \\ 546 \\ \hline \end{array} \qquad \begin{array}{r} 2693 \\ 497 \\ \hline \end{array}$$

Explain the following multiplication and show its variation from the above example:

$$\begin{array}{r} 978 \\ 742 \\ \hline 6846 \\ 41076 \\ \hline 725676 \end{array}$$

When multiplying by number of three figures, notice whether they will conform to this short method.

Improvement Test No. 1

ADDITION

(1)	(2)	(3)	(4)	(5)	(6)
347	503	673	583	299	374
253	196	259	642	417	625
648	683	465	597	633	478
927	294	378	438	208	932
355	628	209	675	735	647
643	729	417	464	489	864
475	892	824	932	217	927
829	580	991	746	453	325
642	777	687	578	198	468

(7)	(8)	(9)	(10)	(11)	(12)
109	253	596	254	138	649
238	639	732	735	476	206
476	727	626	478	609	798
539	486	122	944	297	875
706	898	337	396	845	324
437	394	858	862	303	977
688	234	774	645	765	498
919	996	196	953	748	857
287	758	456	786	293	786
	497	927	479	533	734
	734	674	985	677	699
	678	459	368	749	536

(13)	(14)	(15)	(16)	(17)
23456	64295	27027	49372	62475
34567	25837	31948	16047	83729
45678	73946	98675	89187	14965
56789	28479	47930	35496	73064
67890	94736	26784	57298	82937
12345	26473	35678	34957	98746
34567	97659	70314	79854	20931
89898	67895	97897	65479	78497

Improvement Test No. 1 – (*Continued*)

(18)	(19)	(20)	(21)	(22)
64486	47698	75196	27685	57812
49310	557	27284	72937	96389
73286	92965	976	664	107
437	8674	93829	45129	42986
2298	35087	64108	363	83847
55529	26929	567	59209	918
97384	385	48249	675	64936
267	98769	87366	68341	2786
82885	45846	29154	92896	78137

Improvement Test No. 2

SUBTRACTION

(1)	7.3465	(2)	946.234	(3)	4.7517	(4)	63.2142
	4.6782		698.575		.6432		18.7465
(5)	8.5412	(6)	9.0046	(7)	10.3414	(8)	11.0406
	2.7634		.7629		6.8976		6.4758
(9)	13.2232	(10)	160.125	(11)	126724	(12)	104130
	7.3769		97.268		87948		15543
(13)	111752	(14)	110406	(15)	132232	(16)	130115
	39865		64758		73769		98487
(17)	103414	(18)	183005	(19)	160125	(20)	145050
	68976		95299		97268		87395
(21)	85310	(22)	89203	(23)	74244	(24)	92310
	55766		36348		34779		37597
(25)	56264	(26)	74525	(27)	66052	(28)	96071
	16986		51958		41966		65282
(29)	68310	(30)	73816	(31)	172324	(32)	160115
	37597		33939		98948		74387

Improvement Test No. 3

MULTIPLICATION

(1) $\begin{array}{r}\$82.38\\75\\\hline\end{array}$ (2) $\begin{array}{r}\$56.94\\76\\\hline\end{array}$ (3) $\begin{array}{r}\$82.19\\63\\\hline\end{array}$ (4) $\begin{array}{r}\$79.28\\58\\\hline\end{array}$

(5) $\begin{array}{r}\$87.38\\75\\\hline\end{array}$ (6) $\begin{array}{r}\$98.73\\29\\\hline\end{array}$ (7) $\begin{array}{r}\$47.26\\98\\\hline\end{array}$ (8) $\begin{array}{r}\$91.36\\78\\\hline\end{array}$

(9) $\begin{array}{r}879\\47\\\hline\end{array}$ (10) $\begin{array}{r}479\\29\\\hline\end{array}$ (11) $\begin{array}{r}658\\62\\\hline\end{array}$ (12) $\begin{array}{r}486\\58\\\hline\end{array}$

(13) $\begin{array}{r}465\\74\\\hline\end{array}$ (14) $\begin{array}{r}865\\93\\\hline\end{array}$ (15) $\begin{array}{r}749\\36\\\hline\end{array}$ (16) $\begin{array}{r}759\\85\\\hline\end{array}$

(17) $\begin{array}{r}28379\\74\\\hline\end{array}$ (18) $\begin{array}{r}42635\\47\\\hline\end{array}$ (19) $\begin{array}{r}41729\\29\\\hline\end{array}$ (20) $\begin{array}{r}71529\\58\\\hline\end{array}$

(21) $\begin{array}{r}36258\\62\\\hline\end{array}$ (22) $\begin{array}{r}62538\\39\\\hline\end{array}$ (23) $\begin{array}{r}24683\\85\\\hline\end{array}$ (24) $\begin{array}{r}91734\\63\\\hline\end{array}$

Improvement Test No. 4

DIVISION

(1) $64\overline{)30272}$ (2) $47\overline{)42394}$ (3) $52\overline{)33644}$

(4) $73\overline{)50005}$ (5) $86\overline{)62006}$ (6) $78\overline{)910182}$

(7) $38\overline{)21647}$ (8) $27\overline{)11729}$ (9) $48\overline{)123401}$

(10) $75\overline{)31422}$ (11) $63\overline{)48296}$ (12) $97\overline{)71724}$

(13) $46\overline{)4462}$ (14) $78\overline{)6942}$ (15) $59\overline{)2773}$

(16) $56\overline{)4424}$ (17) $98\overline{)4508}$ (18) $74\overline{)7252}$

Improvement Test No. 5

OPERATIONS WITH FRACTIONS

ADDITION

$$\begin{array}{rrrrrrrrrr}
(1) & 3\frac{7}{12} & (2) & 5\frac{2}{9} & (3) & 6\frac{1}{2} & (4) & 9\frac{5}{6} & (5) & 8\frac{3}{16} \\
 & 8\frac{1}{2} & & 2\frac{1}{3} & & \frac{1}{4} & & 3\frac{3}{8} & & 2\frac{1}{2} \\
 & \frac{2}{3} & & 3\frac{5}{6} & & 7\frac{2}{5} & & 2\frac{2}{3} & & 3\frac{3}{4} \\
 & \underline{2\frac{1}{4}} & & \underline{7\frac{1}{2}} & & \underline{8\frac{3}{4}} & & \underline{7\frac{1}{4}} & & \underline{5\frac{3}{8}}
\end{array}$$

$$\begin{array}{rrrrrrrrrr}
(6) & 7\frac{1}{2} & (7) & 6\frac{5}{12} & (8) & 8\frac{3}{8} & (9) & 7\frac{1}{3} & (10) & 6\frac{3}{16} \\
 & 2\frac{1}{4} & & 3\frac{1}{3} & & 2\frac{1}{24} & & 3\frac{3}{10} & & 2\frac{5}{8} \\
 & 8\frac{5}{8} & & 4\frac{1}{4} & & 9\frac{7}{12} & & 6\frac{2}{15} & & 9\frac{1}{4} \\
 & 3\frac{3}{4} & & 9\frac{5}{6} & & 3\frac{2}{3} & & 5\frac{1}{2} & & 3\frac{5}{32} \\
 & 4\frac{7}{8} & & 2\frac{1}{2} & & 7\frac{1}{4} & & 6\frac{5}{6} & & 8\frac{5}{16} \\
 & \underline{7\frac{3}{4}} & & \underline{6\frac{1}{4}} & & \underline{6\frac{5}{6}} & & \underline{9\frac{2}{5}} & & \underline{7\frac{7}{8}}
\end{array}$$

(11) $4\frac{5}{16}+2\frac{9}{16}+1\frac{1}{16}$

(12) $\frac{21}{24}+\frac{7}{24}+4\frac{1}{24}$

(13) $\frac{14}{17}+1\frac{1}{17}+\frac{3}{17}+5\frac{1}{17}$

(14) $\frac{11}{32}+2\frac{1}{32}+5\frac{5}{32}+\frac{31}{32}$

(15) $4\frac{1}{2}+3\frac{1}{8}+2\frac{1}{4}$

(16) $9\frac{1}{11}+\frac{3}{44}+40\frac{1}{22}$

(17) $172\frac{1}{15}+\frac{31}{30}$

(18) $\frac{30}{32}+4\frac{3}{8}+6\frac{7}{8}$

Improvement Test No. 6

SUBTRACTION

(1) $\begin{array}{r} 28 \\ \underline{6\frac{7}{12}} \end{array}$

(2) $\begin{array}{r} 5\frac{1}{2} \\ \underline{2\frac{3}{4}} \end{array}$

(3) $\begin{array}{r} 8\frac{2}{3} \\ \underline{4\frac{5}{6}} \end{array}$

(4) $\begin{array}{r} 9\frac{1}{5} \\ \underline{4\frac{5}{6}} \end{array}$

(5) $\begin{array}{r} 11\frac{1}{3} \\ \underline{2\frac{4}{7}} \end{array}$

(6) $\begin{array}{r} 17\frac{4}{9} \\ \underline{\frac{1}{2}} \end{array}$

(7) $\begin{array}{r} 12\frac{1}{8} \\ \underline{8\frac{1}{5}} \end{array}$

(8) $\begin{array}{r} 5\frac{5}{9} \\ \underline{4\frac{5}{6}} \end{array}$

(9) $\begin{array}{r} 12\frac{4}{11} \\ \underline{2\frac{1}{2}} \end{array}$

(10) $\begin{array}{r} 14\frac{1}{3} \\ \underline{6\frac{7}{8}} \end{array}$

(11) $\begin{array}{r} 12\frac{1}{5} \\ \underline{9\frac{2}{3}} \end{array}$

(12) $\begin{array}{r} 8\frac{3}{10} \\ \underline{4\frac{5}{6}} \end{array}$

(13) $\begin{array}{r} 9 \\ \underline{2\frac{5}{6}} \end{array}$

(14) $\begin{array}{r} 6\frac{1}{3} \\ \underline{3\frac{2}{3}} \end{array}$

(15) $\begin{array}{r} 12\frac{5}{8} \\ \underline{9\frac{1}{2}} \end{array}$

(16) $\begin{array}{r} 8\frac{2}{5} \\ \underline{1\frac{3}{4}} \end{array}$

(17) $\begin{array}{r} 7\frac{1}{4} \\ \underline{\frac{5}{8}} \end{array}$

(18) $\begin{array}{r} 10\frac{5}{16} \\ \underline{6\frac{3}{4}} \end{array}$

(19) $\begin{array}{r} 9\frac{1}{6} \\ \underline{7\frac{2}{5}} \end{array}$

(20) $\begin{array}{r} 11\frac{1}{3} \\ \underline{8\frac{1}{2}} \end{array}$

(21) $\begin{array}{r} 5\frac{5}{9} \\ \underline{4\frac{5}{6}} \end{array}$

(22) $\begin{array}{r} 13\frac{1}{4} \\ \underline{8\frac{2}{3}} \end{array}$

(23) $\begin{array}{r} 15\frac{1}{2} \\ \underline{9\frac{5}{8}} \end{array}$

(24) $\begin{array}{r} 7\frac{1}{3} \\ \underline{6\frac{7}{10}} \end{array}$

(25) $4\frac{7}{16} - 2\frac{1}{3}$

(26) $16\frac{2}{3} - 8\frac{1}{5} - 2\frac{1}{2}$

(27) $\begin{array}{r} 7\frac{1}{8} \\ \underline{5\frac{1}{3}} \end{array}$

(28) $\begin{array}{r} 7\frac{1}{13} \\ \underline{5\frac{2}{13}} \end{array}$

(29) $\begin{array}{r} 121\frac{11}{20} \\ \underline{61\frac{2}{3}} \end{array}$

(30) $\begin{array}{r} 64\frac{1}{7} \\ \underline{63\frac{1}{7}} \end{array}$

Improvement Test No. 7

MULTIPLICATION

(1) $\frac{1}{2}\times\frac{5}{7}$ (2) $\frac{3}{7}\times\frac{8}{9}$ (3) $\frac{5}{7}\times\frac{14}{15}$ (4) $\frac{4}{9}\times\frac{3}{8}$
(5) $\frac{4}{5}\times\frac{15}{16}$ (6) $18\times\frac{7}{12}$ (7) $\frac{5}{6}\times16$ (8) $\frac{3}{14}\times42$
(9) $28\times\frac{6}{7}$ (10) $63\times\frac{7}{9}$ (11) $2\frac{2}{3}\times9$ (12) $12\times2\frac{5}{8}$
(13) $3\frac{1}{3}\times4\frac{1}{2}$ (14) $3\frac{1}{8}\times3\frac{1}{5}$ (15) $2\frac{5}{6}\times3\frac{3}{5}$
(16) $54\frac{5}{9}\times18$ (17) $72\times9\frac{7}{8}$ (18) $\$2.60\times5\frac{4}{5}$
(19) $2\frac{4}{5}\times6\frac{2}{3}$ (20) $\$1.75\times\frac{7}{9}$ (21) $\frac{4}{15}\times\frac{2}{3}\times\frac{5}{8}$
(22) $1\frac{1}{4}\times2\frac{1}{5}\times7\frac{1}{2}$ (23) $40\frac{2}{3}\times2\frac{2}{5}$ (24) $17\frac{1}{2}\times\frac{3}{8}\times2.5$
(25) $2800\times\frac{3}{7}$ (26) $\frac{3}{5}$ of $8\frac{1}{3}$ (27) $484\times$ (28) $87\frac{1}{2}\times2\frac{2}{5}$
(29) $\frac{4}{7}$ of 91 (30) $16\frac{3}{4}\times\frac{2}{3}$ (31) $\frac{12}{49}\times\frac{7}{8}$ (32) $\frac{42}{65}\times\frac{39}{65}\times\frac{24}{27}$

Improvement Test No. 8

DIVISION

(1) $16\div\frac{2}{3}$ (2) $5\frac{5}{8}\div4$ (3) $6\frac{1}{2}\div2\frac{1}{3}$ (4) $8\div1\frac{1}{4}$
(5) $\frac{5}{7}\div\frac{3}{10}$ (6) $3\frac{1}{4}\div2\frac{5}{6}$ (7) $3\frac{7}{9}\div5\frac{2}{3}$ (8) $9\frac{7}{8}\div8\frac{7}{9}$
(9) $15\frac{2}{7}\div5$ (10) $9\frac{1}{4}\div4\frac{5}{8}$ (11) $500\frac{1}{4}\div9$ (12) $55\frac{1}{8}\div7$
(13) $18\div\frac{3}{4}$ (14) $16\frac{1}{2}\div5$ (15) $24\frac{1}{4}\div6$ (16) $\frac{5}{8}\div\frac{2}{3}$
(17) $400\div1\frac{1}{5}$ (18) $21\div\frac{5}{8}$ (19) $\frac{15}{16}\div\frac{3}{6}$ (20) $16\frac{2}{3}\div2\frac{1}{4}$
(21) $8\frac{1}{2}\div2\frac{5}{6}$ (22) $9\frac{5}{8}\div7$ (23) $3\div1\frac{1}{9}$ (24) $60\div\frac{15}{16}$
(25) $\frac{42}{85}\div21$ (26) $\frac{25}{48}\div\frac{5}{16}$ (27) $45\div3\frac{4}{7}$ (28) $14\frac{2}{3}\div4$
(29) $8\frac{3}{4}\div\frac{7}{12}$ (30) $\frac{18}{35}\div1\frac{4}{5}$ (31) $9\frac{3}{4}\div7\frac{1}{3}$ (32) $64\div\frac{8}{51}$

Improvement Test No. 9

Operations with Decimals

MULTIPLICATION

(1) 27.8 9.63	(2) 4.87 26.9	(3) .865 8.09	(4) 65.9 34.7	(5) 9.38 .698
(6) .657 .298	(7) 4.68 72.6	(8) 96.3 6.89	(9) .529 87.3	(10) 82.9 .347
(11) 638.4 .29	(12) 78.56 3.8	(13) 9.179 .45	(14) .7853 6.7	(15) 823.5 9.3
(16) 5.978 84	(17) 75.46 .96	(18) 3.487 89	(19) 47.89 5.6	(20) 6.947 .47
(21) 4.068 67	(22) .03072 72	(23) 350.6 7.5	(24) 5230 .067	(25) .206 .28
(26) 67.05 3.9				

Improvement Test No. 10

DIVISION

(1) $4.2\overline{)247.41}$ (2) $.78\overline{)203.58}$ (3) $4.6\overline{)29.186}$

(4) $.057\overline{)51.3285}$ (5) $.097\overline{)518.294}$ (6) $8.9\overline{).66305}$

(7) $.96\overline{)192.96}$ (8) $28.6\overline{)8751.6}$ (9) $4.93\overline{)15.2337}$

(10) $57.3\overline{)345.519}$ (11) $19.2\overline{)1539.84}$ (12) $.567\overline{)4.537701}$

Change to decimals and per cent:

(13) $\frac{5}{8}$ (14) $\frac{3}{5}$ (15) $\frac{7}{50}$ (16) $\frac{15}{16}$

(17) $\frac{27}{40}$ (18) $\frac{11}{32}$ (19) $\frac{2}{7}$ (20) $\frac{19}{35}$

(21) $\frac{41}{48}$ (22) $\frac{17}{75}$ (23) $\frac{19}{36}$ (24) $\frac{29}{60}$

Cumulative Tests in Algebra

A

1. In the formula $A=bh$, find b when $A=22.4$ cm., and $h=6.4$ cm.

2. If $h=3$ in., $a=5$ in., $b=6$ in., find the value of $\frac{1}{2}h(a+b)$.

3. Reduce to one term $3a^2b-5a^2b+\frac{1}{2}a^2b$.

4. Subtract $3x-2x^2+5$ from x^2-x-1.

5. If $e=1.2$ in., find S in the formula $S=6e^2$.

6. Find to 2 decimal places the square root of 1.829.

7. What is the volume of a cone, the diameter of the base being 2 in. and the height $4\frac{1}{2}$ in.?

8. Add $3a-2$, $5a+6$, $-2a+1$, $-7-8a$.

9. By substituting in the formula $a^2=b^2+c^2$, determine which of the following sets of numbers could represent the sides of a right triangle: (*a*) 2, 3, 4; (*b*) 4, 5, 6; (*c*) 5, 6, 7.

10. Select the value of x which satisfies the equation:

$$5(x-2)=2x-1;\ x=0,\ 1,\ 2,\ 3,\ 4.$$

B

1. Find the sum of the expressions:

$$2c-5cd+d^2,\ 6cd-d^2,\ -4c^2+cd.$$

2. What must be added to $a-b$ to give $3+2b$?

3. Remove brackets and collect terms:

$$(5x-2y)-(3x+5y)+(-2x+y).$$

4. Find the product of $3a^2-2a+1$ and $-2a$.

5. If $d=-6$, find the value of d^2-d.

6. Take $8-6y+2y^2$ from $7+6y-y^2$.

7. Remove brackets and simplify:

$$5a-3(a-4)+2(-a+1)-(2a-5).$$

8. Simplify: $-4ab(1-a+2b)$.

9. If $t=3$ find the value of $4t^3-6t^2+5$.

10. If $y=2x-5$ and if $x=-3$, find y.

C

1. Write the formula used in finding:
   (*a*) The base of a cone;
   (*b*) The total surface of a cube.

2. Evaluate the formula $V=\frac{1}{3}\pi r^2h$, when $\pi=\frac{22}{7}$, $r=2.5$ in., $h=6.2$ in.

3. Find, to 2 decimal places, the length of the diagonal of a square, the side of which is 3 in.

4. Solve the equations:
   (*a*) $2x-3=5-x$ (*c*) $\frac{1}{4}x-8=0$
   (*b*) $0=\frac{1}{2}y+6$ (*d*) $2(y-1)=3$

5. Find the area of a circular plot of ground, whose diameter is 8 yds.

6. From the Table of Squares, find the value of:
   (*a*) $95^2+16^2+21^2$ (*b*) $3(68)^2$

7. Find the product: $12c^2(-c+2c^2)$.

8. Divide $6y^3-10^2+2y$ by $-2y$.

9. Find which of the roots, $\frac{5}{13}$, $\frac{13}{5}$, 5 satisfies the equation
   $2(a-5)=3(1-a)$.

10. Find the sum of $1.5a-2$; $.75a+3$; $4-.5a$.

D

1. If $b=-2$, find the value of $2b^3$.
2. From 0 subtract $3x-2y+4$.
3. Multiply $5x^2-3xy+2y^2$ by -1.
5. If $3x-2y=8$ and if $x=3$ find y.
6. From the formula $A=\frac{1}{2}h(a+b)$ derive a formula for finding b.
7. Is $+4$ or -4 a root of the following equation?
   $\frac{1}{2}(2a-3)-2(1-a)=8\frac{1}{2}$.
8. If an aviator travels 125 miles per hour, how far would he go in y hours at the same rate?
9. At 3 lb. for 25¢, how many pounds could I buy with x cents?
10. If $a=3$, $b=2$, $c=0$, find the value of $2a^2-3b^2+5c^2$.

E

1. Solve the equations:

(*a*) $\frac{1}{2}c-3=\frac{1}{3}c+2$ (*c*) $5-4(y-2)=0$

(*b*) $\frac{a}{4}-1.3=2.8$ $1-\frac{6x-2}{3}=0$

2. Find which two of the three values 7, —7, 9 satisfy the equation: $x(x-2)=63$.

3. One angle is 4 times another, and the two make up a right angle. Write an equation in x which may be used to find the sizes of the two angles.

4. Jack is 4 years older than Tom, and the sum of their ages is 30 years. Write an equation, solve it, and find the two ages.

5. Add $6(2a-3)$ to $5(4-2)a$, and find the value of the sum when $a=3$.

6. Evaluate $p=2(l+w)$ when $p=23$ cm., $l=7.5$ cm.

7. What must be taken from $2a+3b$ to leave $a-b$?

8. Find the value of $(x+5)^2+(x-2)^2$.

9. Is $(-2a^3)^2$ equal to $-2a^6$, $-4a^5$, $+4a^6$, $+4a^5$, or $2a^6$?

10. Multiply $a-5(a+2)$ by $-3a$.

F

1. From $A=lwh$, derive formulas for l, w, and h.

2. Using $A=p+prt$, find a formula for t.

3. How long does an automobile take to travel x miles at a steady rate of 40 miles per hour?

4. Add the following pairs of expressions:

$13a-12b$	$1-2y+3y^2$	$4-x$
$-2a+7b$	$1\quad-6y^2$	$-5+2x-x^2$

5. What must I add to $5c-3a$ to give $2a-6c$?

6. Give the product of $(2x-3)(x+5)$ and $-3x$.

7. If $A=2x-y$ and $B=-x-3y$, what is $2AB$?

8. If $x=-4$ and $y=5$, find the value of $(x^2-y^2)^2$.

9. One factor of $14a^3-7a^2-28a$ is $+7a$. Find the other.

10. Solve $\frac{y-1}{3}+\frac{4}{3}=\frac{1}{4}(y+4)$.

TABLES OF MEASURE

MEASURE OF TIME

60 seconds (sec.)=1 minute (min.)
60 minutes=1 hour (hr.)
24 hours=1 day (da.)
7 days=1 week (wk.)
12 months=1 year (yr.)
365 days=1 year
366 days=1 leap year
10 years=1 decade
100 years=1 century

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February alone.
To which we twenty-eight assign
Till leap year brings it twenty-nine.

ANGLES AND ARCS

60 seconds (″)=1 minute(′)
60 minutes=1 degree (°)
360 degrees=1 circumference
An angle of 90°=1 right angle
An arc of 90°=1 quadrant
$\pi=3.1416$

COUNTING TABLE

2=1 pair
12=1 dozen
20=1 score
144=1 gross
12 gross=1 great gross

COUNTING PAPER

24 sheets=1 quire
20 quires=1 ream

CANADIAN MONEY

10 mills (m.)=1 cent (¢)
10 cents=1 dime
100 cents=1 dollar ($)

ENGLISH MONEY

4 farthings (far.)=1 penny (d.)
12 pence=1 shilling (s.)
20 shillings=1 pound (£)

METRIC MEASURES OF LENGTH

10 millimetres (mm.)=1 centimetre (cm.)
10 centimetres=1 decimetre (dm.)
10 decimetres=1 metre (m.)
10 metres=1 dekametre (Dm.)
10 dekametres=1 hektometre (Hm.)
10 hektometres=1 kilometre (Km.)

MEASURES OF AREA

100 square millimetres=1 square centimetre
100 square centimetres=1 square decimetre
100 square decimetres=1 square metre
100 square metres=1 square dekametre
100 square dekametres=1 square hektometre
100 square hektometres=1 square kilometre

TABLE OF VOLUMES

1000 cubic millimetres=1 cubic centimetre
1000 cubic centimetres=1 cubic decimetre
1000 cubic decimetres=1 cubic metre

METRIC MEASURES OF WEIGHT

10 milligrams=1 centigram
10 centigrams=1 decigram
10 decigrams=1 gram
10 grams=1 dekagram
10 dekagrams=1 hektogram
10 hektograms=1 kilogram

EQUIVALENTS

1 in.=2.54 cm.
1 m.=39.37 in.; 1.1 yd.
1 Km.=.62 mi.; $\frac{5}{8}$ mi.
1 sq. m.=1.2 sq. yd.
1 Ha.=2.5 acres

1 cu. m.=1.3 cu. yd.
1 litre=.88 qt.
1 Hl.=2.75 bu.
1 Kg.=2.2 lb.
1 metric ton=1.1 tons

LIST OF FORMULAS

$$r = \frac{d}{2}$$

$$p = 2l + 2w$$

$$p = 4s$$

$$A = lw$$

$$A = s^2$$

$$x + y + z = 180$$

$$p = a + b + c$$

$$A = ba$$

$$A = \tfrac{1}{2}ba$$

$$C = 2\pi r$$

$$C = \pi d$$

$$A = \pi r^2$$

$$V = lwh$$

$$V = s^3$$

$$V = s^2h$$

$$V = \pi r^2 h$$

$$d = rt$$

$$i = prt$$

$$A = p\,(1 + rt)$$

$$V = \tfrac{1}{3}\pi r^2 h$$

$$V = \tfrac{1}{3}Bh$$

$$A = Ch$$

$$A = 2\pi rh$$

$$A = 4\pi r^2$$

$$V = \tfrac{4}{3}\pi r^3$$

$$a^2 = b^2 + c^2$$

$$F = \frac{9C}{5} + 32$$

$$A = \tfrac{1}{2}h\,(a + b)$$

$$S = 6e^2$$

TABLE OF SQUARES AND SQUARE ROOTS

Number	*Square*	*Square Root*	*Number*	*Square*	*Square Root*
1	1	1.000	51	2601	7.141
2	4	1.414	52	2704	7,211
3	9	1.732	53	2809	7,280
4	16	2.000	54	2916	7.348
5	25	2.236	55	3025	7.416
6	36	2.449	56	3136	7.483
7	49	2.646	57	3249	7.550
8	64	2.828	58	3364	7.616
9	81	3.000	59	3481	7.681
10	100	3.162	60	3600	7.746
11	121	3.317	61	3721	7.810
12	144	3.464	62	3844	7.874
13	169	3.606	63	3969	7.937
14	196	3.742	64	4096	8.000
15	225	3.873	65	4325	8.062
16	256	4.000	66	4356	8.124
17	289	4.123	67	4489	8.185
18	324	4.243	68	4624	8.246
19	361	4.359	69	4761	8.307
20	400	4.472	70	4900	8.367
21	441	4.583	71	5041	8.426
22	484	4.690	72	5184	8.485
23	529	4.796	73	5329	8.544
24	576	4.899	74	5476	8.602
25	625	5.000	75	5625	8.660
26	676	5.099	76	5776	8.718
27	729	5.196	77	5929	8.775
28	784	5.292	78	6084	8.832
29	841	5.385	79	6241	8.888
30	900	5.477	80	6400	8.944
31	961	5.568	81	6561	9.000
32	1024	5.657	82	6724	9.055
33	1089	5.745	83	6889	9.110
34	1156	5.831	84	7056	9.165
35	1225	5.916	85	7225	9.220
36	1296	6.000	86	7396	9.274
37	1369	6.083	87	7569	9.327
38	1444	6.164	88	7744	9.381
39	1521	6.245	89	7921	9.434
40	1600	6.325	90	8100	9.487
41	1681	6.403	91	8281	9.539
42	1764	6.481	92	8464	9.592
43	1849	6.557	93	8649	9.644
44	1936	6.633	94	8836	9.695
45	2025	6.708	95	9025	9.747
46	2116	6.782	96	9216	9.798
47	2209	6.856	97	9409	9.849
48	2304	6.928	98	9604	9.899
49	2401	7.000	99	9801	9.950
50	2500	7.071	100	10.000	10.000

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